CS 381 Final Exam 9-11:30 Wed Dec 13
Fall 2006 Closed Book Hollister B14

All problems should be straight forward. Partial credit will depend on clarity and conciseness of your answer. Please do not put down correct but irrelevant information.

1. Write a regular expression describing all strings of a's and b's not containing the substring aab.

2. Let $L = \{ a^i b^j c^k \mid k < i \text{ and } k < j \}$. Either construct a context-free grammar for L or prove that L is not a context-free language using the pumping lemma.

3. Given a context-free language L and a regular set R, prove that the set

$$\{ a_1 b_1 b_2 a_2 b_3 b_4 a_3 b_5 b_6 \cdots a_n b_{2n-1} b_{2n} \mid a_1 a_2 \cdots a_n \in \text{ L and } b_1 b_2 \cdots b_{2n} \in R \}$$

is a context-free language.

4. Let L be the set of all valid computations of a Turing machine M. Using the operations of h, h^{-1}, and intersection with a regular set, explain how to convert L to L' where L' consists of only the first two ID's of the valid computation.

5. Is the set L={(M,x)|Turing machine M does not halt on input x} a regular set? a cfl? a recursive set? an r.e. set? not r.e.? Give short proofs of your answers. You may use any theorem proved in class.

6. Consider a reduction of problem A to problem B. What is the most precise claim you can make about problem B for each of the following situations?

a) A is NP-complete and the reduction is in polynomial time.

b) A is in polynomial time and the reduction is also in polynomial time.

c) A is NP-complete and the reduction is in Pspace.

d) A is in nondeterministic polynomial time and the reduction is in polynomial time.

e) A requires exponential time and the reduction is in polynomial time.

f) A is Pspace complete and the reduction is in Pspace.

7. What is a quantified Boolean formula? (Ten words or less)
Given an algorithm for deciding if a QBF is true. Is the QBF problem in P?, NP?, Pspace?