CS381 Final Exam Friday Dec 12, 2003
Fall 2003 Olin 255 9:00-11:30

This is a $2\frac{1}{2}$ hour in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. Thank you.

1. Is it decidable for regular sets R_1 and R_2 whether $R_1 \subseteq R_2$? Justify your answer.

Answer: $R_1 \subseteq R_2$ iff $R_1 \cap \overline{R}_2 = \Phi$. But $R_1 \cap \overline{R}_2$ is a regular set and emptiness for regular sets is decidable.

2. Write a context-free grammar for the compliment of $\{ww \mid w \in (a+b)^*\}$.

Answer:

$$S \to AB \mid BA \mid O$$
$$A \to aAa \mid aAb \mid bAa \mid bAb \mid a$$
$$B \to aBa \mid aBb \mid bBa \mid bBb \mid b$$
$$O \to aaO \mid abO \mid baO \mid bbO \mid a \mid b$$

3. Let $L \subseteq (a+b)^*$ be a context-free language. In each string interchange the order of a and b in each occurrence of ab. Is the resulting language context free? Give a proof of your answer.

Examples

$$aabb \to abab$$
$$ababab \to bababa$$
$$bababa \to bbabaa$$

Answer:
Define h_1, h_2, and R as follows

$$h_1(a) = a \qquad h_2(a) = a$$
$$h_1(b) = b \qquad h_2(b) = b$$
$$h_1(c) = ab \qquad h_2(c) = ba$$

$$R = (a+b+c)^* - (a+b+c)^* ab (a+b+c)^*$$

Then the desired set is $h_2\left(h_1^{-1}(L) \cap R\right)$. The class of cfl's is closed under homomorphisms. inverse homomorphisms, and intersection with a regular set. Thus $h_2\left(h_1^{-1}(L) \cap R\right)$ is a cfl.

4. If one can list the elements of a set in order, then must the set be recursive? Prove your answer.

Answer: Yes. If the set is finite then it clearly is recursive. Assume that the set is infinite. To determine if x is in the set enumerate elements of the set in order until either x appears or a string beyond x in the ordering appears. Since the set is infinite one or the other most occur thus determining if x is or is not in the set.

5. Is the class of Turing machines that accept the empty set recursive, r.e. or not r.e.? Justify your answer.
Answer: Not r.e. The class of Turing machines that accept non empty sets can be enumerated. The set is not recursive since non empty is a non trivial property of the r.e. sets. Thus its complement, the class of Turing machines that accept non empty sets, cannot be r.e.