

This is a 50-minute in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. **Grading will take into account the clarity of your answer.**

1. Let $L \subseteq (a + b + c)^*$ consist of all strings with an equal number of a's, b's, and c's. Prove that L is not a context-free language.

2. Prove that the class of context-free languages is closed under inverse homomorphisms

3. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, b, F)$ be a Turing machine. Write a context-free grammar for the set

$$\{ID_i \# ID_j^R \mid \text{where } ID_j \text{ follows from } ID_i \text{ by one move}\}.$$

Here # is a special symbol not used in any ID. You can ignore the special cases where the symbol scanned by the tape head is the first symbol of the input or the blank at either end of the input.

4. Prove that the halting problem is undecidable. That is, there is no algorithm that halts on all inputs (M, x) and answers whether Turing machine M halts when started on x.

5. Prove that the set $\{M \mid L(M) = \Phi\}$ is not recursively enumerable.