

This is a 2 and ½ hour in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. Thank you.

1. Let  $R \subseteq (a + b)^*$  be a regular set. Consider the set consisting of all strings that can be obtained from strings in  $R$  by deleting two  $b$ 's. Is this set regular? Give rigorous proof of your answer.
2. Let  $R \subseteq (a + b + c)^*$  be a regular set. Rearrange the symbols in each string of  $R$  so that all  $a$ 's appear first, then the  $b$ 's and then the  $c$ 's.
  - a) Is the resulting set regular?
  - b) Is it context free?
3. Prove or disprove that  $\{a^i b^j c^k d^l \mid \text{either } i = k \text{ or } j = l\}$  is a context-free language.
4. Prove or disprove each of the following:
  - a) The class of context-free languages is closed under intersection.
  - b) The class of context-free languages is closed under homomorphism.
  - c) The class of context-free languages is closed under inverse homomorphism.
5. Prove that the halting problem for Turing machines,  $\{(M, x) \mid M \text{ halts on input } x\}$ , is undecidable.
6. For each of the following conditions, give an example of a set or prove that no such set exists.
  - a) Both the set and its complement are recursive.
  - b) The set is recursive but its complement is not.
  - c) The set is recursively enumerable and its complement is recursive.
  - d) The set is recursively enumerable and its complement is also recursively enumerable.
  - e) The set is recursively enumerable but its complement is not recursively enumerable.
  - f) Neither the set nor its complement are recursively enumerable.