CS 381 Homework 6 Solutions

1.

$L = L0 \cap L1$
$= \{ w\#w(0 + 1)\# \mid w \in (0 + 1)^* \}$

Describe the set $L^* \cap 0\#L^*(0+1)^*$:

$L^* \cap 0\#L^* (0+1)^*$ is the set
$\{ 0 \# w_0 \# w_1 \# w_2 \# \dots \# w_{2n} \#$, where $w_0 = 0(0+1)$, $w_n = w_{n-1}(0+1)$, and $n>=0 \}$
This is another alternating blocks problem. Each block forces the next block to equal w(0+1), and the string must start with 0# due to the right side of the intersection.

Modify the set so that it has an odd number of blocks:

$L^* (0 + 1)^*\# \cap 0\#L^*$
Since each instance of L has 2 blocks, this set is guarenteed to have an odd number of blocks.

Prove that the language of balanced parenthesis is not regular

Proof by the Pumping Lemma: Suppose we select w from L such that $w = (^n)^n$ where n is the n from the pumping lemma. Now we know that xy can only contain opening parenthesis since $|xy| \leq n$. When we pump on this string its easy to see that we will only increase the number of opening parenthesis, and since the pumped string contains a different number of opening and closing parenthesis, they cannot possibly be balanced.

Show that $\{0^n1^m0^n \ |m \text{ and } n \text{ arbitrary integers}\}$ *is not regular*

Proof by the Pumping Lemma: Select w = $0^n1^n0^n$, where n is the n from the pumping lemma. This means that xy may only contain 0's, and thus when we pump xy^kz, then we will have $0^{n+m}1^n0^n$, where m is dependant on how the initial zeros are divided between x and y. Since this string is not in the original set, the language is not regular.

Show that $\{0^n1^{2n}|n \geq 1\}$ *is not regular.*

Proof by Pumping Lemma: Suppose we select w from L such that $w = 0^n1^{2n}$ where n is the n from the pumping lemma. Now any choice of xy must include be all zeros, because $|xy| \leq n$, by the definition of the pumping lemma. Thus, when we pump on this string, we will only increase the number of zeros, and we will be left with the string $0^{n+m}1^{2n}$ for some m dependant on how we break up the initial zeros into x and y. This is not in the original set, so the language is not regular.

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Problem 3 [Exercise 4.1.4 in book]

4.1.4:

b) If the adversary picks $n = 3$, then we cannot pick a w of length at least n.

d) L = {01*0*1}
Assume L is regular.
By pumping lemma, $\exists$ n of pumping lemma
Select w, $|w| \geq n$

We will choose $w = 01^n0^n1$
By lemma: w can be written as xyz and adversary chooses the following breakdown:
$x = 0$
$y \in 1^+$
z will take on the remainder of the string

$xy^iz \in L$ //whatever i we pick, xy^iz will be in the language and we lose
=> L is regular.

CS 381 – Homework 6 – Question 4 – Solution

4.3.2

The first step to finding out whether there are 100 strings in a regular expression would be to convert the expression in to a DFA. Once we have the DFA, there is one main thing to notice: if our DFA has n nodes then if we can generate a string of length between n and $2n$ then the DFA has infinite strings. Otherwise, we can use brute force to generate all strings of length less than n and count them. If there are more than 100 of them, then we return true, otherwise we return false.

Thus, the main observation of this problem is that we have to bound the length of our strings by n, otherwise it is possible to search forever.

Problem 4.4.2

(a) The table of states that are distinguishable from one another:

A								
x	**B**							
x	x	**C***						
	x	x	**D**					
x		x	x	**E**				
x	x		x	x	**F***			
	x	x		x	x	**G**		
x		x	x		x	x	**H**	
x	x		x	x		x	x	**I***

This gives us the following sets of equivalent states:
(A,D,G)
(B,E,H)
(C,F,I)

(b) We build the following DFA by combining the original nine states into three states that represent the equivalent states from part (a):