Intro to Theory of Computing	**481 Prelim 2**
CS481 Fall 2005	**November 4**

This is an in-class examination.

1. Of the following languages state which are regular, context-free but not regular, not context free. Give 1/2 line informal arguments. No need to give the actual application of pumping lemma or detailed grammar/PDA. ($5 \times 5 = 25$).

   (a) $\{a^m b^n \mid 5m - 3n = 24, m, n \geq 0\}$.

   Answer . Context free, can be show to be equivalent to $\{a^n b^n\}$ by homomorphism, inverse homomorphism and intersection with regular.

   (b) $\{a^m b^n \mid 5m + 3n = 24, m, n \geq 0\}$.

   Answer . Regular since finite.

   (c) $\{a^i b^j c^k d^l \mid j = k \wedge i = l\}$.

   Answer . Context free as we can design PDA, push down $a^i b^j$. j and k can be verified to be equal, and then i and l.

   (d) $L(G)$ where $G = \{S \rightarrow aS \mid Sb \mid bSa \mid \varepsilon\}$.

   Answer . Regular as the language accepted is $\{a, b\}^*$.

   (e) $\{a^i b^j c^k \mid i \neq j \wedge j \neq k \wedge k \neq i\}$.

   Answer . Not context free. Intuitively, cannot check all three. Formally can take $a^n b^{n+n!} c^{n+2(n!)}$ where n is the pumping lemma constant, and then pump.

2. Show that if L is regular then the following language is regular by constructing a NFA for the language. (25)

$$\text{cycle}(L) = \{w \mid \exists x, y \in \Sigma^*, w = xy \text{ such that } yx \in L\}.$$

   Answer. Start with a DFA $M = (Q, \Sigma, s, \delta, F)$ for L. Design an NFA for the new language. The new NFA has two copies of original DFA. Initially, the first copy is started with a guess of the state q. The NFA then runs this copy of the DFA on the first portion x of the input string. If it reaches a final state, then the second copy of the DFA is started from start state with the remaining portion y of the string. If it ends in the state q in which the first copy had initially started out, then we accept.

3. If $L = \{ww^R \mid w \in \{a,b\}^*\}$, then is the language $\overline{L} = \Sigma^* - L$ context free ? If yes, give a grammar for it. Else prove that it is not context-free. (25)

   Answer. It is context free. The following grammar generates it. Either the string is of odd length, or it is of even length and there is some index i such that the two positions i^{th} from start, and i^{th} from end have non-equal symbols.

$$
\begin{aligned}
S &\to O|E \\
O &\to aOO|bOO|a|b \\
E &\to aTb|aEa|bEb|bTa \\
T &\to E|\varepsilon
\end{aligned}
$$

4. Show that the following language is not context free using the pumping lemma. (25)

$$\{w\#x \mid w \text{ is a substring of } x \text{ where } w, x \in \{a,b\}^*\}.$$

Answer. Let L be the above language. Define $L' = L \cap \{ba^+b^+a\#ba^+b^+a\} = \{ba^ib^ja\#ba^ib^ja \mid i,j \geq 0\}$. We apply pumping lemma on this language to show that it is not context free. Let use take $z = ba^nb^na\#ba^nb^na$ and let $uvwxy$ be the decomposition of the string. If v and x lies in entirely the first half or the second half of the string, or contains the symbol $\#$, then we are done. The only other interesting case is when w contains $\#$ and v lies in the first half and w the second half. But then, v cannot reach the first block of a's in first half, and w must be limited to the first block of a's in second half. It is easy to see that pumping destroys the structure.