CS381 Second Mid Term Friday, Nov. 4, 2005
Fall 2005 Olin 155 9:05-9:55 am

1. Is the language $L=\{a^i b^j c^i d^j \mid i \geq 1, j \geq 1\}$ a context-free language? If yes give a Chomsky normal form context-free grammar for it. If not use the pumping lemma to prove that it is not a context-free language.

No it is not a context-free language. Assume it is. Let n be the integer of the pumping lemma. Select $z=a^n b^n c^n d^n$. Write z=uvwxy with $|vwx| \leq n$. Then vx cannot contain both a's and c's nor can it contain both b's and d's. Thus uv^2wx^2y has an unequal number of a's and c's or an unequal number of b's and d's and is not in L, a contradiction. Therefore L is not a context-free language.

2. What is the specific class of languages defined by grammars in which each production is of one of the following forms:

- A variable goes to epsilon $A \rightarrow \varepsilon$
- A variable goes to a terminal $A \rightarrow a$
- A variable goes to a string consisting of two terminals $A \rightarrow ab$
- A variable goes to a string consisting of a terminal followed by a variable $A \rightarrow aB$
- A variable goes to a string consisting of two terminals followed by a variable. $A \rightarrow abB$

Regular sets. Grammars with productions of the form $A \rightarrow aB$, $A \rightarrow a$, and $A \rightarrow \varepsilon$ can generate all and only regular sets. Productions of the form $A \rightarrow ab$ can be replaced by two productions of the forms $A \rightarrow aB$ and $B \rightarrow b$. Productions of the form $A \rightarrow abB$ can be replaced by introducing a new variable C and two productions $A \rightarrow aC$ and $C \rightarrow bB$.

3. Let $L \subseteq \{a,b\}^*$ be a context-free language and let Final(L)$=\{y \mid \exists x \;\; xy \in L\}$. Prove that Final(L) is a context-free language or that Final(L) is not a context-free language using closure properties that preserve context-free languages.

Final(L) is a context-free language. Let h_1 be the homomorphism

$$h_1(a)=a \quad h_1(b)=b \quad h_1(\hat{a})=a \quad h_1\left(\hat{b}\right)=b\,.$$

Let R be the regular set $\left(\hat{a}+\hat{b}\right)^*(a+b)^*$ and let h_2 be the homomorphism

$$h_2(a)=a \quad h_2(b)=b \quad h_2(\hat{a})=\varepsilon \quad h_1\left(\hat{b}\right)=\varepsilon\,.$$

Then $Final(L)=h_2\left(h_1^{-1}(L)\cap R\right)$ and thus is context free since the class of context-free languages is closed under h, h^{-1} and intersection with regular sets.

4. Let $M=(Q,\Sigma,\Gamma,\delta,q_0,Z_0,F)$ be a multi-state pushdown automaton that accepts by empty stack. Let $M'=(Q',\Sigma',\Gamma',\delta',q_0',Z_0',F')$ be the equivalent one state pushdown automaton. Specify Γ' and δ' precisely.

$\Gamma'=Q\times\Gamma\times Q$.

Let $Q'=\{1\}$.

a) $\delta'(1,a,[q,A,p])$ contains $(1,[r,B,s][s,C,p])$ for each $s\in Q$ if $\delta(q,a,A)$ contains (r,BC).

b) $\delta'(1,a,[q,A,p])$ contains $(1,\varepsilon)$ if $\delta(q,a,A)$ contains (p,ε).