Intro to Theory of Computing	**481 Homework 7**
CS481 Fall 2005	**October 14**

1. 5.1.7.

2. Write a CFG for following languages. Give clear explanations/intuitions about your grammar.

   (a) $w \in (a+b)^*$, $\#a(w) = \#b(w)$.

   (b) Let $b(n)$ denote the binary representation of $n \geq 1$, leading zeros omitted. For example, $b(5) = 101$ and $b(12) = 1100$. Let $rev(w)$ be the reverse of the string w. For example, $rev(b(12)) = 0011$. \$ is a symbol not in $\{0, 1\}$. Write a CFG for the following language

   $$\{rev(b(n))\$b(n+1) \mid n \geq 1\}$$

3. A symmetric linear grammar is one for which the productions are of the following form . A, B, C and X are non-terminals, a and b are terminals (that may or may not be distinct).

   $$A \to \varepsilon$$
   $$B \to aXb$$
   $$C \to a$$

   A language L is a *symmetric linear language* if $L = L(G)$ for some symmetric linear grammar G.

   (a) Give a symmetric linear language that is not regular.

   (b) Show that all regular languages are symmetric linear languages. *Hint:* This might be trickier than you think. Remember that non-terminals can represent set (or ...) of states.

   (c) Show that all symmetric linear languages over a single letter alphabet are regular.

4. Write down a CFG for the complement of the following language $\{(1^i0^i)^j \mid i, j \geq 0\}$. Here w^n is a string of n consecutive w's.