CS 481 Homework 2
due Friday September 9

1. Do each problem on a separate set of sheets and please remember to write your name, net-id and problem number on each sheet.

2. Suppose the set of input symbols Σ contains k elements. Define the language L as follows:

$$L = \{w \in \Sigma^* | \exists a, b \in \Sigma, a \neq b, w \text{ does not contain } a \text{ or } b\}$$

   It is easy to define a finite automaton for L having number of states that is exponential in k. Define a NFA for L having only $O(k^2)$ states.

3. Let x and y be strings and let L be any language. We say that x and y are distinguishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L. Otherwise, if for every string z, $xz \in L$ if and only if $yz \in L$, we say that x and y are indistinguishable by L. If x and y are indistinguishable by L we write $x \equiv_L y$.

   (a) Prove that $\equiv_L$ is an equivalence relation i.e. is reflexive, symmetric and transitive.

   (b) Suppose L is a regular language and let X be a set of k strings $\{x_1, \dots x_k\}$. Suppose we have that for each pair $i, j \leq k$, such that $i \neq j$, $x_i \not\equiv_L x_j$. Then prove that the DFA that accepts L must have at least k states.

4. Define $L = \{0^n 1 0^n | n \geq 0\}$. Use L along with the operations of union, concatenation, and closure to generate all strings that are **not** of the form $\{01, 01001, 010010001...\}$ i.e. strings where delimiter 1 separates out consecutive number of zeroes.

5. Problem $4.2.8$.