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| **Intro to Theory of Computing** | **481 Homework 13** |
| **CS481 Fall 2005** | **December 2** |

Please write your name and net-id on the upper right corner of every page.

1. $L$ is a CFL, $D, D'$ are DCFLs and $R$ is a regular set. Which of the following are decidable and which are not. Prove your answers.

   (a) $L = R$.

   (b) $L \subseteq R$.

   (c) $D = R$.

   (d) $D \cap D' = \varnothing$.

2. Show the following theorem : Let $P$ be a property of languages. Define $L_P = \{M \mid L(M) \text{ satisfies } P\}$. $P$ is said to have *containment property* if for all languages $L$ in $P$ and for all r.e. $L' \supseteq L$, $L'$ is also in $P$. Show that if $P$ *violates* the containment property, then $L_P$ is not r.e. *Hint.* Note that $L_r$ and $L_{nr}$ both violate the containment property.

$$L_r = \{M \mid L(M) \text{ is recursive }\}.$$

$$L_{nr} = \{M \mid L(M) \text{ is non-recursive }\}.$$

3. Show that the $\{M \mid L(M) \cap L_u \neq \varnothing\}$ is r.e. Also show that $\{M \mid L(M) - L_u \neq \varnothing\}$ is not r.e.

4. We just briefly covered oracle TMs in class. Some more material is available in Kozen's book pages 274-281, although for the following problem, you just need to know the definition.

   Suppose you are given an oracle that will always give both "yes" and "no" answers to questions of the form "Is $L(M)$ a regular language? ". Show how to use such an oracle to decide (i.e. machine must halt) questions of the form "Is $L(M)$ finite ?". Remember that even if $L(M)$ is regular, the description of $M$ might not look anything like a FA, it might perform complicated operations to accept a regular set.