Notation

$\varepsilon, \{\varepsilon\}, \Phi$

$L_1 \bullet L_2 = \{xy \mid x \in L_1 \; y \in L_2\}$

$L^* = \{\varepsilon\} \bigcup L \bigcup L^2 \bigcup L^3 \bigcup \cdots$

2^S set of all subsets

$\{0^n 1 0^n 1 \mid n \geq 1\}$ and $\{0^n 1 0^n 1 \mid n \geq 1\}^*$

Concepts

fa
nfa
e-nfa
e-closure
regular set
regular expression
induction
definition of $h \; h^{-1}$
closure properties of regular sets
- Union, dot, star
- complement machine construction
- intersection $L_1 \bigcap L_2 = \overline{\overline{L_1} \bigcup \overline{L_2}}$ or cross product machine construction
- $h \; h^{-1}$
- reversal

prove set not regular
valid computation of fa

Constructions

cross product construction
subset construction
hat technique
$h\left(h^{-1}(L) \bigcap R\right)$
nfa to fa subset construction
fa to regular expression R_{ij}^k
regular expression to fa
valid computation of fa
Write regular expression from English description
- often break string down into pieces

pumping lemma
minimization of states in fa

Examples

shuffle

Touched on

countably infinite
noncountably infinite
diagonalization
there exist non regular sets

Context-free languages

context free grammar
pda
acceptance by empty store, final state
one state pda
all regular sets are context-free languages
The regular sets are properly contained in the class of context-free languages, i.e. there exist context-free languages that are not regular

Examples of cfl's

$\{a^i b^j \mid i \neq j\} = \{a^i b^j \mid i < j\} \cup \{a^i b^j \mid i > j\}$

$\{a^i b^j c^k \mid \text{either i<j or j<k}\}$

Constructions

empty store to final state
final state to empty store
cfg to empty store
many state to one state
one state to cfg

Normal forms

no useless variables
if ε not in L(G) can eliminate ε-productions
if ε in L(G) need $S \to \varepsilon$
eliminate unit productions
Chomsky normal form $A \to BC \quad A \to b$

Pumping lemma

not cfl's $\{a^n b^n c^n \mid n \geq 1\} \quad \{a^i b^j c^k \mid i < j < k\}$

Closure properties

substitution implies union, concatenation, star, homomorphism
reversal
inverse homomorphism
intersect with regular set

Not closed under

intersection
complement

Decision algorithms
- membership
- emptiness

Undecidable
- equivalence
- equivalent to Σ^*
- emptiness of intersection

Efficient membership algorithm

Concepts
- diagonalization
- recursive set
- recursively enumerable set
- decidable
- Turing machine
- computability

More powerful models
- multi tape
- multi track
- nondeterministic

Weaker models
- semi infinite tape
- two pushdown store
- 4-counter machine
- 2-counter machine $2^i 2^j 5^k 7^l$

L_D
halting problem
class of recursive sets closed under complement
class of r.e. sets not closed under complement
listing strings in r.e. set
If L and $\overline{L}$ are both r.e. then L and $\overline{L}$ are both recursive
If L can be enumerated in order, then L is recursive
Can we enumerate names of all recursive sets? (Depends on definition of name.)
Rice's Theorem: Every nontrivial property on the r.e. sets is undecidable.
concept of reduction

Decidability for cfl's
- set of valid computations of Tm is intersection of two cfl's
- set of invalid computations is a clf

Undecidable

$L(G_1) \bigcap L(G_2) = \Phi$

$L(G) = \Sigma *$

$L(G_1) = L(G_2)$ *equvilance*

$L(G_1) \subseteq L(G_2)$

$R \subseteq L(G)$

Rado's sigma function

$\{M \mid L(M) \text{ infinite}\}$ not r.e. and complement not r.e.

Every r.e. set is the homomorphic image of a recursive set

P and NP

- complete problems for NP
  - 3-CNF satisfiability
  - clique
  - Hamilton circuit