CS381 | Second Mid Term Solutions | Friday Nov 7, 2003
Fall 2003 | | Olin 255 9:05-9:55

This is a 50-minute in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. Thank you.

1. Give a context-free grammar for the set $\{a^i b^j \mid i \le j \le 2i\}$.
Solution: $S \to aSb \mid aSbb \mid \varepsilon$

2. Prove or disprove that $L = \{a^n b^m c^n d^m \mid n \ge 1, m \ge 1\}$ is a context-free language.
Solution: Not context free by pumping lemma. Assume L is a cfl. Let k be integer of pumping lemma. Select $z = a^k b^k c^k d^k$. Suppose z is written uvwxy where $|vwx| \le n$ and $vx \ne \varepsilon$. Note vwx cannot contain both a's and c's nor can it contain both b's and d's. Thus, uv^2wx^2y will either have a different number of a's and c's or a different number of b's and d's and will not be of the form $a^n b^m c^n d^m$. Hence uv^2wx^2y is not in L, a contradiction. Therefore L is not a cfl.

3. Describe in English how to convert a many state pushdown automaton to a one state pushdown automaton.
Solution: The pushdown store alphabet of the one state machine will be $Q \times \Gamma \times Q$ where Q is the set of states of the multi-state machine and Γ is its pushdown store alphabet. The one state pda will store the current state of the multi-state pda in the first component of the top stack symbol. When the pda grows its store by writing a new symbol on the store it places the state in the first component of the top symbol and guesses the state it will be in when it erases the top symbol exposing the symbol just below. The guess is stored in the symbol just below the top. To verify that the guess is correct it will also store the guess as the third component of the symbol on the top of the store. Whenever a symbol is erased from the store a check is made that the third component is the new state since that is the state that is in the first component of the symbol below which will become the new top symbol.

4. Let L_1 be a context-free language. Suppose L_2 is obtained from L_1 by adding and deleting a finite number of strings. Is L_2 a context-free language? Justify your answer.
Solution: Let F_1 be the finite set of strings added to L_1 and F_2 the finite set of strings removed. F_1 and $\Sigma^* - F_2$ are regular sets. Since the class of cfl's is closed under union and intersection with regular sets

$$L_2 = \left(L_1 \cup F_1\right) \cap \left(\Sigma^* - F_2\right)$$

is a cfl.