CS381 First Mid Term Solution Friday Oct 10, 2003
Fall 2003 Olin 255 9:05-9:55

This is a 50-minute in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. Thank you.

There are many ways to do each problem. The answers given below are only one of many possible solutions.

1. Let L be the set of strings of 0's and 1's with an even number of 0's. Strings with zero 0's have an even number of 0's. Write a regular expression for L.

Answer: (1*01*01*)*+1*

2. Consider the set

$$\left\{0^i 1 0^{2i} 1 \mid i \geq 1\right\}^* \cap 01\left\{0^i 1 0^{2i} 1 \mid i \geq 1\right\}^* 0^* 1$$

Write down a string of length 19 in the set.
Answer: 0100100001000000001
What is the length of the shortest string in the set of length greater than 19?
Answer: 19+16+1+32+1=69 $01001 0^4 1 0^8 1 0^{16} 1 0^{32} 1$

3. Let $L_1 \subseteq (a+b)^*$ be a set of strings. In each string in L_1 delete every b immediately following an a to get the set L_2. Using $h, h^{-1} \cap R$ applied to L_1, write an expression for the resulting set of strings L_2.
Answer: Let

$$h_1(a) = a \quad h_1(b) = b \quad h_1(\hat{b}) = b$$
$$h_2(a) = a \quad h_2(b) = b \quad h_2(\hat{b}) = \varepsilon$$

Then $L_2 = h_2\left(h_1^{-1}\left(L_1\right) \cap b^*(a + a\hat{b}b^*)^*\right)$.

4. Let $L \subseteq (a+b)^*$ be the set of strings which scanned from left to right the number of a's never exceeds the number of b's. Is L regular or not? Give a proof of your answer.

Answer: Not regular. Assume L is regular and let n be the integer of the pumping lemma. Consider the string $b^n a^n$. No matter how $b^n a^n$ is broken up into the form xyz with $|xy| \leq n$ and $y \neq \varepsilon$, y will be in b^+. Set k of pumping lemma to 0, i.e. delete y. The resulting string xz must be in L but xz has more a's than b's. Therefore L is not a regular set.