

**Notation** $\varepsilon, \{\varepsilon\}, \Phi$  $L_1 \sqcup L_2 = \{xy \mid x \in L_1, y \in L_2\}$  $L^* = \{\varepsilon\} \cup L \cup L^2 \cup L^3 \cup \dots$  $2^S$  set of all subsets $\{0^n 10^n \mid n \geq 1\}$  and  $\{0^n 10^n \mid n \geq 1\}^*$ **Concepts**

fa

nfa

e-nfa

e-closure

regular set

regular expression

induction

definition of  $h, h^{-1}$ 

closure properties of regular sets

Union, dot, star

complement machine construction

intersection  $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$  or cross product machine construction $h, h^{-1}$ 

reversal

prove set not regular

valid computation of fa

**Constructions**

cross product construction

subset construction

hat technique

 $h(h^{-1}(L) \cap R)$ 

nfa to fa subset construction

fa to regular expression  $R_{ij}^k$ 

regular expression to fa

valid computation of fa

Write regular expression from English description

often break string down into pieces

pumping lemma

**Examples**

shuffle

**Touched on**

countably infinite  
 noncountably infinite  
 diagonalization  
 there exist non regular sets

**Context-free languages**

context free grammar  
 pda  
 acceptance by empty store, final state  
 one state pda  
 all regular sets are context-free languages  
 The regular sets are properly contained in the class of context-free languages, i.e. there exist context-free languages that are not regular

**Examples of cfl's**

$\{a^i b^j \mid i \neq j\} = \{a^i b^j \mid i < j\} \cup \{a^i b^j \mid i > j\}$   
 $\{a^i b^j c^k \mid \text{either } i < j \text{ or } j < k\}$

**Constructions**

empty store to final state  
 final state to empty store  
 cfg to empty store  
 many state to one state  
 one state to cfg

**Normal forms**

no useless variables  
 if  $\varepsilon$  not in  $L(G)$  can eliminate  $\varepsilon$ -productions  
 if  $\varepsilon$  in  $L(G)$  need  $S \rightarrow \varepsilon$   
 eliminate unit productions  
 Chomsky normal form  $A \rightarrow BC \quad A \rightarrow b$

**Pumping lemma**

not cfl's  $\{a^n b^n c^n \mid n \geq 1\} \quad \{a^i b^j c^k \mid i < j < k\}$

**Closure properties**

substitution implies union, concatenation, star, homomorphism  
 reversal  
 inverse homomorphism  
 intersect with regular set

**Not closed under**

intersection  
 complement

**Decision algorithms**

- membership
- emptiness

**Undecidable**

- equivalence
- equivalent to  $\Sigma^*$
- emptiness of intersection

**Efficient membership algorithm**