

**Write your name here:** \_\_\_\_\_

**1.** Let  $A \subset \Sigma^*$  be a nonempty regular set and  $z \in A$  a fixed string. Prove that

$$\{x \in \Sigma^* \mid \text{either } x \in A \text{ or } x = z^n \text{ for some integer } n \geq 0\}.$$

is a regular set. You may use the fact that  $B = \{z\}$  is a regular set without proving it.

**2.** Show that the following set is not regular:

$$A = \{a^n b^m \mid m \geq n\}.$$

You may use the property  $(\neg P)$ : For all  $k \geq 0$ , there exist strings  $x, y, z$  such that  $xyz \in A$ ,  $|y| \geq k$ , and for all  $u, v, w$  with  $y = uvw$  and  $v \neq \epsilon$ , there exists an  $i \geq 0$  such that  $xuv^i wz \notin A$ .

**3.** Let  $A = \{a^n b^n \mid n \geq 0\}$ . True or false: For every  $B \subseteq \{a, b\}^*$  with  $A \subseteq B$ ,  $B$  is not regular.