CS 381	**Introduction to Theory of Computing**	Summer 2002
Homework 2		Due: Tue, June 4, 2002

You must justify (prove or explain) every statement you make.

1. Kozen, p. 320, number 13

2. Kozen, p. 321, number 18

3. a) Let $\Sigma = \{0, 1\}$. Construct an NFA that accepts a string if and only if its first and last symbols are the same. Do not accept any strings of length less than 2. For example, your NFA should accept 01010010 and 11 but not 01001 or 1 or ϵ.

b) Use the subset construction to construct a DFA equivalent to the NFA in (a).

c) Let Σ be an arbitrary alphabet. Suppose you have a DFA, M, which accepts exactly the set of strings $x \in \Sigma^*$ such that the substring consisting of the first k symbols of x is the same as the substring consisting of the last k symbols of x, for some fixed $k \geq 1$. That is,

$$L(M) = \{x \in \Sigma^* \mid \text{prefix}(x, k) = \text{suffix}(x, k)\}.$$

Prove that M must contain at least $|\Sigma|^k$ states. Hint: Suppose M has fewer states and use the Pigeonhole Principle to get a contradiction.

4. Kozen, p. 302, number 3. Use the hint for the problem on page 302. Your solution should follow this outline fairly closely.

5. Show that each of the following sets is not regular.

a) $\{a^n b^m \mid n, m \geq 1 \text{ and } 2n = 3m\}$

b) $\{x \in \{a, b, c\}^* \mid |x| = n^2 \text{ for some integer } n \geq 0\}$

c) $\{a^n b a^m b a^{n+m} \mid n, m \geq 1\}$