| CS 381 | **Introduction to Theory of Computing** | Summer 2002 |
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| Homework 1 | | Due: Wed, May 29, 2002 |

You must justify (prove or explain) every statement you make.

**1.** Write each of the following sets in terms of $A = \{0\}$, $B = \{1\}$, and union, concatenation, power, and asterate.

a) $\{0, 1\}^+$

b) All strings over $\{0, 1\}$ having at most one 0.

c) All strings over $\{0, 1\}$ not having two consecutive 0's.

**2.** Given a string $x$ over an alphabet $\Sigma$, the reversal of $x$ is written as $x^R$ and intuitively is $x$ written backwards. Formally, the definition is

i If $|x| = 0$, then $x^R = x$. I.e., $\epsilon^R = \epsilon$.

ii If $|x| = n + 1 > 0$, then $x = ya$ for some $a \in \Sigma$, and $x^R = ay^R$.

a) Use induction on $|y|$ to show that for any strings $x, y \in \Sigma^*$, $(xy)^R = y^R x^R$.

b) Use induction on $|x|$ to show that $(x^R)^R = x$.

In your induction, be very clear about labeling the basis step, your induction hypothesis, and the induction step, and give clear justification for each claim you make. If you need help with induction, then ask for it now!

**3.** Design DFA's for each of the following sets. In each case, give both the table form and the transition diagram.

a) The set of strings in $\{0, 1, 2, \ldots, 9\}^*$ that **do not** contain the substring 381.

b) The set of strings in $\{a, b\}^*$ of the form $a^n b^m a^r$ with $n + m + r$ even and $m \geq 1$.

c) The set of strings in $\{0, 1, 2\}^*$ that are base 3 representations (leading zeros permitted) of multiples of 5.

**4.** For the following pair of DFA's, use the product construction to produce the DFA accepting (a) the intersection and (b) the union of the sets accepted by these automata. $S$ indicates the initial state and $F$ indicates an accept state.

| | $a$ | $b$ |
|---|---|---|
| $S1$ | 2 | 3 |
| $F2$ | 3 | 1 |
| $F3$ | 1 | 2 |

| | $a$ | $b$ |
|---|---|---|
| $SF1$ | 3 | 2 |
| 2 | 1 | 3 |
| $F3$ | 2 | 1 |

5. a) Let $M = (\{s,t\},\{a\},\delta,s,F)$ be a 2-state DFA accepting $aaa$. List all the possible transition diagrams for $M$ (including those with inaccessible states).

b) Give an infinite set of strings that all of the automata in (a) will accept.

c) Let $N = (S,\Sigma,\delta,s,F)$ be a DFA with $|S| = n$, $|\Sigma| = m$, and where $s$ can be any element of $S$. How many such $N$ are possible (allowing inaccessible states)? Give your answer in terms of $n$ and $m$.