Which of the following statements holds for every three languages L_1, L_2, L_3.

i) $(L_1 \cap L_2)L_3 = L_1L_3 \cap L_2L_3$
FALSE.
Proof by counterexample:
$L_1 = \{a\}$
$L_2 = \{b\}$
$L_3 = \{c\}$
$(L_1 \cap L_2)L_3 = \{\}\{c\} = \{c\}$
$(L_1L_3 \cap L_2L_3) = \{ac\} \cap \{bc\} = \{\}$
$\{c\} \neq \{\}$

ii) $(L_1^* = L_1^*L_1^*$
TRUE.
To prove this claim, we must prove a: $(L_1^* \subseteq L_1^*L_1^*$ and b: $(L_1^*L_1^* \subseteq L_1^*$.
a) Take any $x \in L_1^*$ and show $x \in L_1^*L_1^*$. Observe $x = x\epsilon$. $x \in L_1^*$ by assumption and $\epsilon \in L_1^*$ by definition of L_1^* therefore $x \in L_1^*L_1^*$.
b) Take any $x \in L_1^*L_1^*$ and show $x \in L_1^*$. Observe $x \in L_1^*L_1^*$ means $x = yz$ where $y \in L_1^*$ and $z \in L_1^*$. By definition of L_1^* this means that $y \in L_1^n$ and $z \in L_1^m$ for some non-negative integers n and m. This means that $yz \in L_1^{\{}n + m\}$, which in turn means that $x = yz \in L_1^*$ because by definition $L_1^{\{}n + m\} \in L_1^*$.

iii) $(L_1 \cup L_2) \cap L_3 = L_1 \cup (L_2 \cap L_3)$ FALSE.
Proof by counterexample:
$L_1 = \{a\}$
$L_2 = \{b\}$
$L_3 = \{c\}$
$(L_1 \cup L_2) \cap L_3 = \{a, b\} \cap \{c\} = \{\}$
$L_1 \cup (L_2 \cap L_3) = \{a\} \cup \{\} = \{a\}$
$\{\} \neq \{a\}$