CS 381	**Introduction to Theory of Computing**	Homework 5
Summer 2000		Due Date: Wed June 28, 2000

1. For strings $x, y \in \Sigma^*$, we define the shuffle operator $x \parallel y$ to be the set of strings obtained by mixing the symbols in x and y in all ways possible, while preserving the order of the symbols that come from the same string. You can think of $x \parallel y$ as inserting the symbols of y before, between, or after symbols in x, while preserving the order of symbols in x.

   Example:
   $$ab \parallel cd = \{abcd, acbd, acdb, cabd, cadb, cdab\}$$

   Define
   $$A \parallel B = \bigcup_{x \in A, y \in B} x \parallel y.$$

   (a) Show that if L is a CFL and R is regular, then $L \parallel R$ is a CFL. (**Justify** your answer.)

   (b) Give an example of two CFL's L_1 and L_2, such that L_1, L_2 are not regular, but $L_1 \parallel L_2$ is a CFL.

   (c) Show that if L_1, L_2 are CFL's, $L_1 \parallel L_2$ is not necessarily a CFL.

2. Kozen, page 334, problem 74, point (b).

3. Prove whether the following sets are CFL's or not:

   (a) $\{x \in \{a, b, c\}^* \mid \#a(x) = \#b(x) = \#c(x)\}$

   (b) $\{a^i b^j c^k \mid i, j, k \geq 0,\ i > j \text{ or } j > k\}$

   (c) $\{a^n b^m c^k d^l \mid 2n = 3l \text{ and } 5k = 7m\}$

   (d) $\{a^m b^n \mid m, n \geq 0,\ 5m + 3n \equiv 24 \pmod{2000}\}$

4. Are the following sets CFL's or not? Prove your answers.

   (a) $\{a^{2^n} b^l \mid 2^n \equiv l \pmod{381}\}$

   (b) $\{a^n b^l \mid n^2 \equiv l \pmod{381}\}$