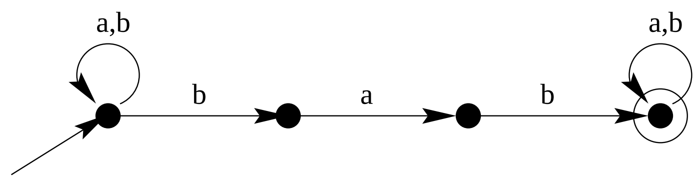
CS 381	**Introduction to Theory of Computing**	HOMEWORK 2
SUMMER 2000		DUE DATE: TUE JUNE 6, 2000

1. The following NFA accepts strings in $\{a,b\}^*$ containing the substring bab. Convert this automaton to an equivalent deterministic one using the subset construction. Do not write down inaccessible states.



2. Are the following pairs of regular expressions over $\Sigma = \{a, b\}$ equivalent? If yes, prove the equility using the algebraic relations given in class; if no, then give a counterexample.

   (a) $(a+b)^*$ and $a^* + a^*b^*$

   (b) $(a+b)^*$ and $(a^*b^*a^*)^*$

   (c) $\emptyset\emptyset^*$ and $\varepsilon\varepsilon^*\varepsilon$

   (d) $a^*(bba)^*bb$ and $a^*b(bab)^*b$

   (e) $(ab+a)^*a$ and $a(ba+a)^*$

3. Let A be a regular set accepted by the DFA $M = (Q, \Sigma = \{a,b\}, \delta, s, F)$. Construct a finite automaton M' that accepts strings from A with exactly two errors. (i.e. exactly two symbols are changed $a \to b$ or $b \to a$).

4. Given two DFA's, M_1 and M_2, we can formally generate a DFA M_3 s.t. $L(M_3) = L(M_1) \cup L(M_2)$. We argued in class that the set of final states of M_3 should be $F_3 = (Q_1 \times F_2) \cup (F_1 \times Q_2)$. We have also indicated that the automaton M_3 could be obtained by using De Morgan's law $\Big(L(M_1) \cup L(M_2) = \sim \big((\sim L(M_1)) \cap (\sim L(M_2))\big)\Big)$.

   (a) Show that using De Morgan's law the set of final states of M_3 can be written as $F_3' = (Q_1 \times Q_2) \setminus ((Q_1 \setminus F_1) \times (Q_2 \setminus F_2))$.

   (b) Show **FORMALLY** that $F_3 = F_3'$.

5. Given a non-empty alphabet Σ, $x \in \Sigma^*$, and $k \geq 1$, define operator $rfs(x,k)$ to be the string obtained from x by repeating the first symbol of x exactly k times.

   Examples:
   $rfs(abc,1) = abc$ $\quad$ $rfs(abc,3) = aaabc$ $\quad$ $rfs(\varepsilon,k) =$undefined

   Given a regular set A and a fixed natural number k, prove that the set

   $$A_k = \{rfs(x,k) \mid x \in A\}$$

   is regular.