CS 381 **Introduction to Theory of Computing** Homework 1
Summer 2000 Due Date: Wed May 31, 2000

1. Design DFA's for each of the following sets:
(a) The set of strings in $\{0, 1, 2, 3, \ldots, 9\}^*$ containing the substring 2000.
(b) The set of strings in $\{a\}^*$ whose length is a multiple of 2 or 5.
(c) The set of strings in $\{a, b\}^*$ of the form $a^n b^m$, whose $n + m$ is a multiple of 4 and $m \geq 1$.

2. Consider the following two DFA's:

	a	b
→ 1	1	2
2F	2	1

M_1

	a	b
→ 1	2	3
2	3	1
3F	1	2

M_2

Use the product construction to generate the automata accepting (a) the intersection; (b) the union of the two sets accepted by these automata.

3. (a) Let $M = (\{s, t\}, \{a\}, \delta, s, F)$ be a 2-state DFA accepting aa. List **ALL** possible DFA's M (including those with inaccessible states) and indicate the sets they accept.
(b) Give an infinite set of strings that all the automata given in (a) will accept.
(c) Let M' be a 2-state DFA on $\Sigma = \{a, b\}$ accepting baa. Give an infinite set of strings that M' must accept. Justify your answers.

4. In the land of Krogs, the only accepted currency is the kroll. All prices in Krogland are integer and the government imposes a flat 7 kroll sales tax on every transaction. The government is on the verge of a monetary reform. It intends to withdraw all outstanding bills and coins, except for the 3 and 5 kroll coins. Will Krogs be able to pay

for everything they buy with a combination of these coins? Prove your claim!

5. For $k \geq 1$ and $p \geq 2$, let

$$A_{k,p} =^{def} \left\{ x \in \{0, 1, \ldots, p-1\}^* \mid x \text{ is a } p\text{-ary representation of a multiple of } k. \right\}$$

We gave a DFA for the set $A_{3,2}$, the multiples of three in binary, and proved it correct. Generalize the construction and proof for arbitrary k and p.