CS 381	**Introduction to Theory of Computing**	PRELIM 1
SUMMER 2000		JUNE 12, 2000

This exam is open book, open notes, but you may not share books or notes with other students during the exam. During the exam you are not allowed to give or receive help, or to cooperate with anybody else. Write your answers in the exam booklet. Show all your work and clearly indicate your answers. Read the text carefully and make sure you do not overlook details. Write legibly. We reserve the right to ignore illegible answers.

Good luck!

Write your name in the space below and on your booklet after you have read the text above. At the end of the exam, turn in this sheet together with your booklet.

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1. **(10 points)** Safe deposit boxes at your local financial institution, Deposit Fraud Associates, Inc. (DFA), have combination locks. The locks have four buttons, marked with symbols α, β, γ, and δ. You open the lock by pressing these buttons, one at a time, to form the secret password. The lock has a device that checks whether the password that has been input satisfies the formal requirements imposed by the bank. If it does, a super-secret device will forward the password to the computer that will decide whether the lock can be opened or not. We know nothing about the super-secret device, but we know that each password has to satisfy the following conditions:

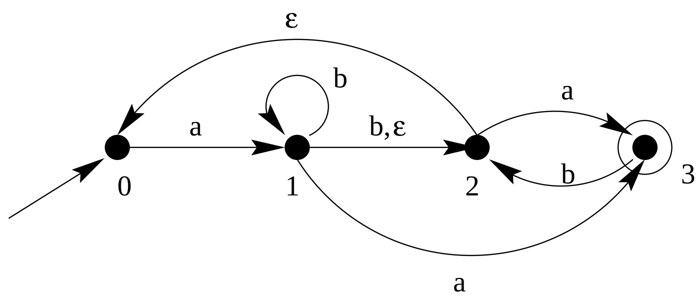
   - Each password starts with α.
   - Each password ends with γ.
   - No password can start with two α's.
   - All symbols must appear in each password.

   There are no other rules that specify formal requirements for passwords. Passwords of any length satisfying these conditions must be accepted.

   DFA, Inc. has asked you to design the device that checks whether passwords have the right form. For practical reasons, they want you to build a deterministic device. They will take care of manufacturing, but you must deliver a complete blueprint (diagram) for this device.

(Turn over.)

2. (13 points) Consider the following NFAε.



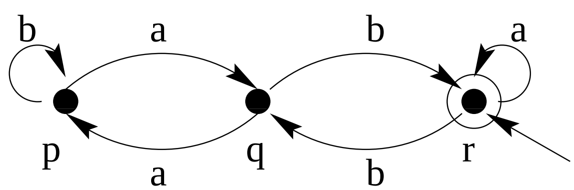
(a) Convert the NFAε above to an NFA with no ε-transitions. Give the full diagram of the resulting NFA.

(b) Convert the NFA you got in (a) into an equivalent DFA.

(c) "Eyeball" the DFA you got in (b) and give the regular expression describing the language of the DFA. Simplify the regular expression as much as you can.

Note: For (a) and (b), apply the algorithms given in class. Show all your work. And in the diagrams, show all accessible states.

3. (12 points) Consider the automaton below:



(a) Compute $\alpha_{rr}^{\{p,q,r\}}$.

(b) Show that $\alpha_{rr}^{\{p,q,r\}} = (a + b(ab^*a)^*b)^*$.

Hint: Choose the intermediate states carefully, so that you minimize the amount of work you have to do.

4. (16 points) Let Σ be an alphabet and $x \in \Sigma^*$, s.t. $x = c_1c_2c_3 \cdots c_k$, $k \geq 0$. We define the operator **deleven**(x) to generate all strings that are obtained from x by deleting 0, 1, 2, or more symbols, but only those that are in the *even* positions of the original string.

Examples:
deleven$(\varepsilon) = \{\varepsilon\}$
deleven$(abcd) = \{abcd, acd, abc, ac\}$
deleven$(abc) = \{abc, ac\}$

(Continued on next page.)

Given a set A in Σ^*, we define

$$\textbf{deleven}(A) = \bigcup_{x \in A} \textbf{deleven}(x).$$

Show (possibly by constructing a finite automaton) that if A is regular, $\textbf{deleven}(A)$ is regular. **Justify** your construction.

5. (16 points) Consider a (given) alphabet Σ containing more than one symbol. Let A be a non-empty subset of Σ^*. Define set B as follows:

$$B = \{x^{|y|} \mid x, y \in A\}.$$

Choose A so that set B is

(a) regular;

(b) non-regular.

Prove the correctness of your choices!

End of Exam