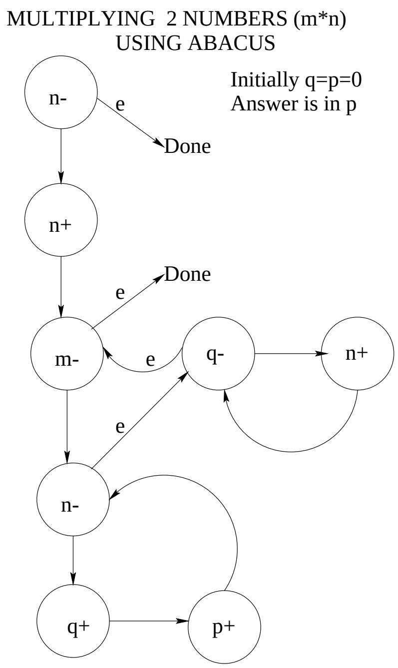
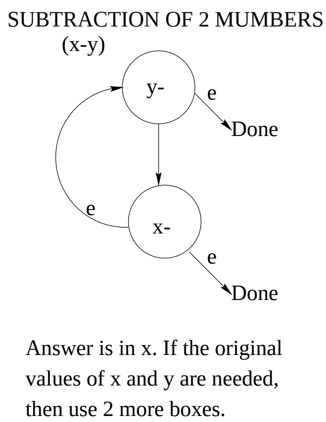
CS 381 Fall 2000
Solutions to Homework 1

Handout #1





Handout #2

Problem 1

Proof of $\emptyset^+ = \emptyset$: **Claim** $\emptyset^i = \emptyset \; \forall \, i \geq 1$. This is obviously true for $i = 1$. Assume this is true for $i \leq n$. Consider $i = n + 1$. By definition $\emptyset^{n+1} = \emptyset \, \emptyset^n = \emptyset \, \emptyset$ (by induction hypothesis) $= \emptyset$. Hence it is true for all i.

Now, by definition $\emptyset^+ = \bigcup_{i \geq 1} \emptyset^i$. The RHS consists of only empty sets and union of countably finite empty sets is empty. Hence $\emptyset^+ = \emptyset$.

By definition, $\emptyset^* = \emptyset^0 \bigcup \emptyset^+ = \emptyset^0$ (from the previous proof) , But by definition $\emptyset^0 = \{\epsilon\}$. Therefore $\emptyset^* = \{\epsilon\}$.

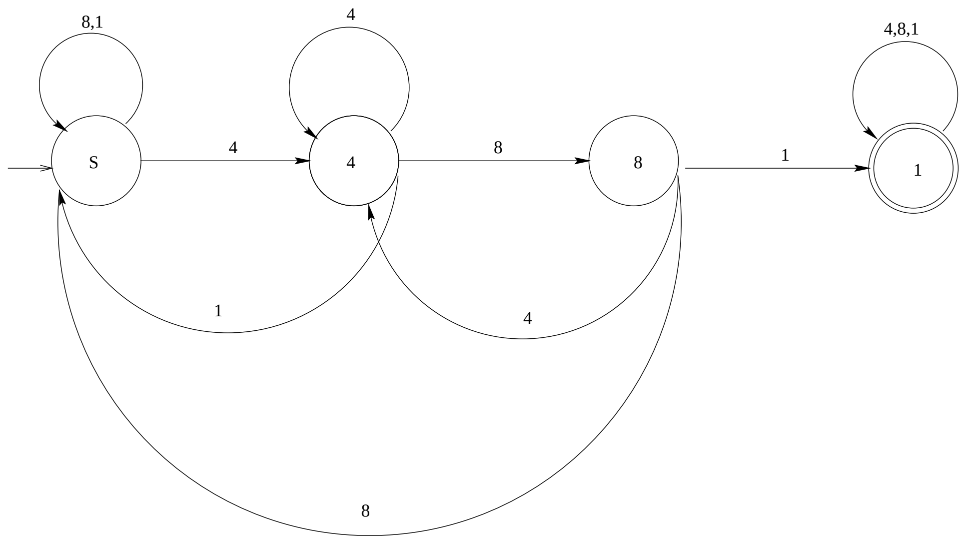
a) YES
b) YES
c) NO
d) YES
e) YES
f) NO

Problem 2

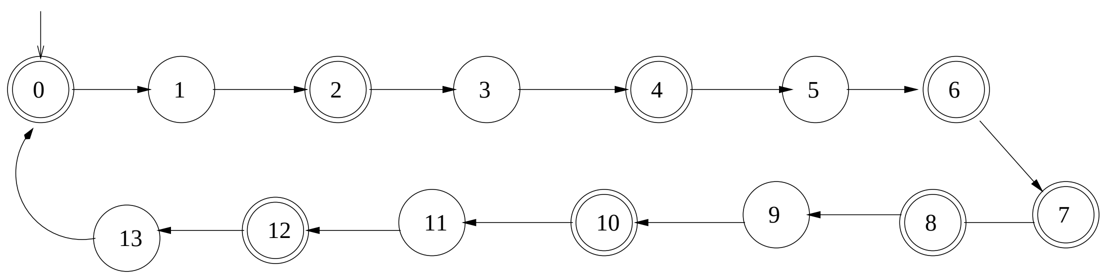
a) $(A \bigcup B)^*$
b) $(AB)^*$
c) A^*B^*
d) $\bigcup_{i \geq 0} A^i B^i$

Handout #3

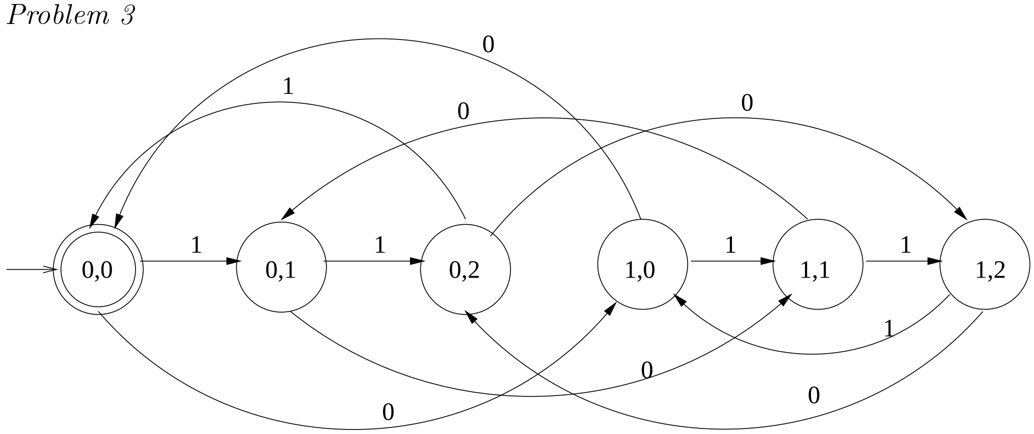
Problem 1



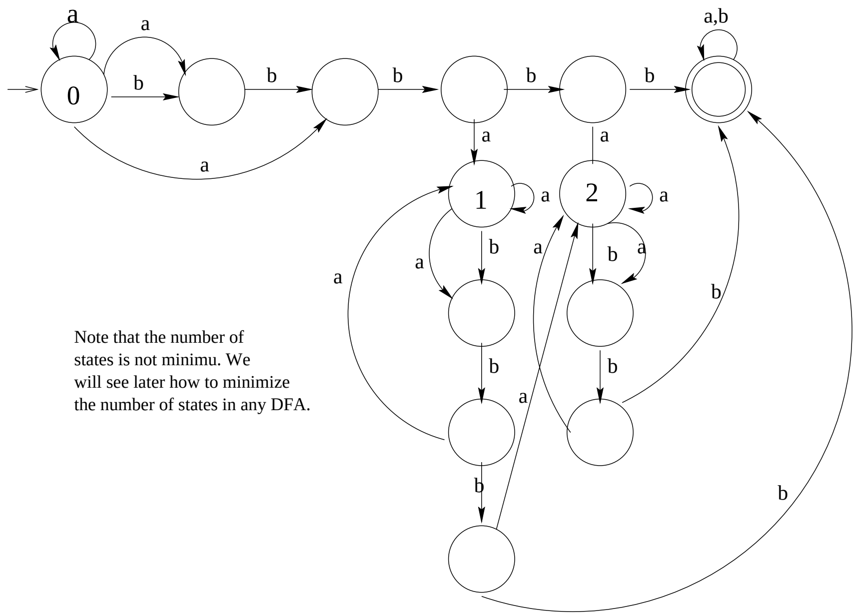
Problem 2



Problem 3



Problem 4



Problem 5