CS 372
Homework 3
Due date:
In class Thusrday Feb. 28, 2008
By email Sunday 11:59 p.m. to Robert Xiao (rkx2@cornell.edu)
use subject line (HWK#3 – CS372)

SHOW YOUR WORK FOR ALL QUESTIONS

1 – Determine the truth value of each of these statements if the domain of each variable consists of all real numbers

a) $\forall x \exists y\ (x^2 = y)$
b) $\forall x \exists y\ (x = y^2)$
c) $\exists x \forall y\ (xy = 0)$
d) $\exists x \exists y\ (x + y \neq y + x)$
e) $\forall x\ (x \neq 0 \rightarrow \exists y\ (xy = 1))$
f) $\exists x \forall y\ (y \neq 0 \rightarrow xy = 1)$
g) $\forall x \exists y\ (x + y = 1)$
h) $\exists x \exists y\ (x + 2y = 2 \wedge 2x + 4y = 5)$
i) $\forall x \exists y\ (x + y = 2 \wedge 2x - y = 1)$
j) $\forall x\ \forall y\ \exists z\ (z = (x+y)/2)$

2 Let F(x,y) be the statement "x can fool y", where the domains consists of all people in the world. Use quantifiers to express each of these statements:

a) Everybody can fool Fred
b) Evelyn can fool everybody
c) Everybody can fool somebody
d) There is no one who can fool everybody
e) Everyone can be fooled somebody
f) No one can fool Fred and Jerry
g) Nancy can fool exactly two people
h) There is exactly one person whom everybody can fool
i) No one can fool himself or herself
j) There is someone who can fool exactly one person besides himself or herself

3. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a) $\neg\forall x\forall y P(x, y)$
b) $\neg\forall y\exists x P(x, y)$
c) $\neg\forall y\forall x(P(x, y) \wedge Q(x, y))$
d) $\neg(\exists x\exists y\neg P(x, y) \wedge \forall x\forall y Q(x, y))$
e) $\neg\forall x(\exists y\forall z P(x, y, z) \wedge \exists z\forall y P(x, y, z))$

4. Show that $\forall$x P(x) $\vee\forall$x Q(x) and $\forall$(x) (P(x) $\vee$ Q(x)) are not logically equivalent.

5. For each of these arguments, explain which rules of inference are used for each step.

a) Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.

b) There is someone in this class who has been in Barcelona. Everyone who goes to Barcelona visits Picasso Museum. Therefore someone in this class has visited Picasso Museum.

6. Use resolution refutation to prove:

$$[(Q \rightarrow \neg P)] \rightarrow [(Q \rightarrow P) \rightarrow \neg Q]$$