CS 372
Homework 2
Due date: Thusrday Feb. 21, 2008
(At the beginning of class)
Reading: Chapter 7, Russell & Norvig

SHOW YOUR WORK FOR ALL QUESTIONS

1 – Consider the logical operator NOR. The proposition p NOR q is true when both p and q are false, and it is false otherwise. The operator NOR is denoted by ↓.

a) Construct a truth table for the logical operator p↓q.
b) Use a truth table to show that p↓p is equivalent to ¬p.
c) Use a truth table to show that (p↓q) ↓(p↓q) is logically equivalent to p∨q.

2 – Show that ¬p→(q→r) and q→(p∨r) are logically equivalent. (Use the equivalences below to show your work).

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

3 – Write the following statements in the form p →q (assume the statements follow the semantics of propositional logic and not just informal language; check your lecture notes)

a. It is necessary to pass a multiple choice test to get a driver's license.
b. Winds from the south imply a spring thaw.
c. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
d. Willy gets caught whenever he cheats.
e. You can access the website only if you pay a subscription fee.
f. Getting elected follows from knowing the right people.
g. Carol gets seasick whenever she is on a boat.
h. If you keep your lectures notes, they will be useful reference in your future courses.

4. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Formalize the knowledge base using propositional logic and explain your reasoning.

5 – Are these system specifications consistent? (Encode your knowledge base using propositional logic).

If the file system is not locked then the new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If the new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then the new messages will be sent to the message buffer. New messages will not be sent to the message buffer.

Assume:

L- the file system is locked
Q - new messages will be queued
N - the system is functioning normally
B - new messages will be sent to the message buffer.

6 – Assume the knowledge base contains only the sentence in each question.

6.1. $(p \wedge q)$ How many models are there? List them. Does the knowledge base entail p? Justify your answer.

6.2. $(a \vee b) \wedge (\neg a \vee b) (a \vee \neg b) \wedge (\neg a \vee \neg b)$ How many models are there? List them. Does the knowledge base entail *a*? Justify your answer.

6.3. $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$ How many models are there? List them. Does the knowledge base entail p? Justify your answer.