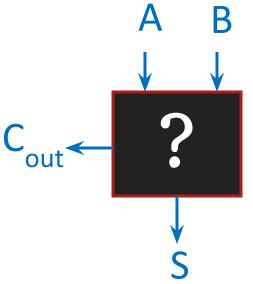
Review: Adder Circuit

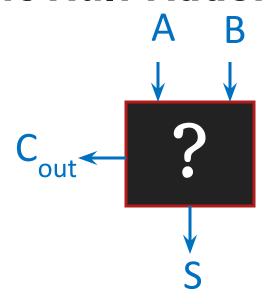
CS 3410: Computer System Organization and Programming

Fall 2025



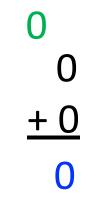
- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

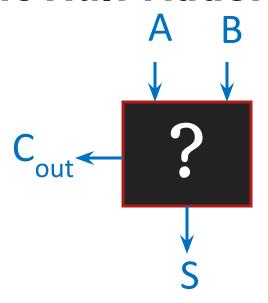
S = one input equals 1 C_{out} = two inputs equal 1



- A B Cout S
- 0 0 0
- 0 1
- 1 0
- 1 1

- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

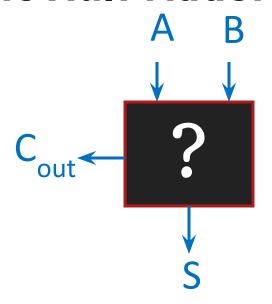




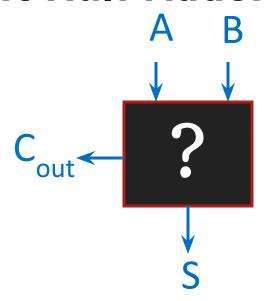
- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

$$\begin{array}{ccc}
0 & 0 \\
0 & 0 \\
+ 0 & + 1 \\
0 & 1
\end{array}$$



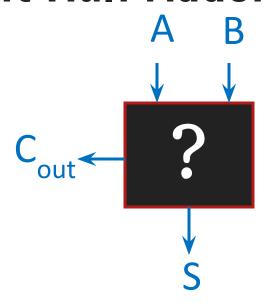


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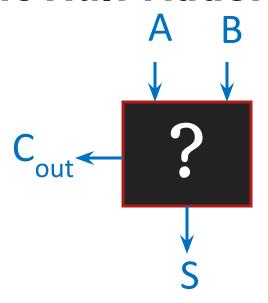




- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

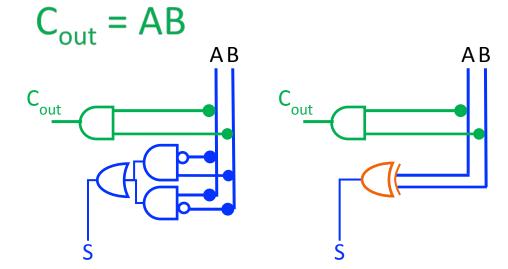
$$S = \overline{A}B + A\overline{B}$$

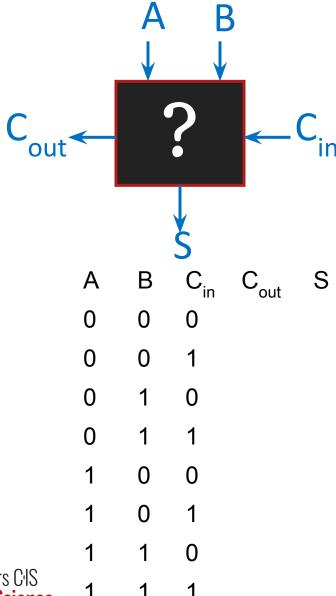




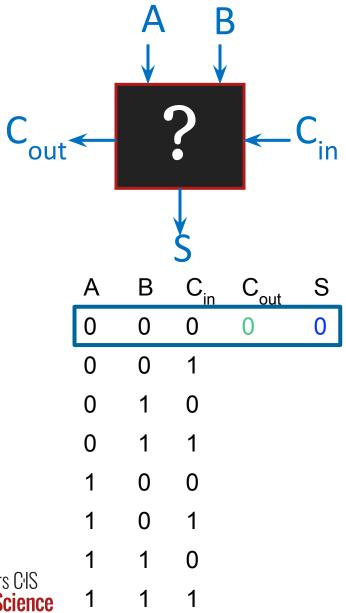
- Adds two 1-bit numbers
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- No carry-in

$$S = \overline{A}B + A\overline{B} = A \oplus B$$



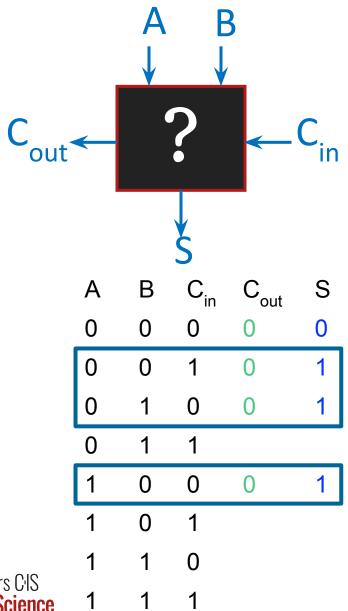


- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits



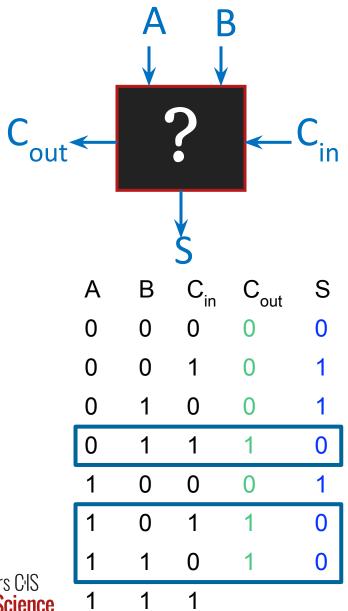
- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

$$0 + 0 + 0 = 0$$



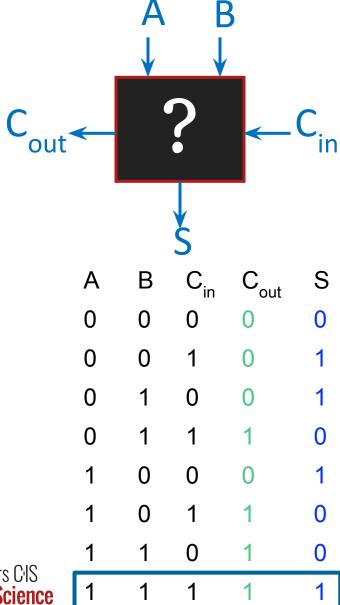
- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

$$1 + 0 + 0 = 1$$



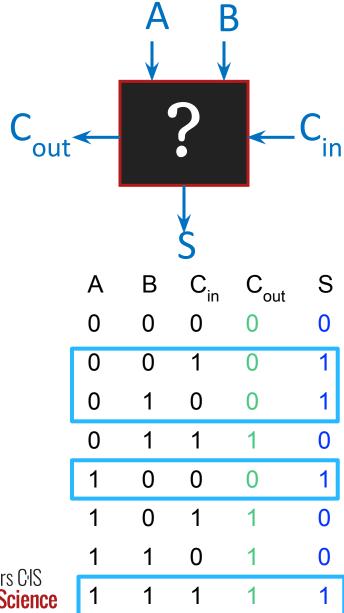
- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

$$1 + 1 + 0 = 2_{10} = 10_2$$



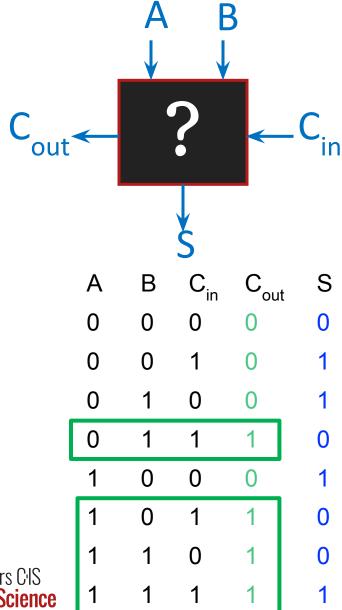
- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

$$1 + 1 + 1 = 3_{10} = 11_{2}$$



- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded
- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

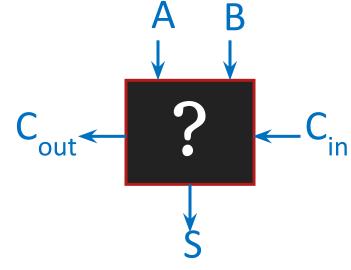
$$S = \overline{AB}C + \overline{A}B\overline{C} + A\overline{BC} + ABC$$



- Adds three 1-bit numbers
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- Can be cascaded
- Fill in Truth Table
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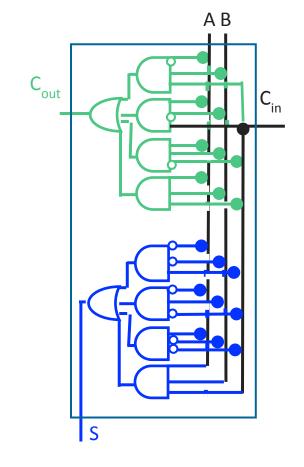
$$S = \overline{ABC} + \overline{ABC} + A\overline{BC} + ABC$$

$$C_{out} = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

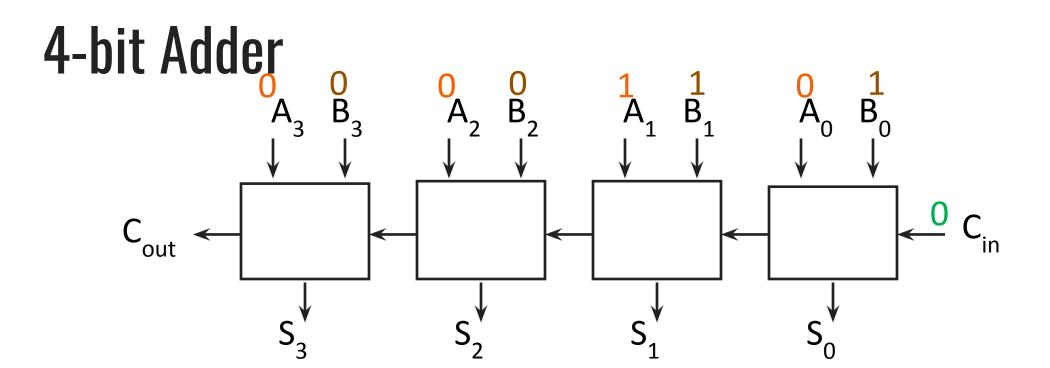


Α	В	C_{in}	C_out	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0

$S = \overline{ABC} + \overline{ABC} + A\overline{BC} + ABC$ $C_{out} = \overline{ABC} + A\overline{BC} + AB\overline{C} + ABC$

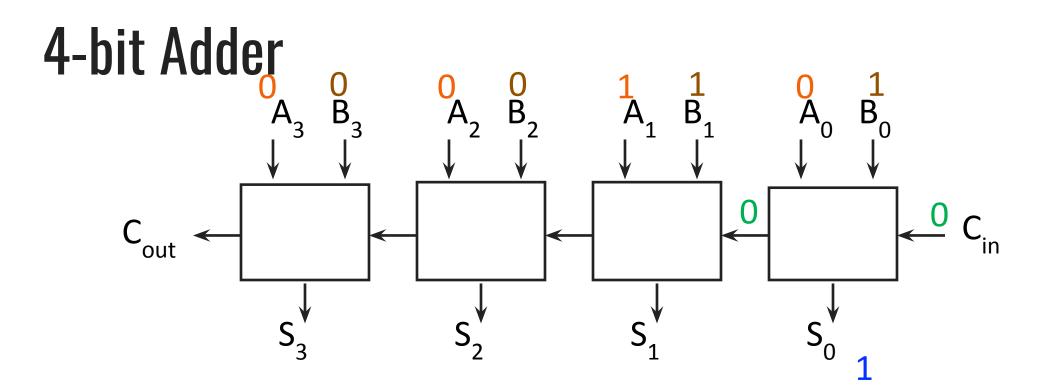






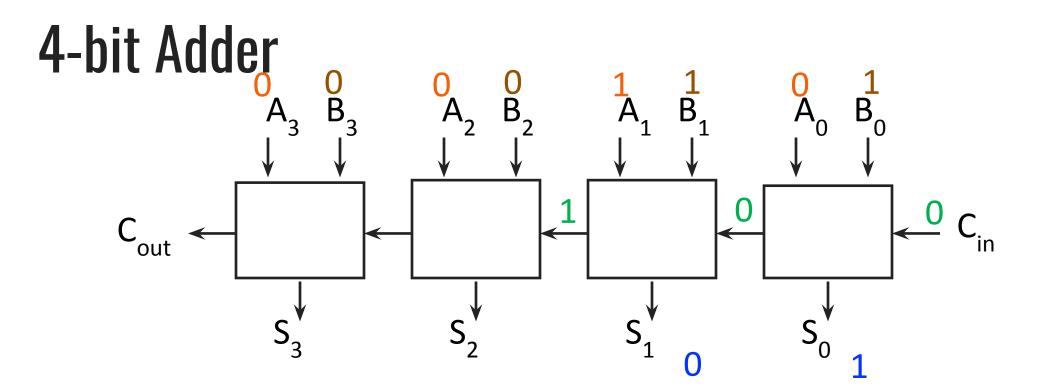
- Adds two 4-bit numbers, along with carry-in
- Computes 4-bit result and carry out
- 3 + 2 = 5
- Carry-out \square result > 4 bits



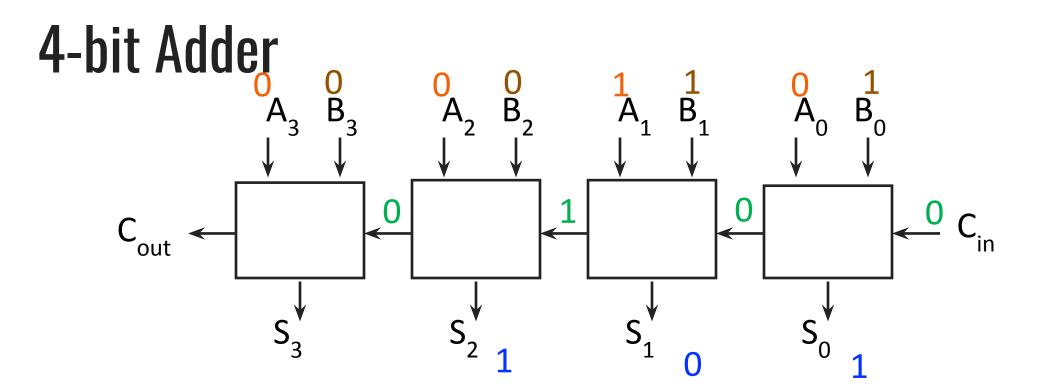


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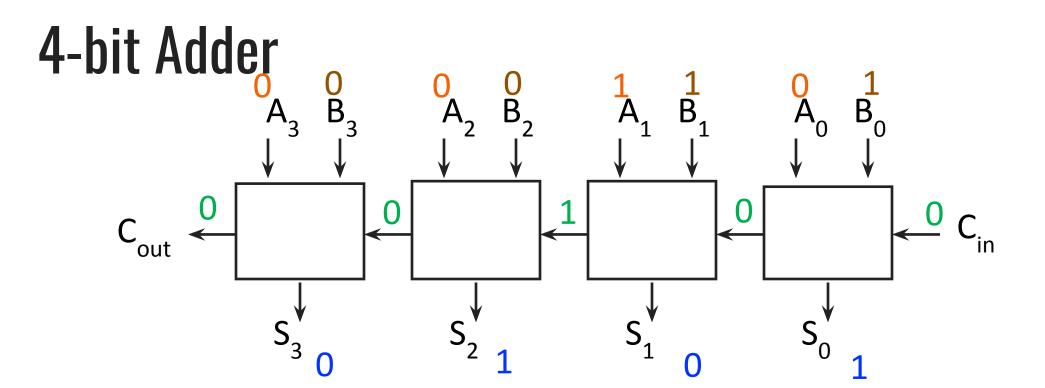




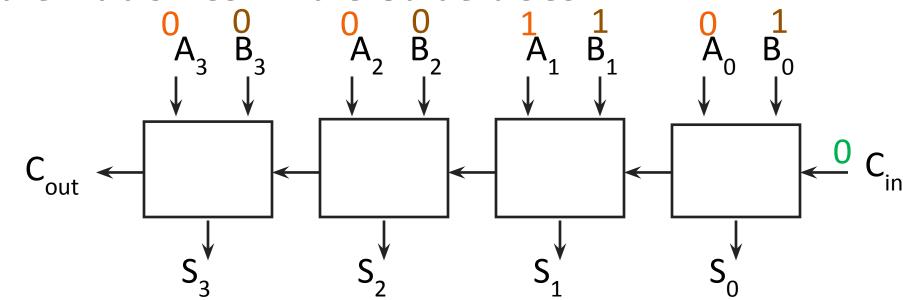
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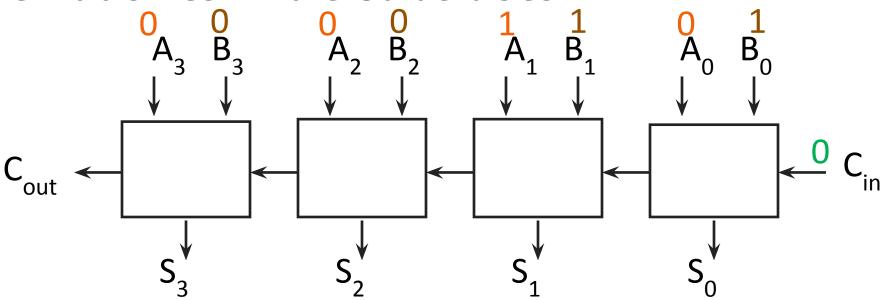
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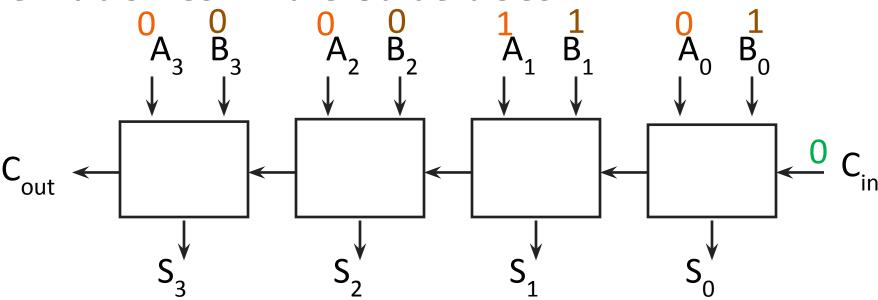
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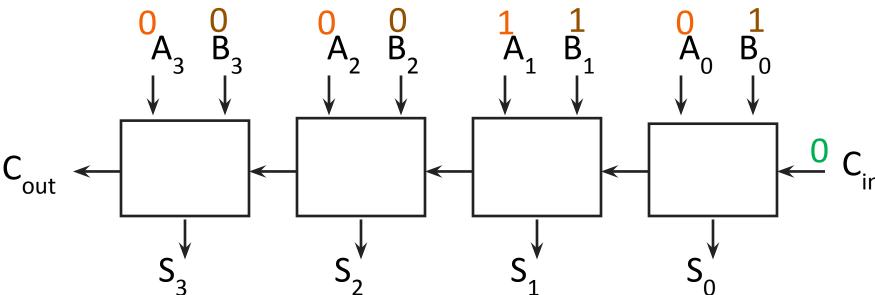
• What if we want to subtract instead?



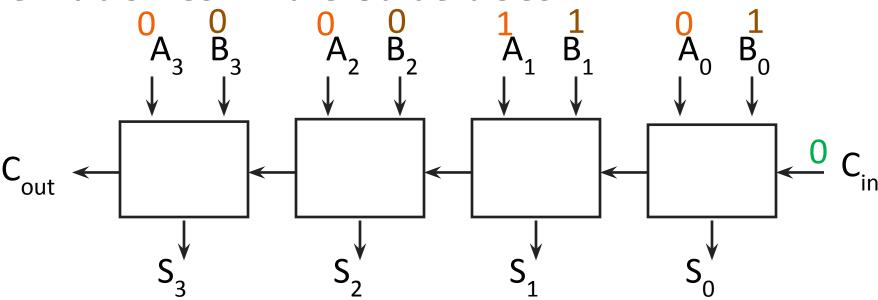
- What if we want to subtract instead?
- How do we calculate 3 2 = 1?



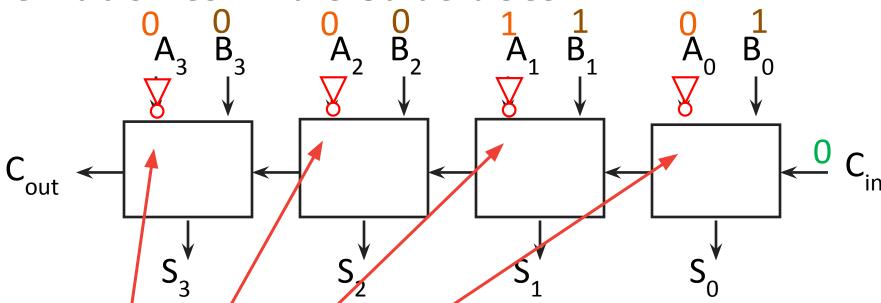
- What if we want to subtract instead?
- How do we calculate 3 2 = 1?
- We know how to calculate 3 + (-2) = 1



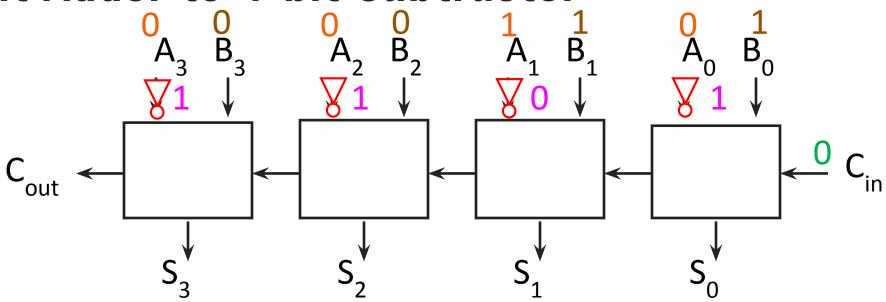
- What if we want to subtract instead?
- How do we calculate 3 2 = 1?
- We know how to calculate 3 + (-2) = 1
- How do we negate a two's complement number?



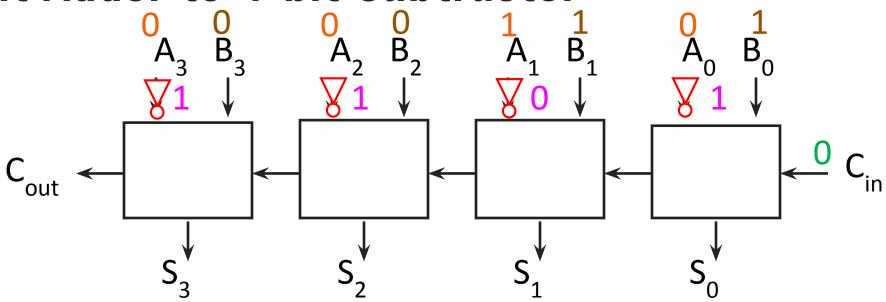
- What if we want to subtract instead?
- How do we calculate 3 2 = 1?
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- -2: !(0010) + 1 = 1101 + 1 = 1110



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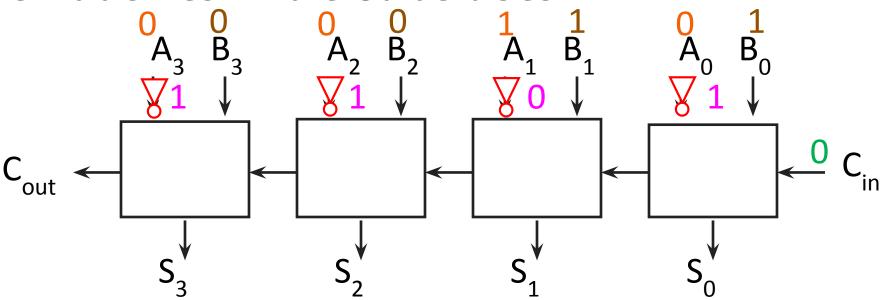


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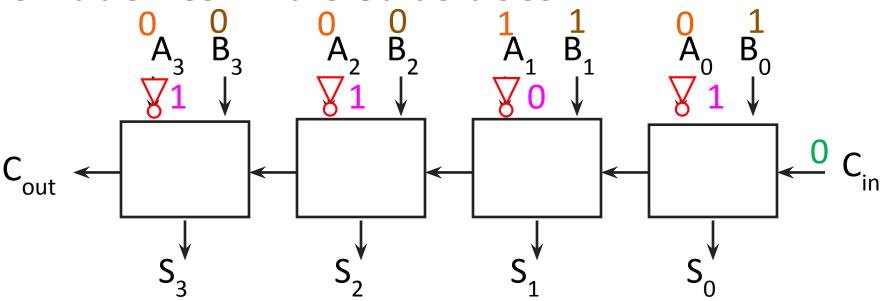


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- 3 2 = 3 + (-2)



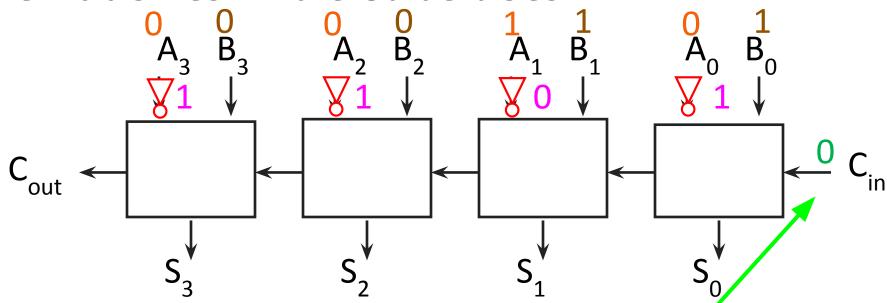


- How do we calculate 3 2 = 1?
- We know how to calculate 3 + (-2) = 1
- -2: !(0010) + 1 = 1101 + 1 = 1110
- $3 2 = 3 + (-2) = 3 + 1101_2 + 1$



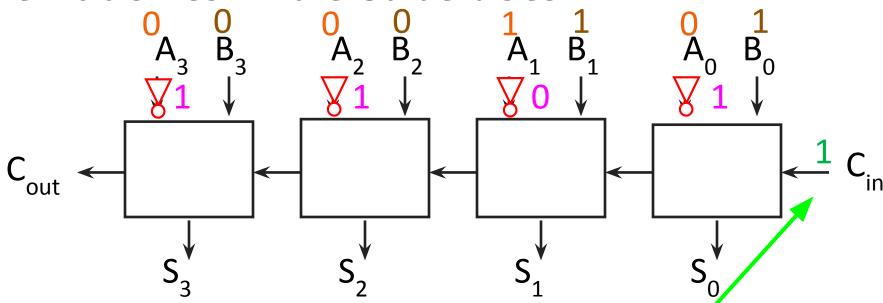
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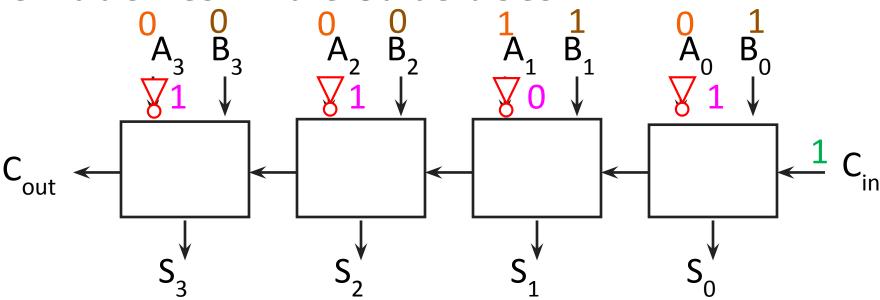


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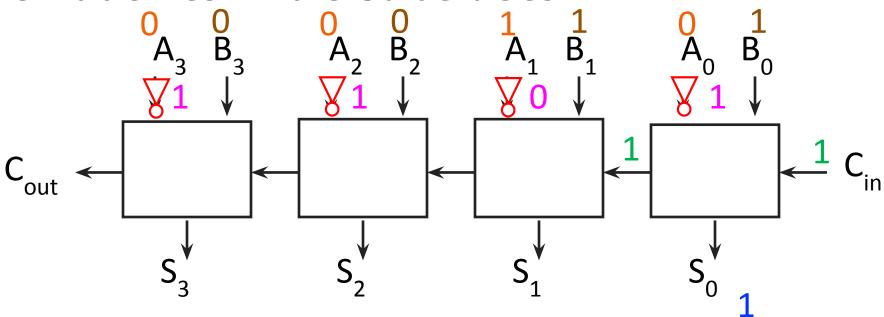




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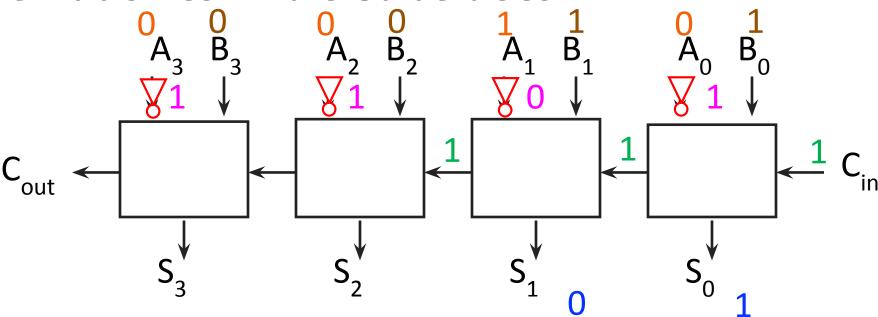


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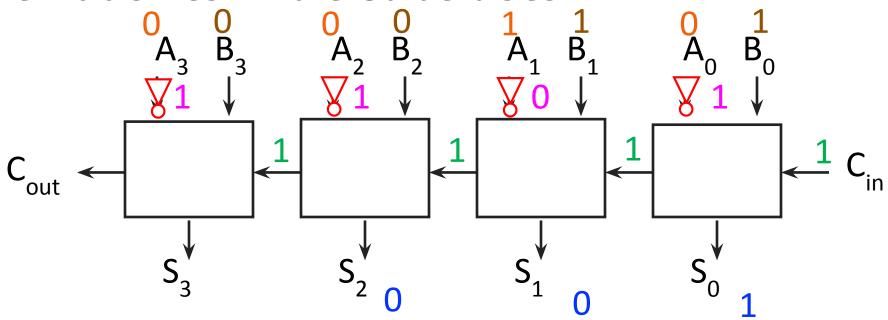


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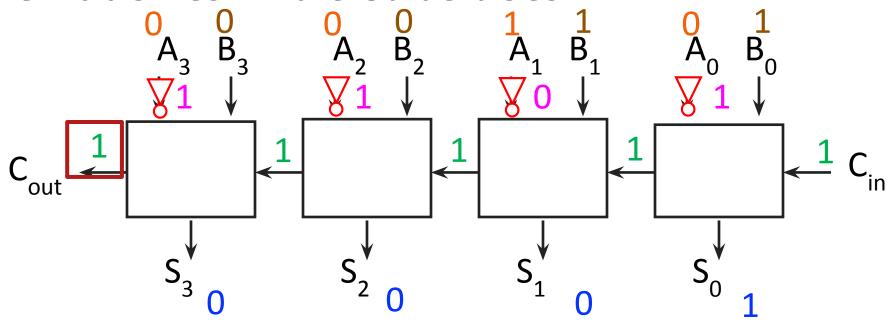




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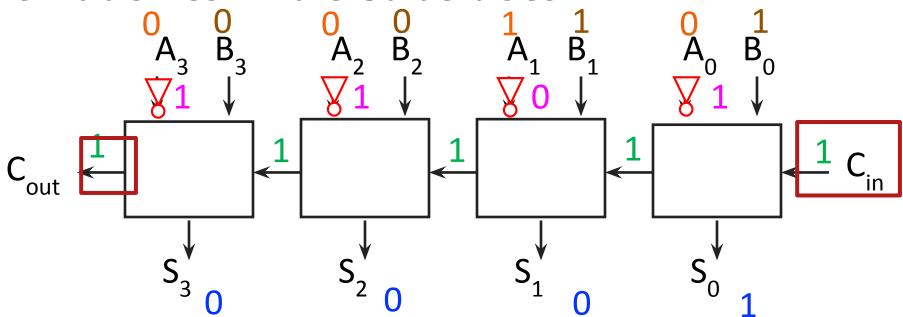
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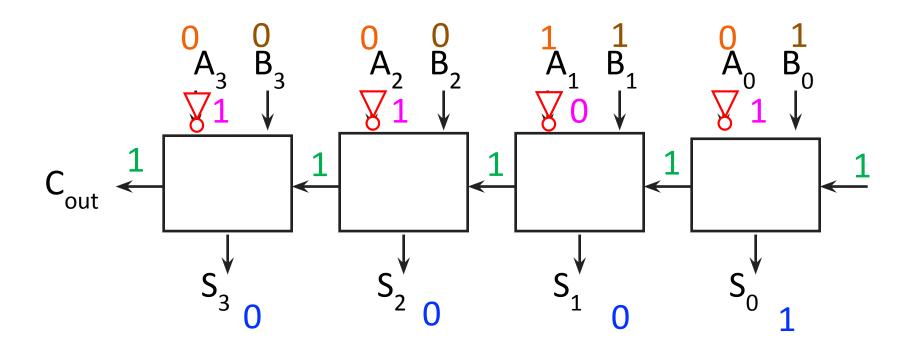
Overflow?



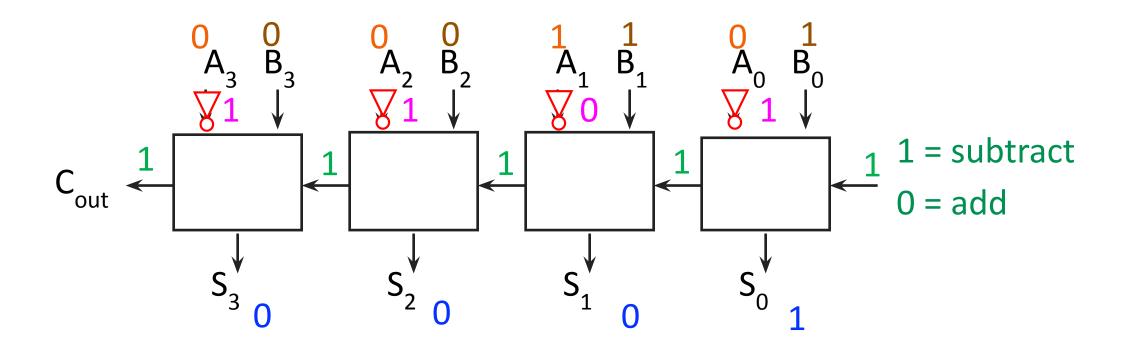
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Overflow?

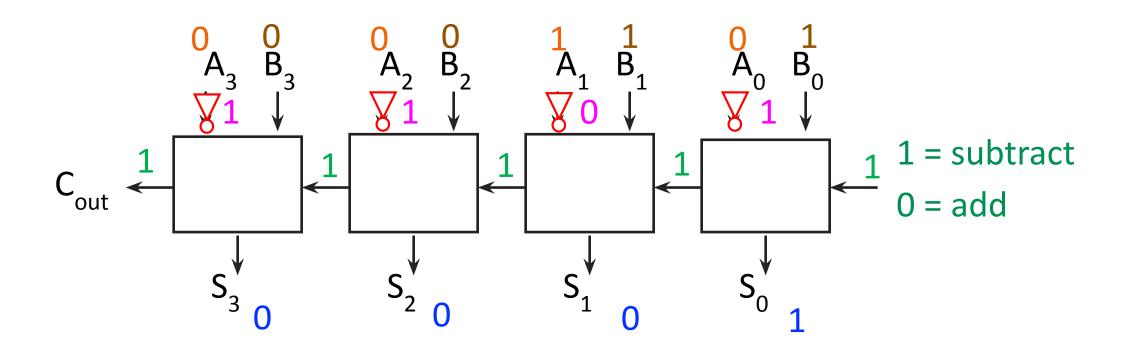
No.
$$C_{in} = C_{out}$$



• Can we add and subtract with the same circuit?

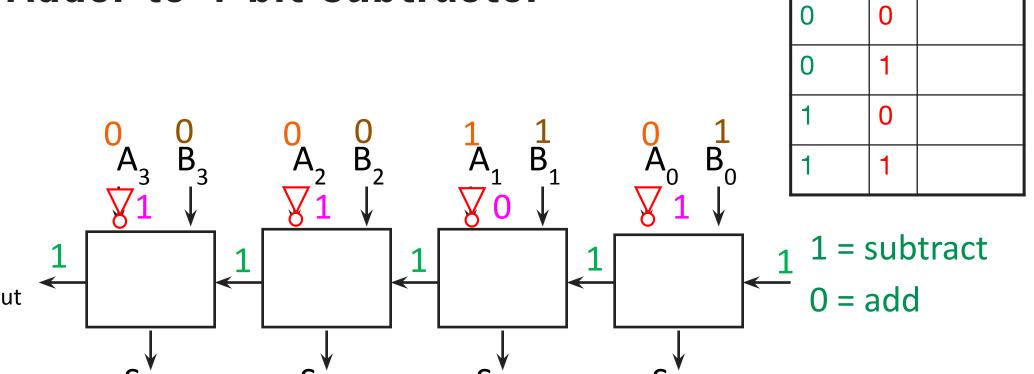


• Can we add and subtract with the same circuit?



- Can we add and subtract with the same circuit?
- How can we disable the inverter?





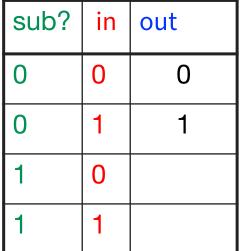
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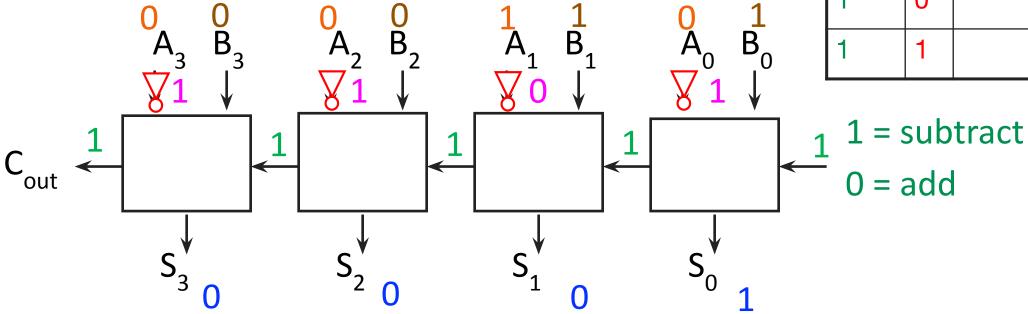


sub?

in

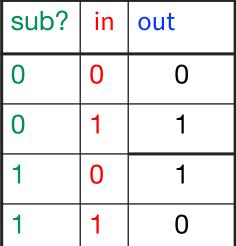
out

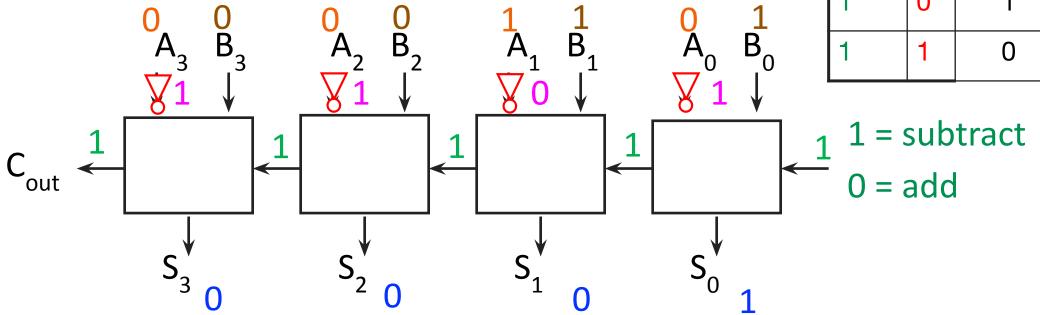




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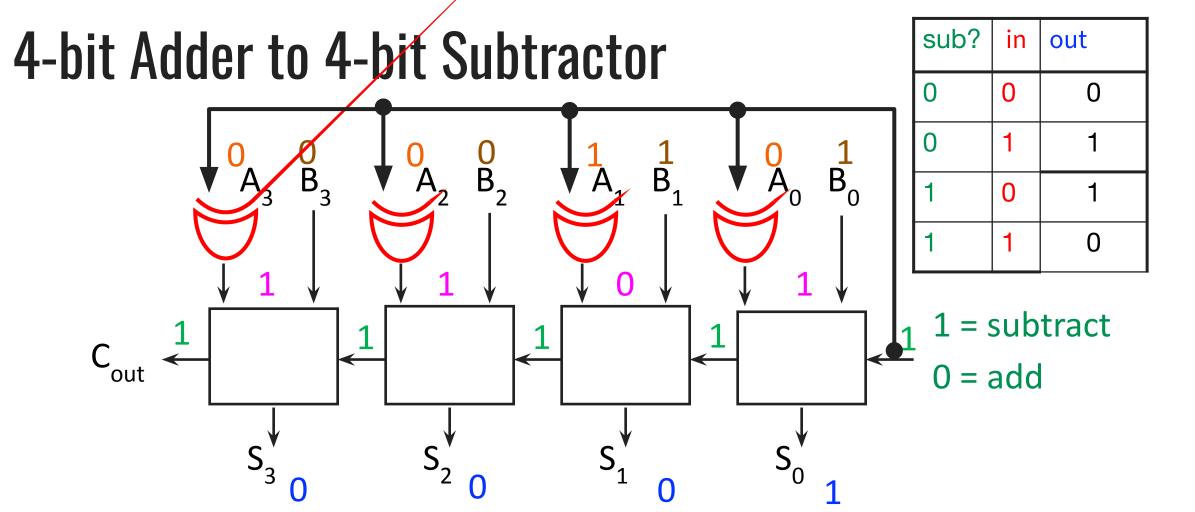






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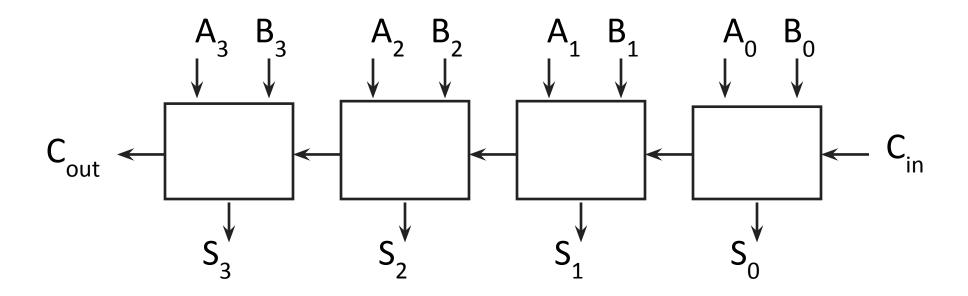


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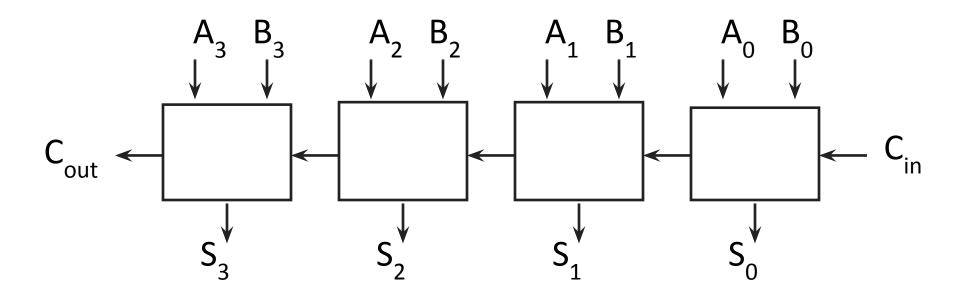
State

CS 3410: Computer System Organization and Programming

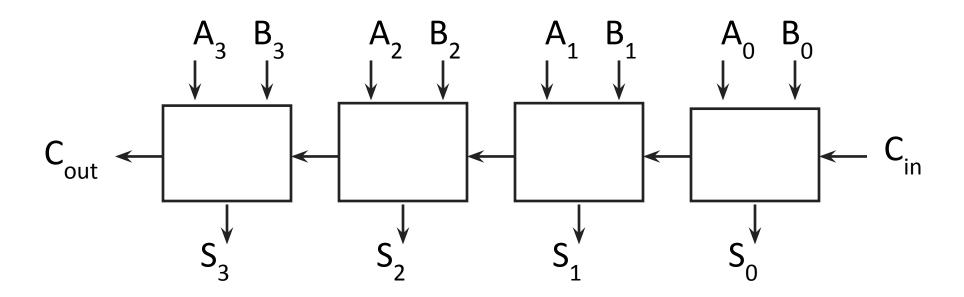


• So far, gates compute instantaneous



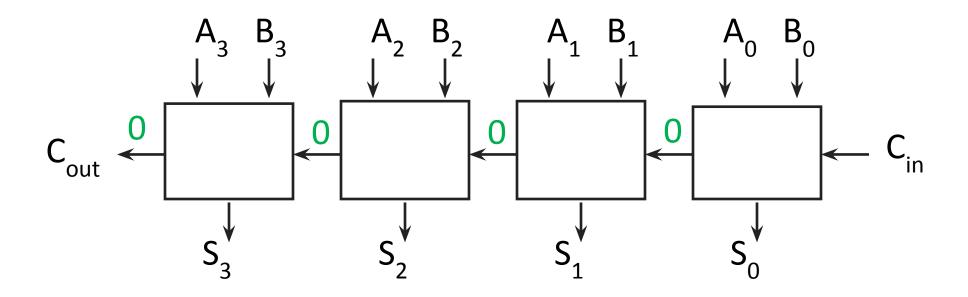


- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.



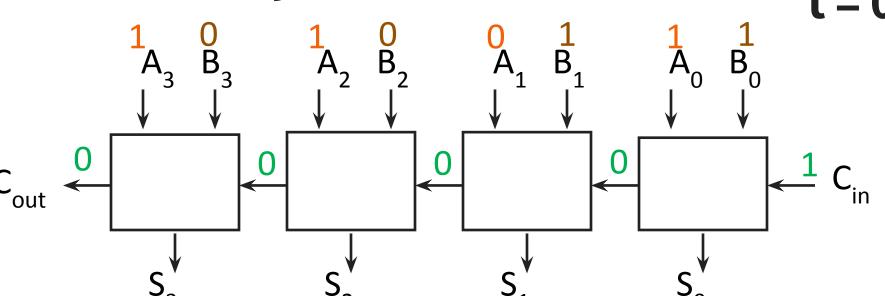
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- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder





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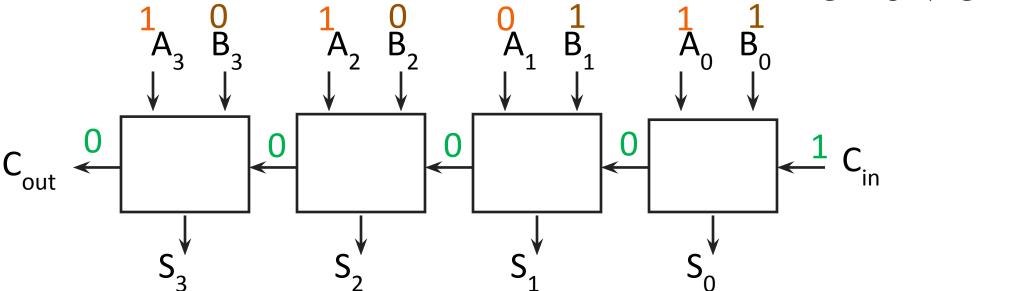




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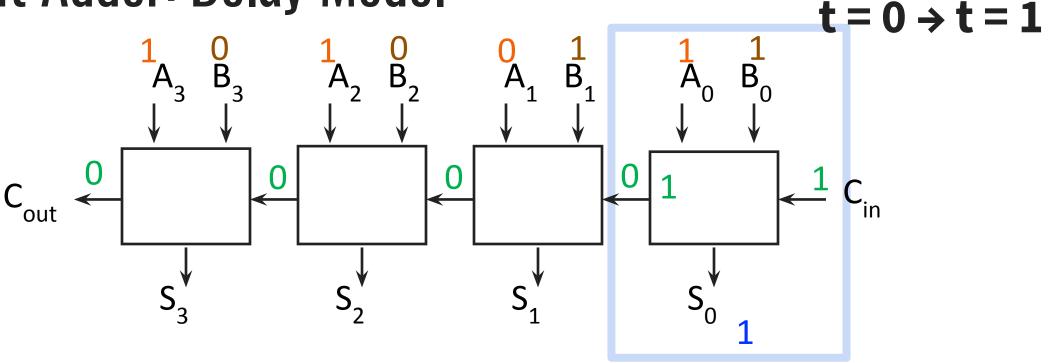


$$t=0 \rightarrow t=1$$



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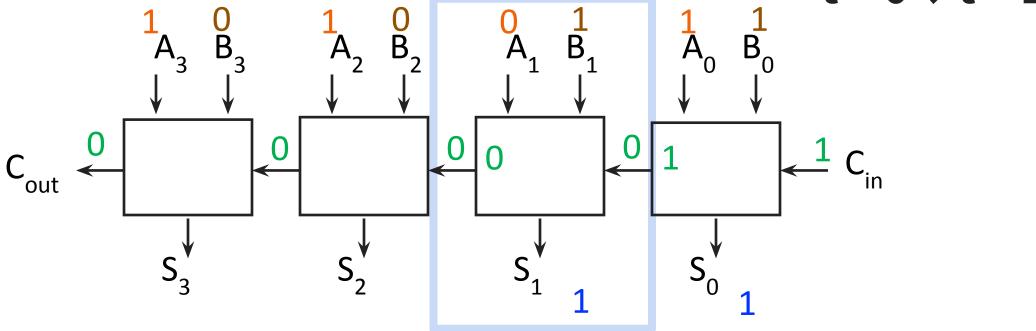




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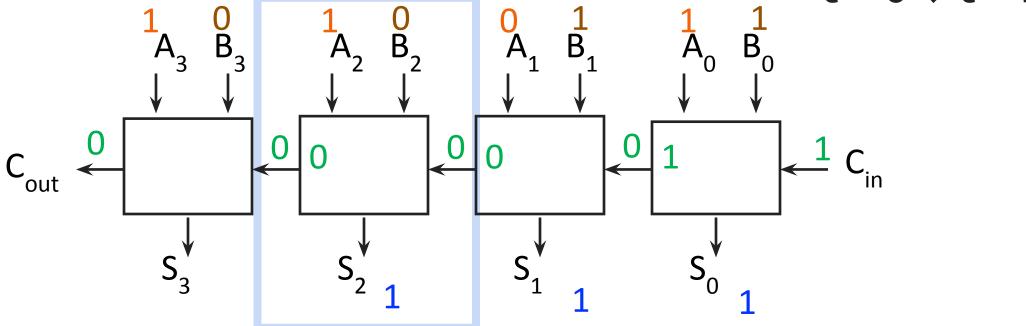
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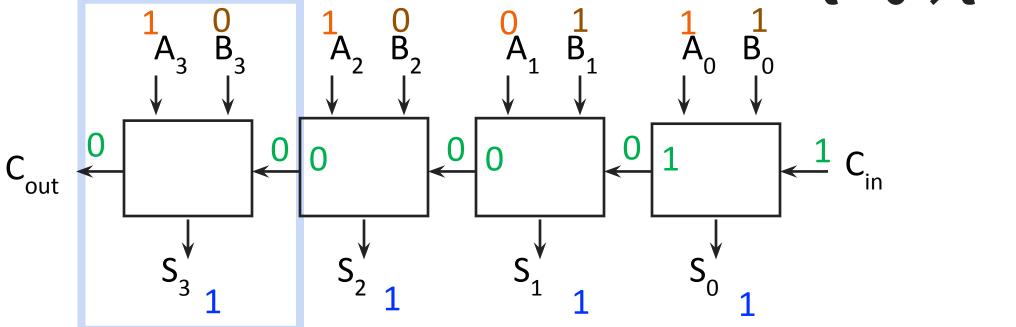
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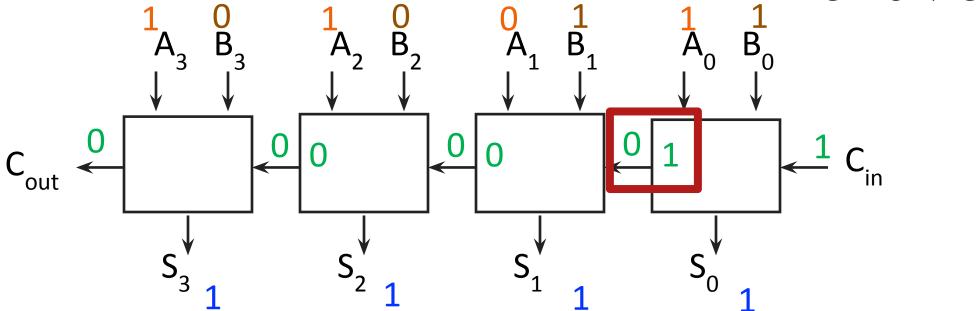
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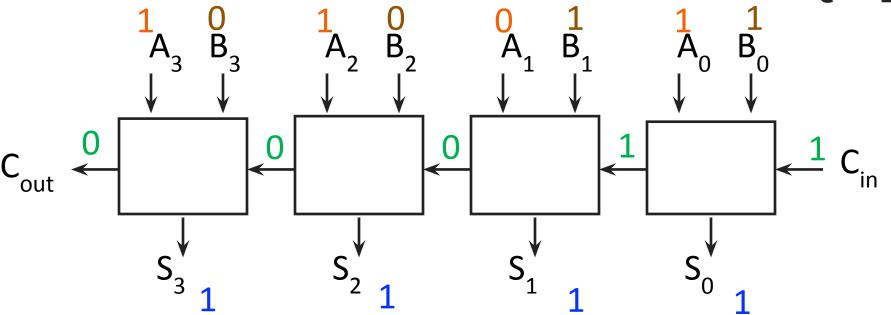
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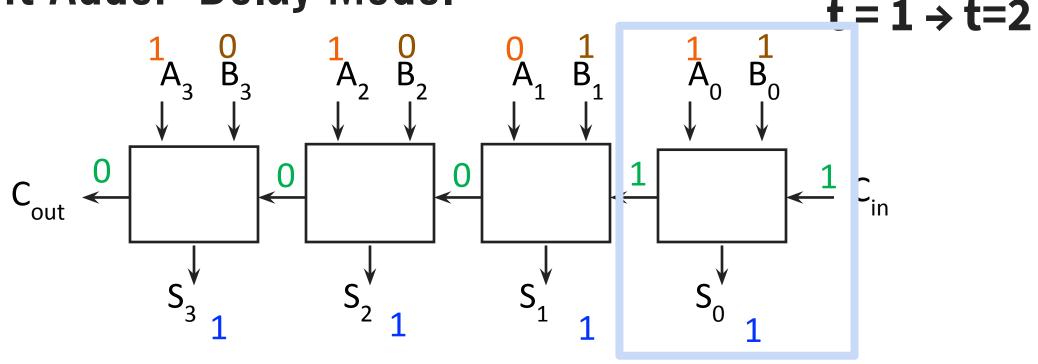


t = 1



- So far, gates compute instantaneous
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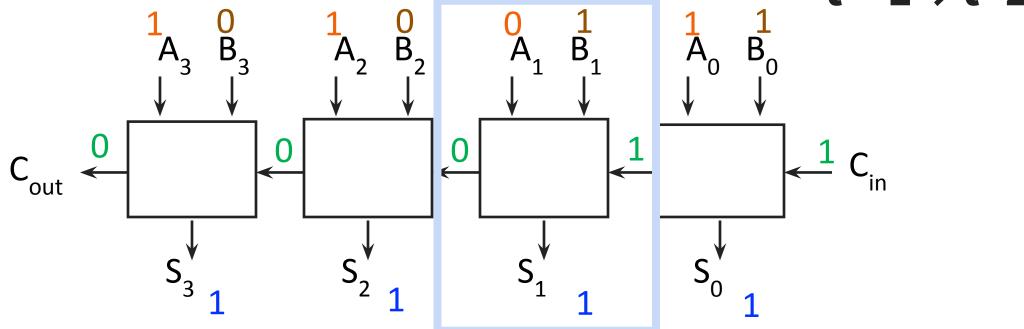


- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
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stays the same!

$$t = 1 \rightarrow t = 2$$



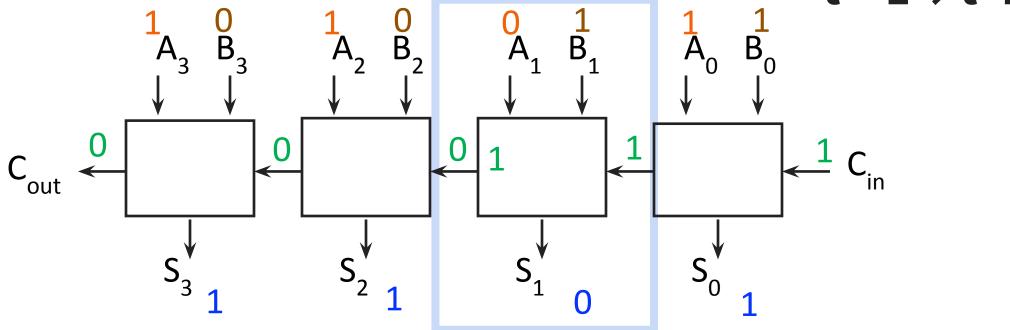
• So far, gates compute instantaneous

needs to be recomputed

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



$$t = 1 \rightarrow t = 2$$



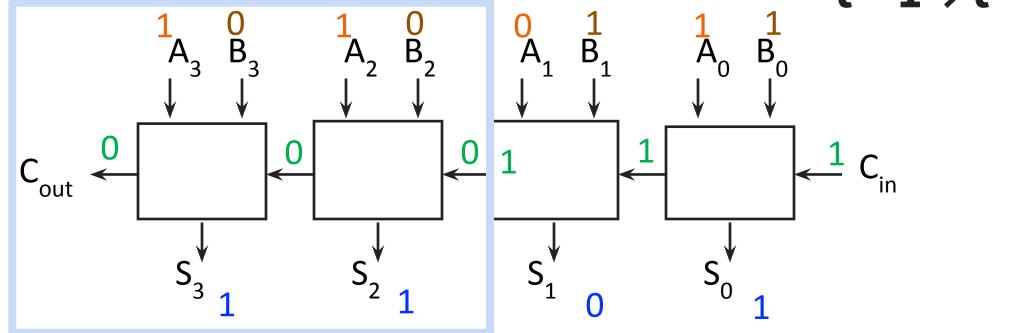
• So far, gates compute instantaneous

needs to be recomputed

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



$$t = 1 \rightarrow t = 2$$



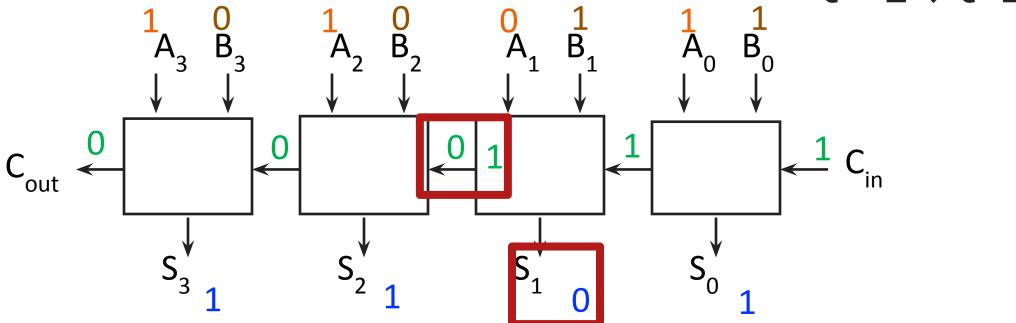
• So far, gates compute instantaneous

stays the same!

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder

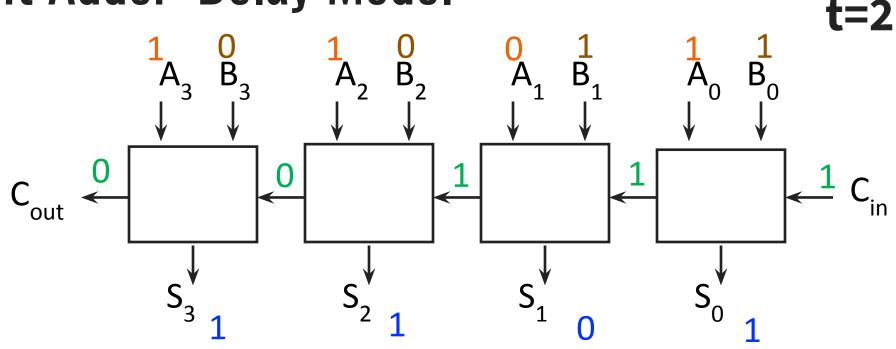


 $t = 1 \rightarrow t = 2$



- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder

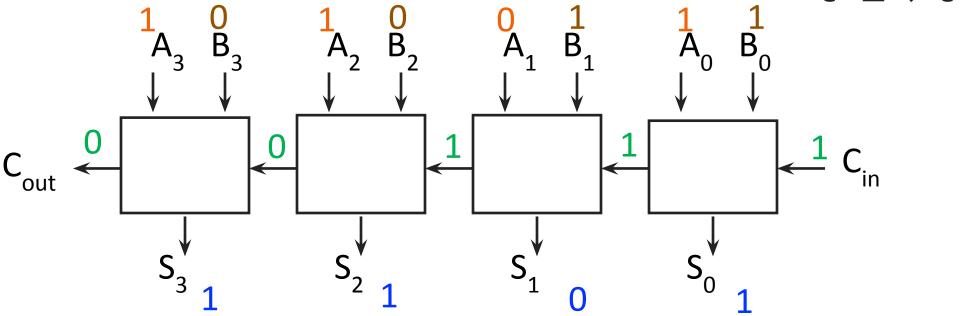




- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder

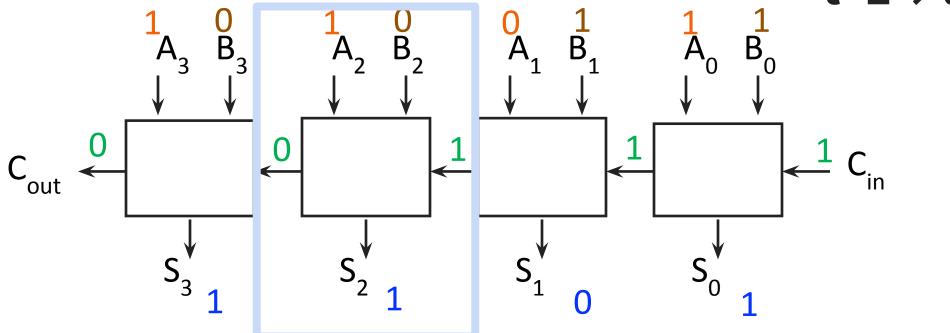


t=2 → t=3



- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder





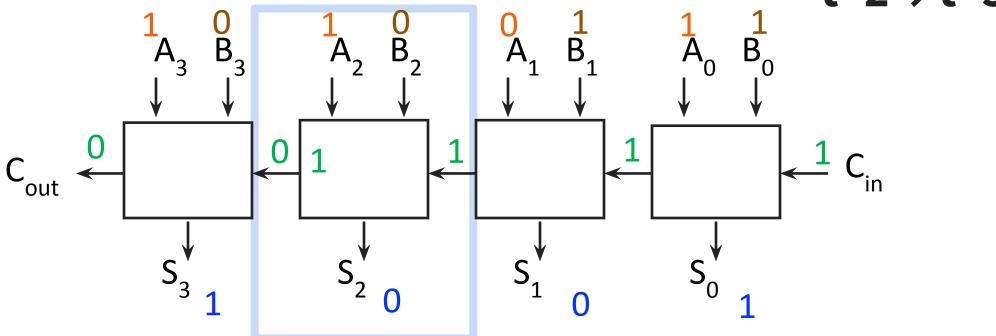
• So far, gates compute instantaneous

• In reality, there is a delay, because it takes time to compute.

needs to be recomputed

• Simple model: it takes 1 unit of time to propagate results through a 1-bit adder





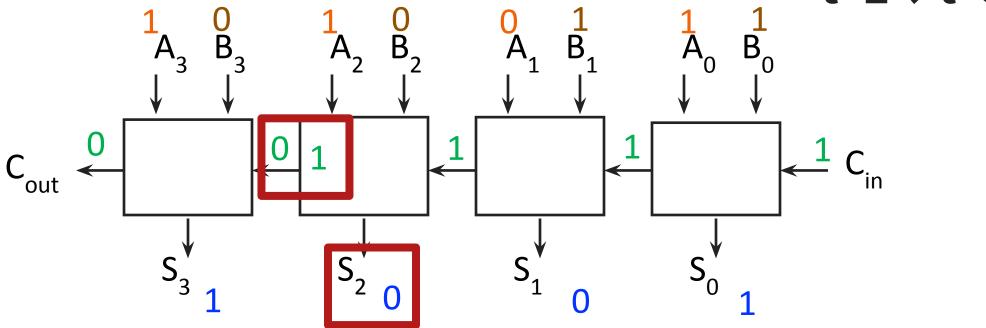
• So far, gates compute instantaneous

needs to be recomputed

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder

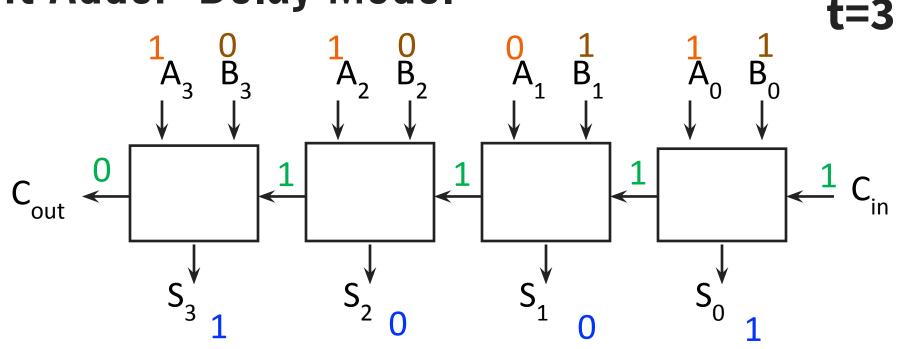


t=2 → t=3



- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder

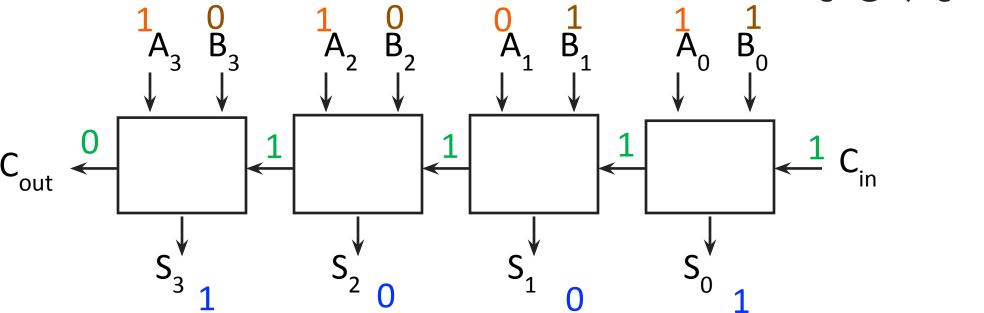




- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



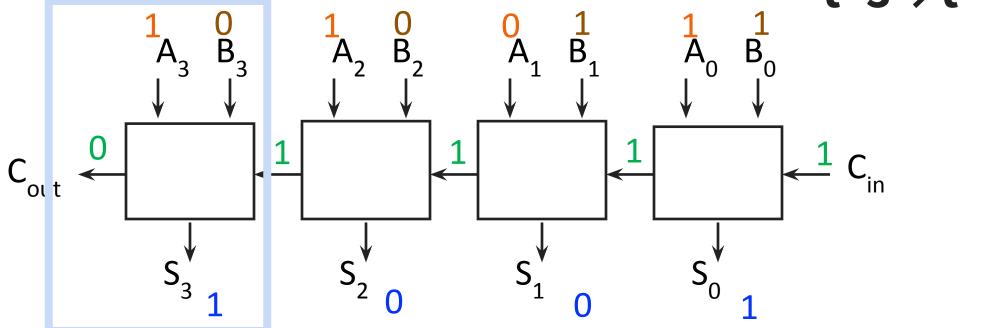
 $t=3 \rightarrow t=4$



- So far, gates compute instantaneous
- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



needs to be recomputed

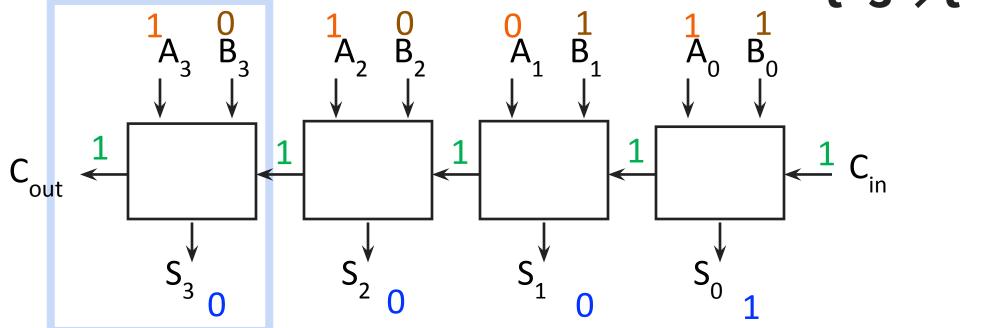


• So far, gates compute instantaneous

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



4-bit Adder: Delay Model



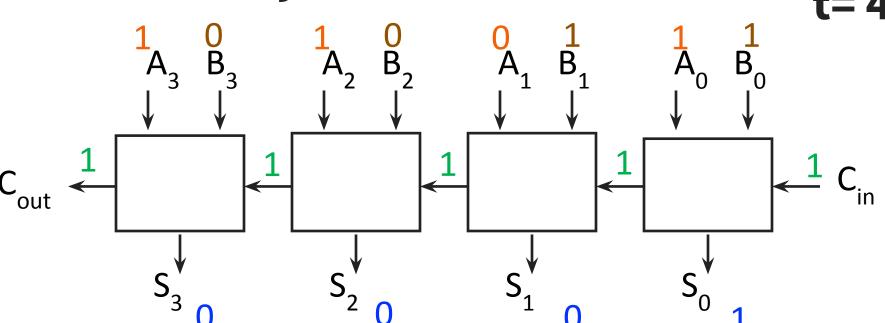
• So far, gates compute instantaneous

needs to be recomputed

- In reality, there is a delay, because it takes time to compute.
- Simple model: it takes 1 unit of time to propagate results through a 1-bit adder



4-bit Adder: Delay Model



• No more changes to the adder inputs \rightarrow we reached a fixed point.

	Cout	S
t = 0	?	?
t = 1	0	1111 ₂
t = 2	0	1101 ₂
t = 3	0	1001 ₂
t = 4	1	00012
t = 5	1	00012



	Cout	S
t = 0	?	?
t = 1	0	111 1 ₂
t = 2	0	11 01 ₂
t = 3	0	1 001 ₂
t = 4	1	0001
t = 5	1	0001

- Lower bits are ready first.
- The correct value is only available after 4 time units.

	Cout	S
t = 0	?	?
t = 1	0	111 1 ₂
t = 2	0	11 01 ₂
t = 3	0	1 001 ₂
t = 4	1	0001
t = 5	1	0001

- Lower bits are ready first.
- The correct value is only available after 4 time units.
- In reality, the delays are more complicated than our model.

	Cout	S
t = 0	?	?
t = 1	0	111 1 ₂
t = 2	0	11 01 ₂
t = 3	0	1 001 ₂
t = 4	1	0001 ₂
t = 5	1	0001 ₂

- Lower bits are ready first.
- The correct value is only available after 4 time units.
- In reality, the delays are more complicated than our model.
- We assumed that all inputs to the
 4-bit adder arrive at t = 0.

	Cout	S
t = 0	?	?
t = 1	0	111 1 ₂
t = 2	0	11 01 ₂
t = 3	0	1 001 ₂
t = 4	1	0001 ₂
t = 5	1	0001 ₂

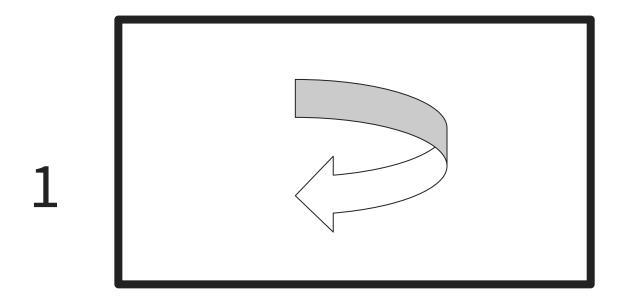
- Lower bits are ready first.
- The correct value is only available after 4 time units.
- In reality, the delays are more complicated than our model.
- We assumed that all inputs to the
 4-bit adder arrive at t = 0.
- Need state to synchronize.

How can we store information in a binary circuit?

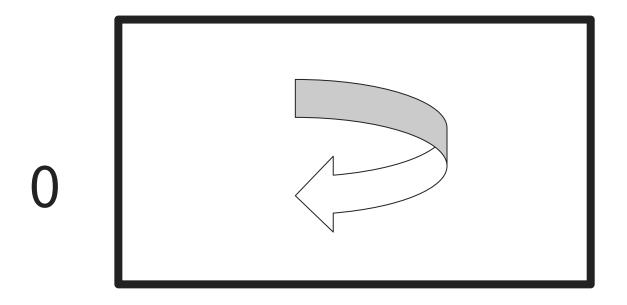




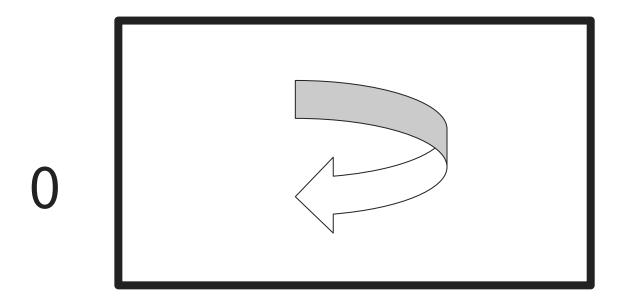




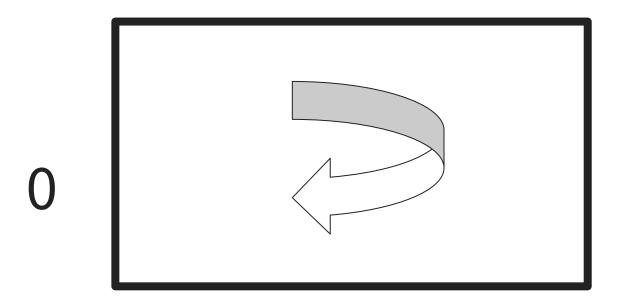








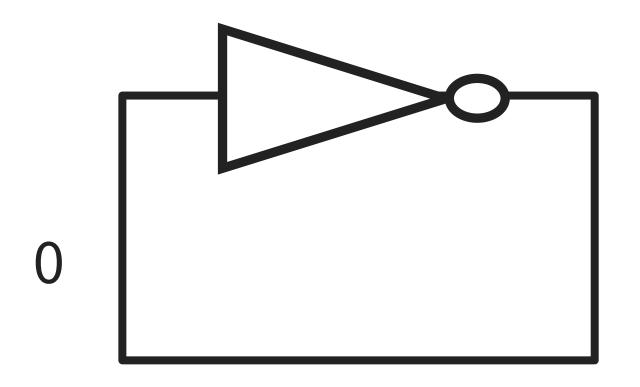
We are storing a charge. This is how your DRAM main memory works.



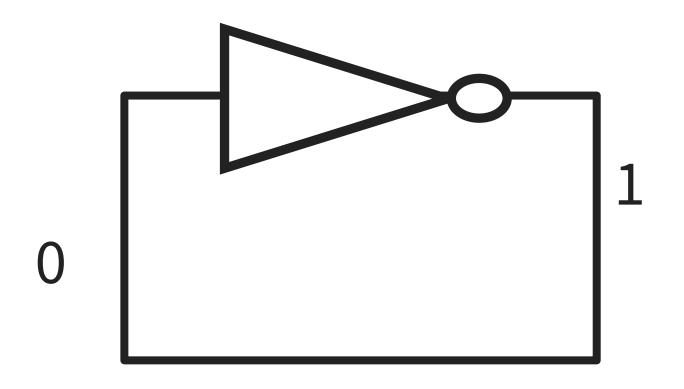
We are storing a charge. This is how your DRAM main memory works.

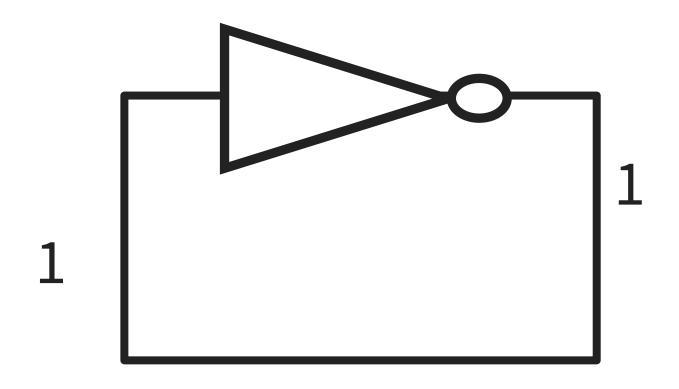
Problem: Charge disappears over time.



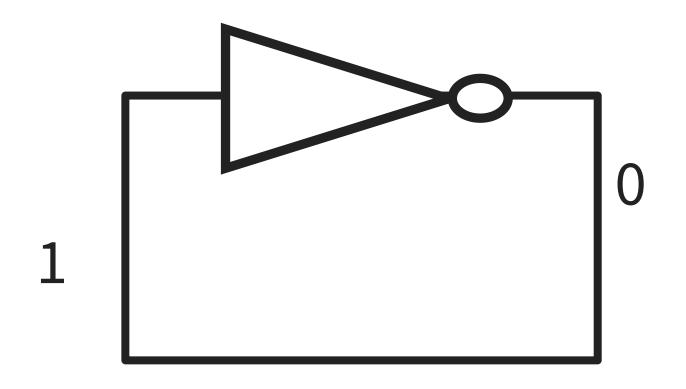


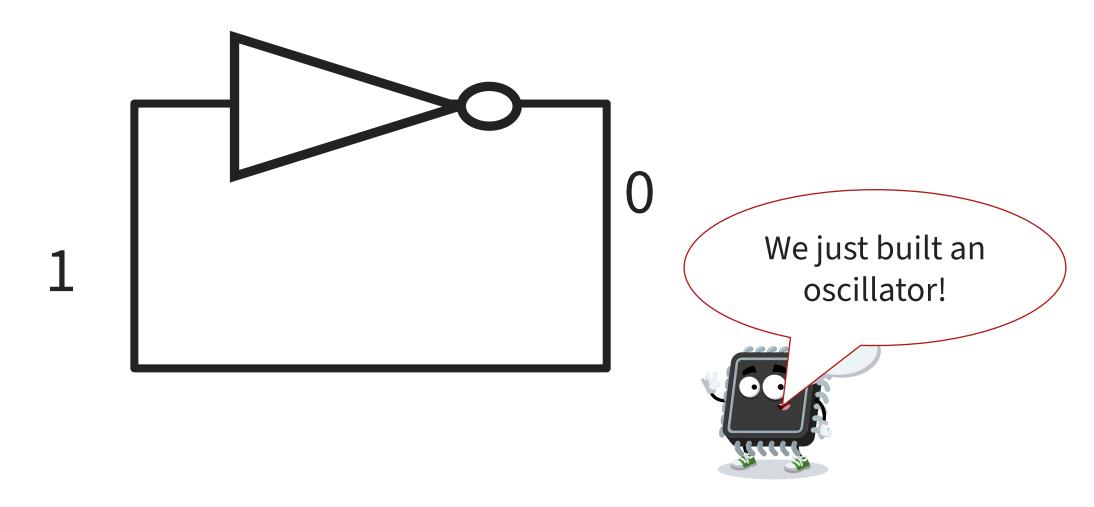




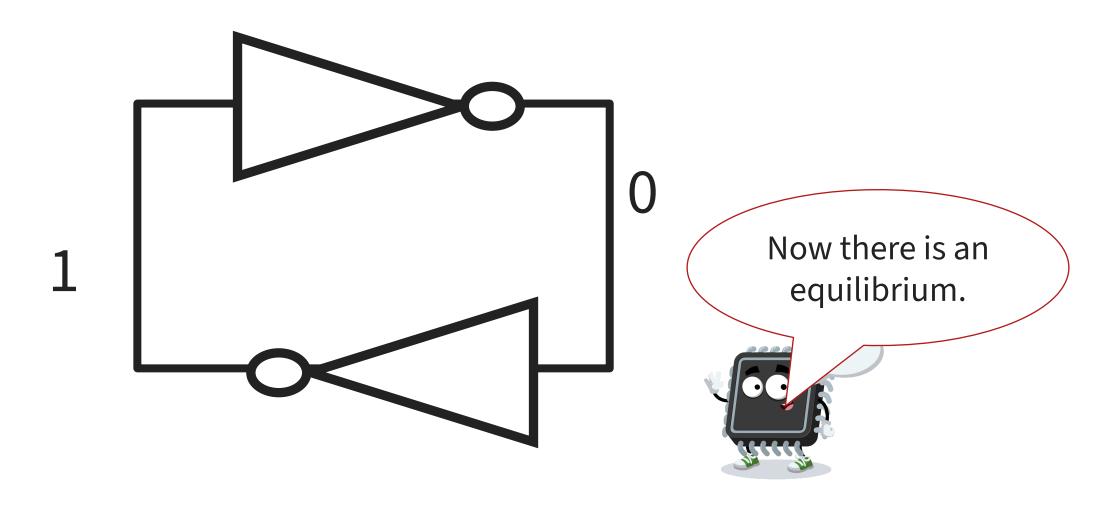




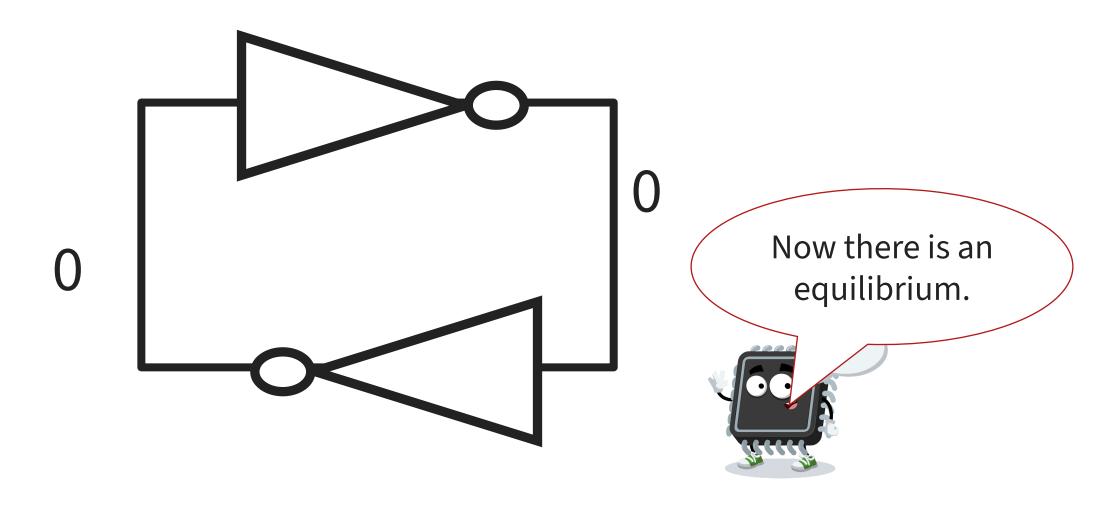




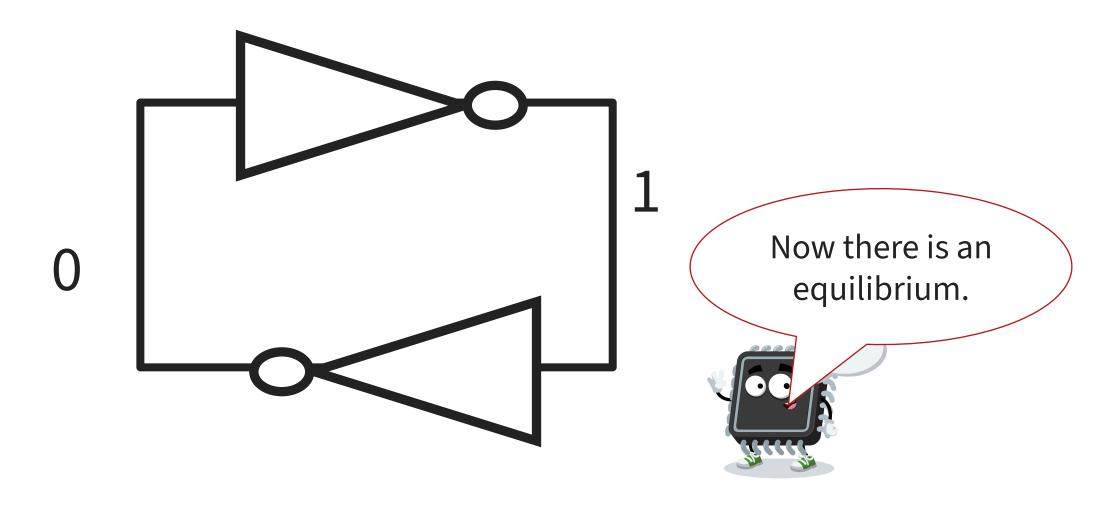




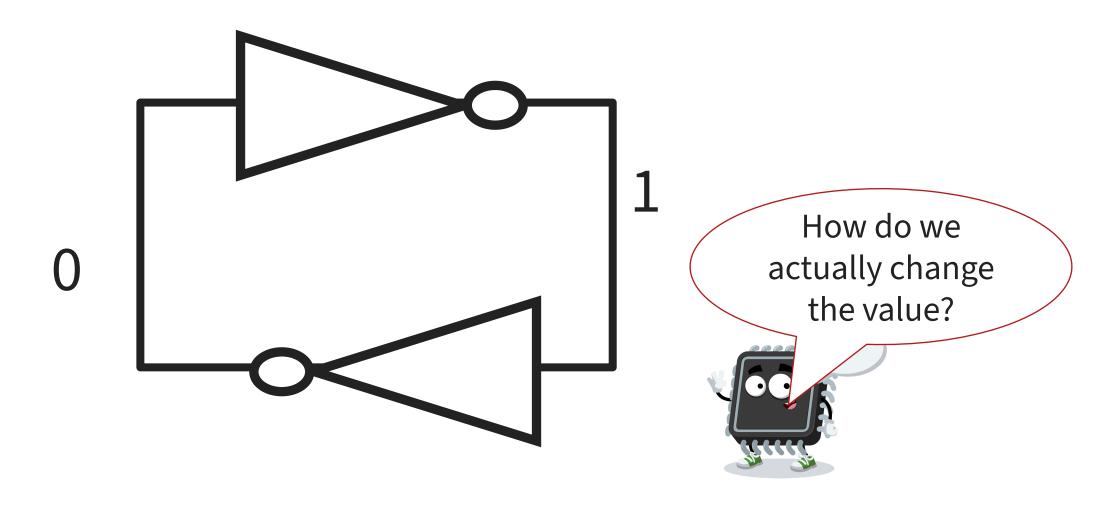




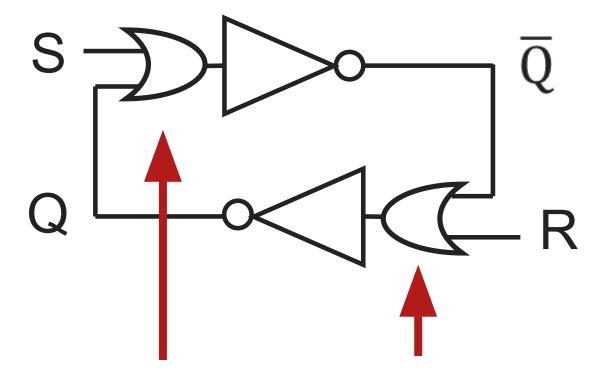






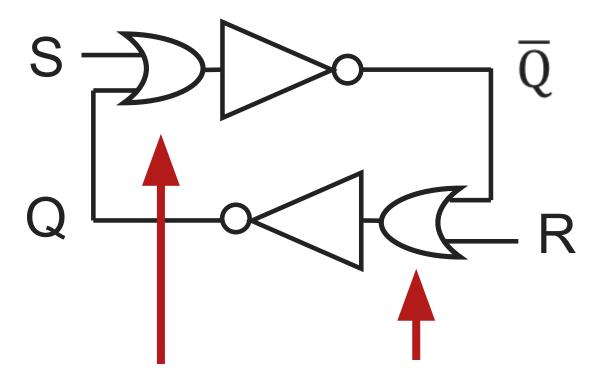






Add two OR gates.

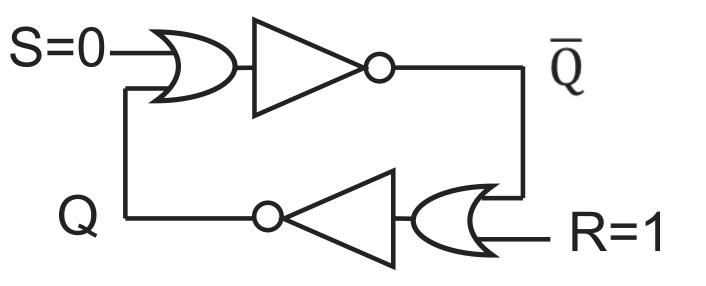




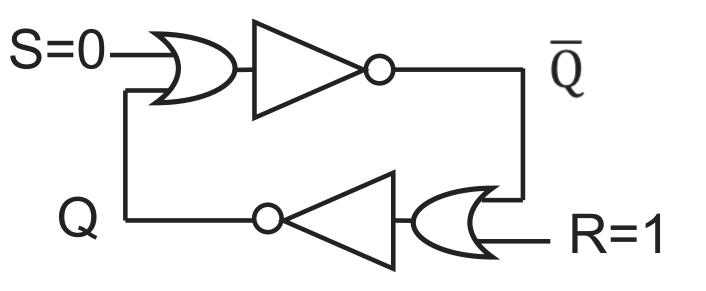
Add two OR gates.

S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		





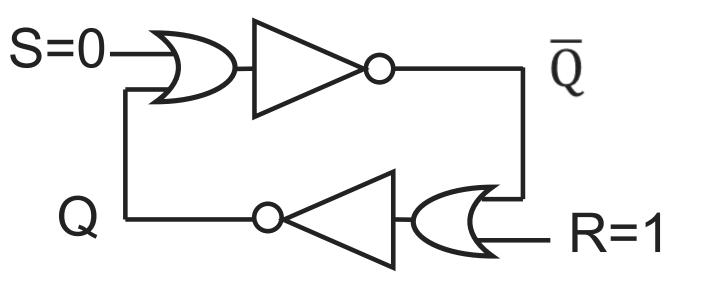
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		



S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Where do we start our analysis?

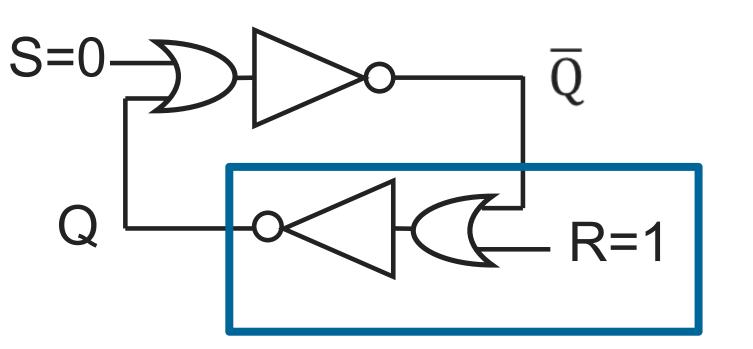




S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Where do we start our analysis?

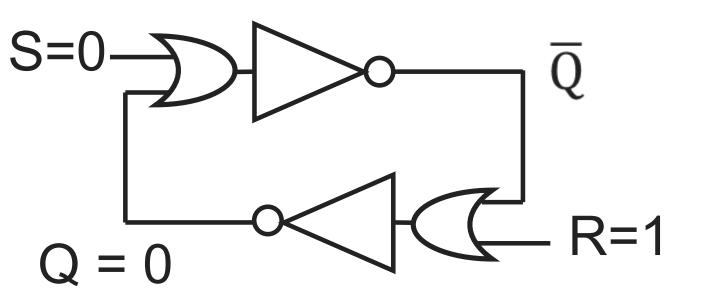




S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Where do we start our analysis?

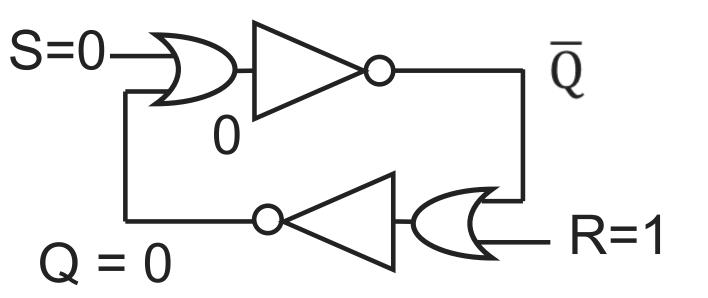




S	R	Q	$\neg Q$
0	0		
0	1	0	
1	0		
1	1		

Where do we start our analysis?

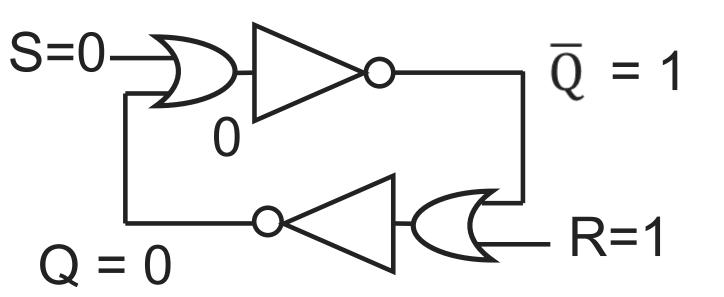




S	R	Q	¬Q
0	0		
0	1	0	
1	0		
1	1		

Where do we start our analysis?

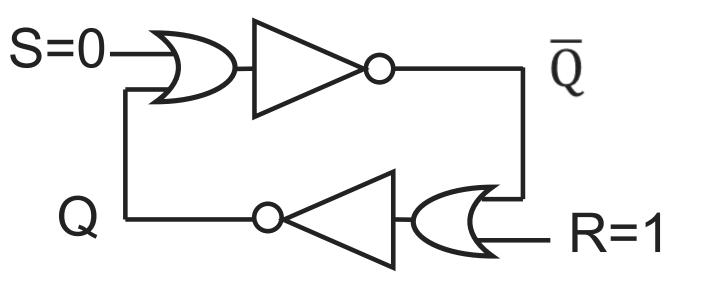




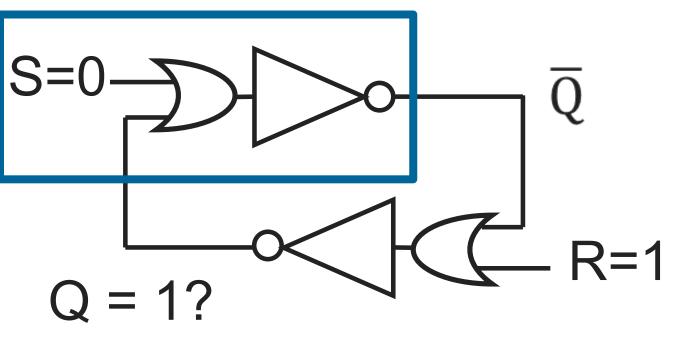
S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		

Where do we start our analysis?

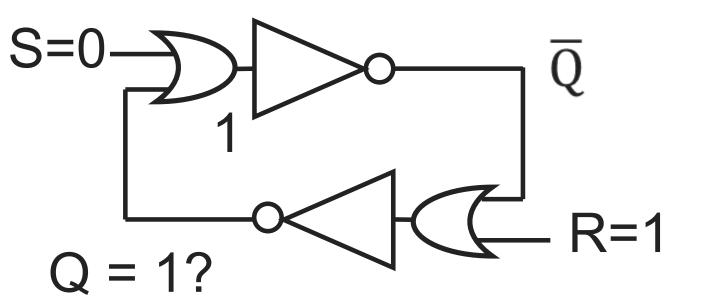




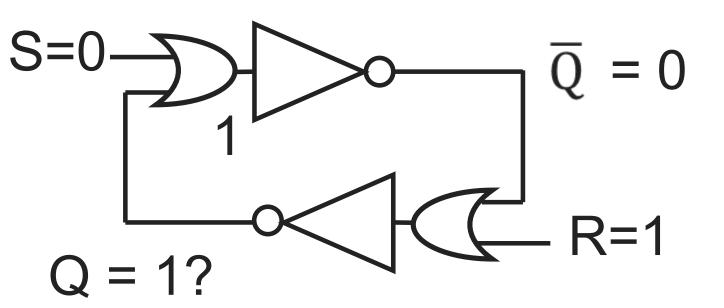
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		



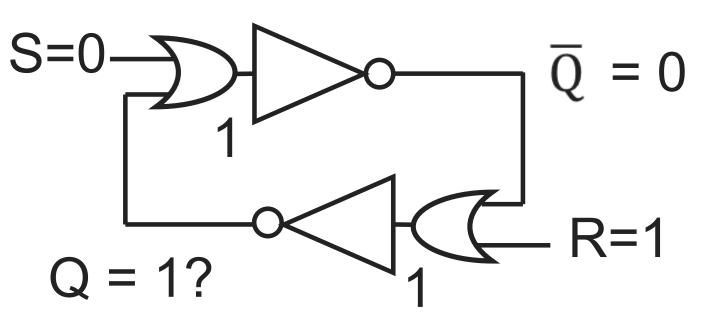
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		



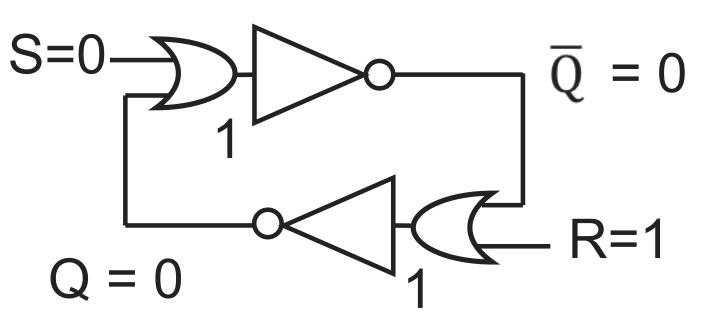
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		



S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

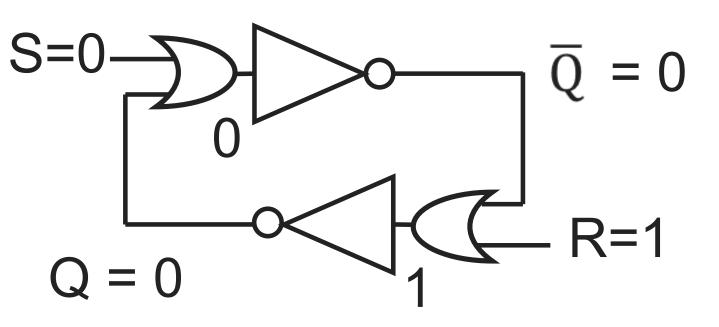


S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		



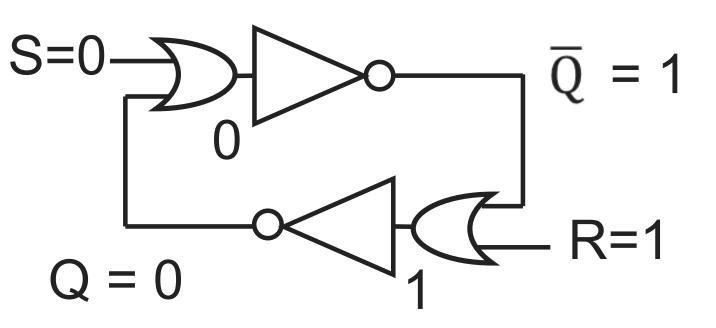
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Alternative: Just guess!



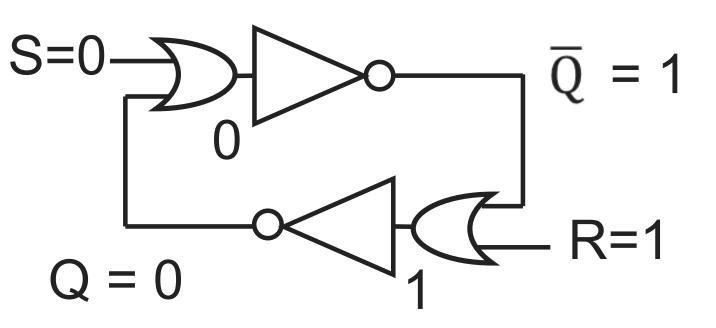
S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Alternative: Just guess!



S	R	Q	$\neg Q$
0	0		
0	1		
1	0		
1	1		

Alternative: Just guess!



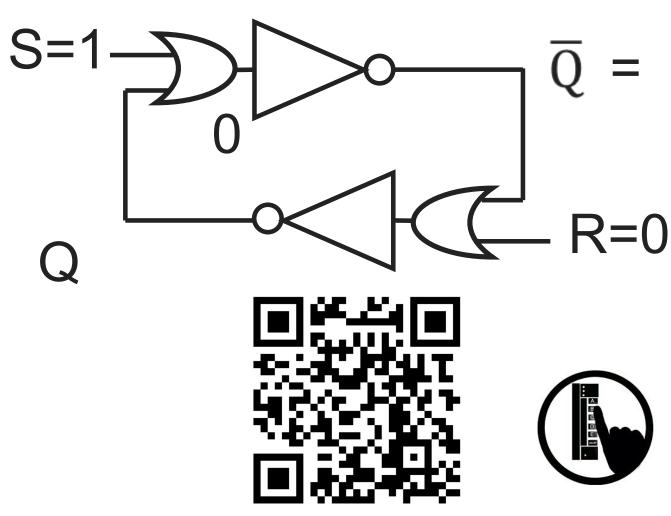
S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		

Alternative: Just guess!

S = 0, R = 1 is a **stable** state.

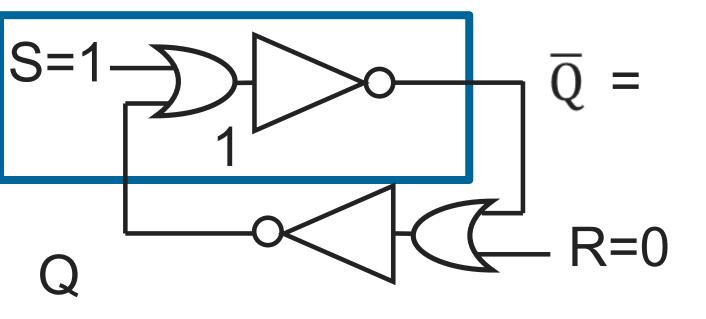
We always converge to the same state.





S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		

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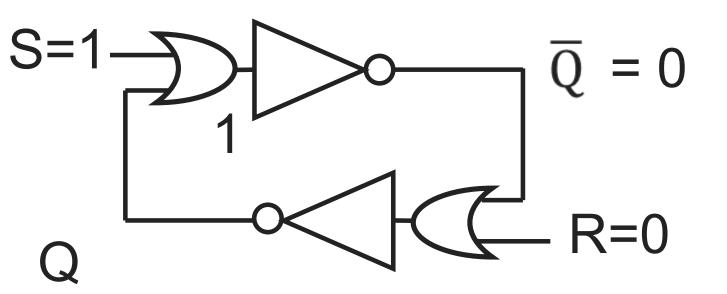


S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		

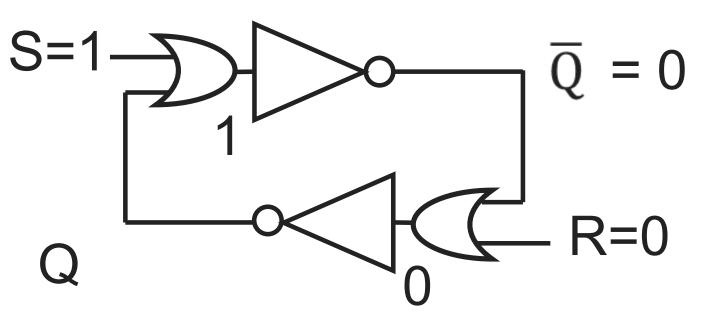
Where do we start our analysis?

Remember: $1 \lor a = 1$

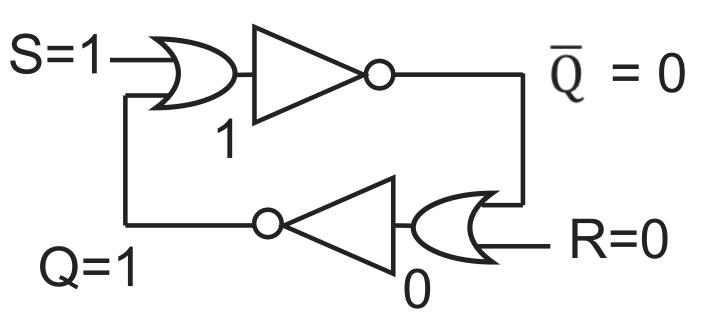




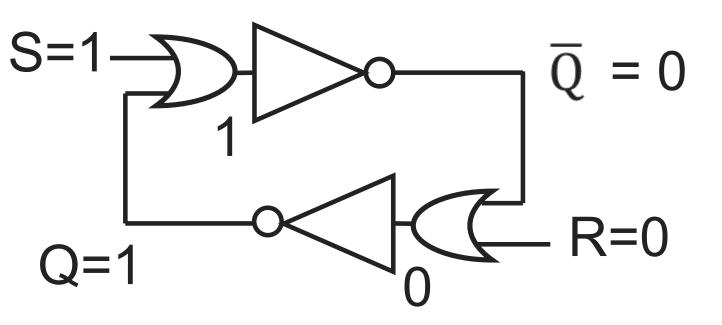
S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		



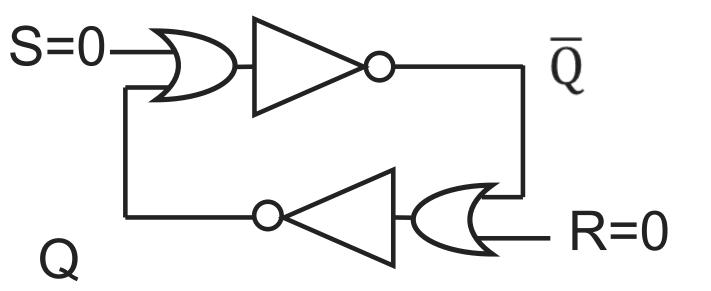
S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		



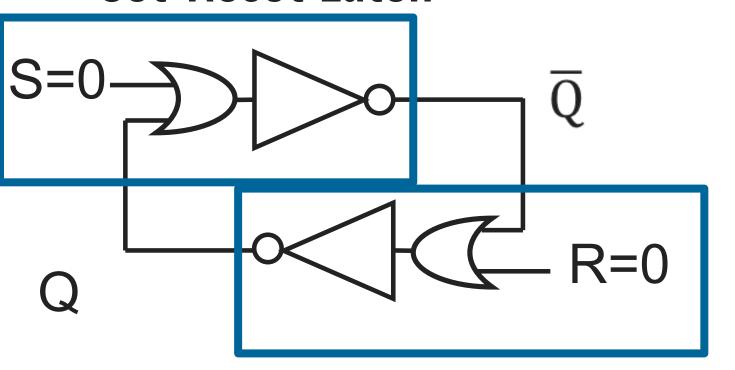
S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0		
1	1		



S	R	Q	$\neg Q$
0	0		
0	1	0	1
1	0	1	0
1	1		

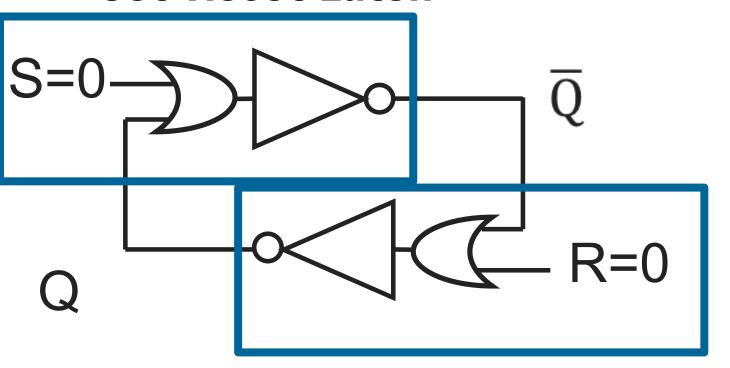


S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		



S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		

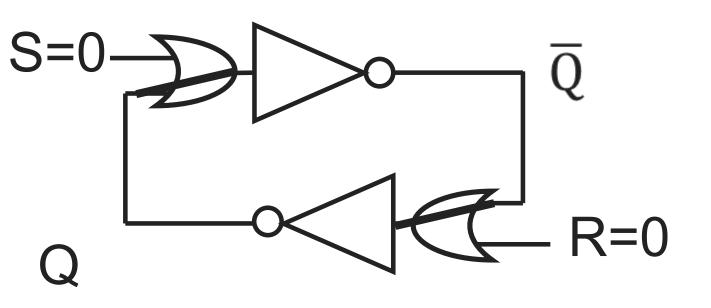
Our "OR trick" no longer works.



S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		

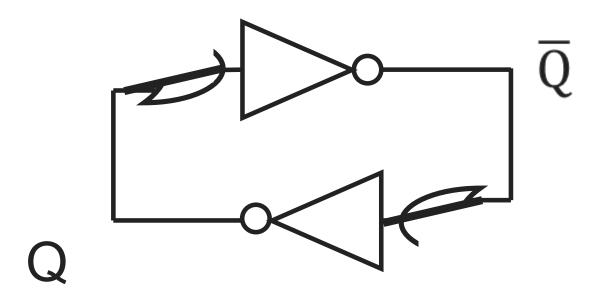
Our "OR trick" no longer works. Remember: 0 \(\nabla \) a = a





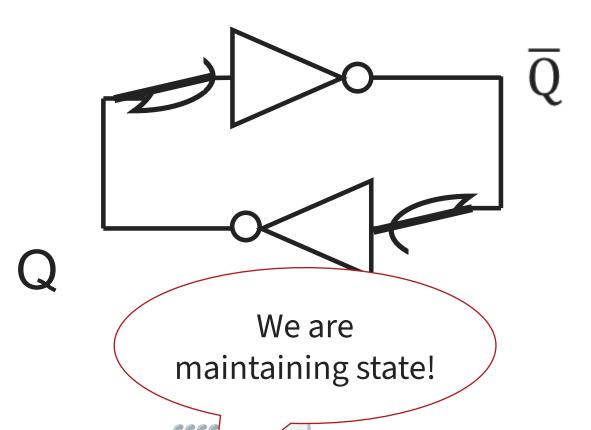
S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		

Our "OR trick" no longer works. Remember: 0 \(\nabla \) a = a



S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		

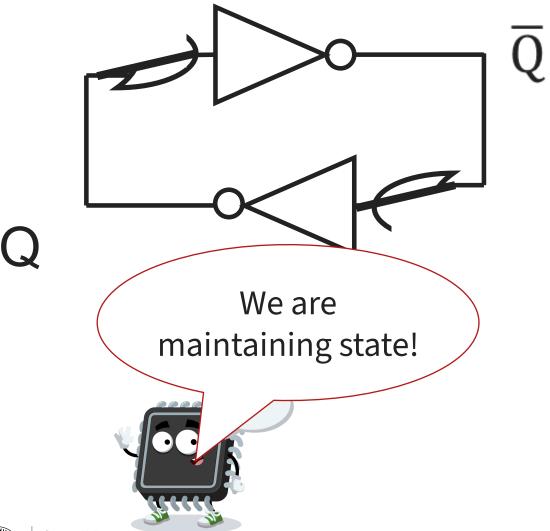
Our "OR trick" no longer works. Remember: 0 \(\nabla \) a = a



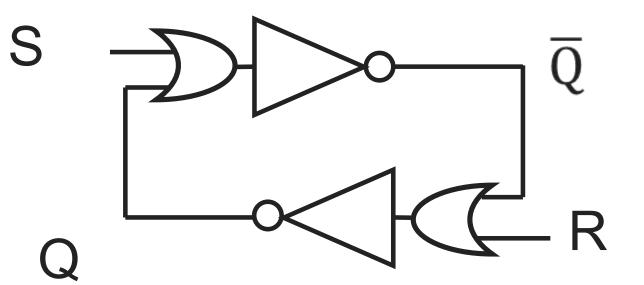
S	R	Q	¬Q
0	0		
0	1	0	1
1	0	1	0
1	1		



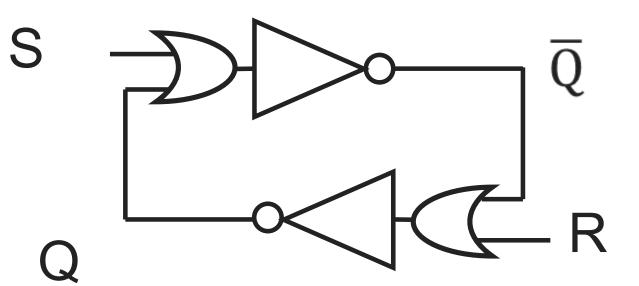




S	R	Q	¬Q
0	0	Q_{t-1}	$\neg Q_{t-}$
			1
0	1	0	1
1	0	1	0
1	1		



S	R	Q	¬Q
0	0	Q _{t-1}	$\neg Q_{t-}$
			1
0	1	0	1
1	0	1	0
1	1		

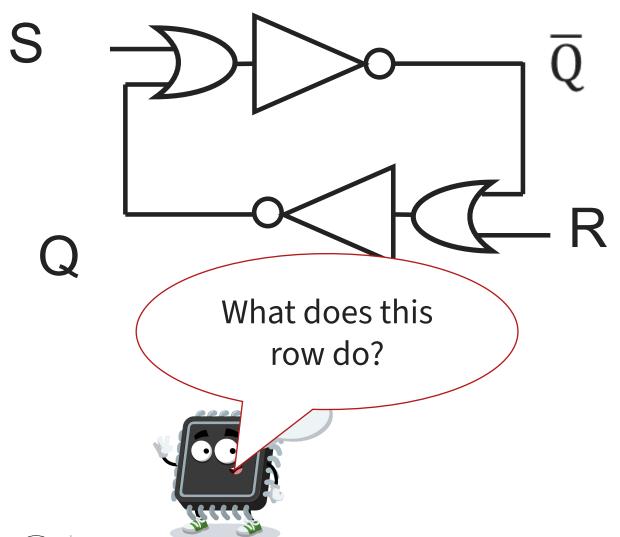


S	R	Q	¬Q
0	0	Q _{t-1}	$\neg Q_{t-}$
			1
0	1	0	1
1	0	1	0
1	1		

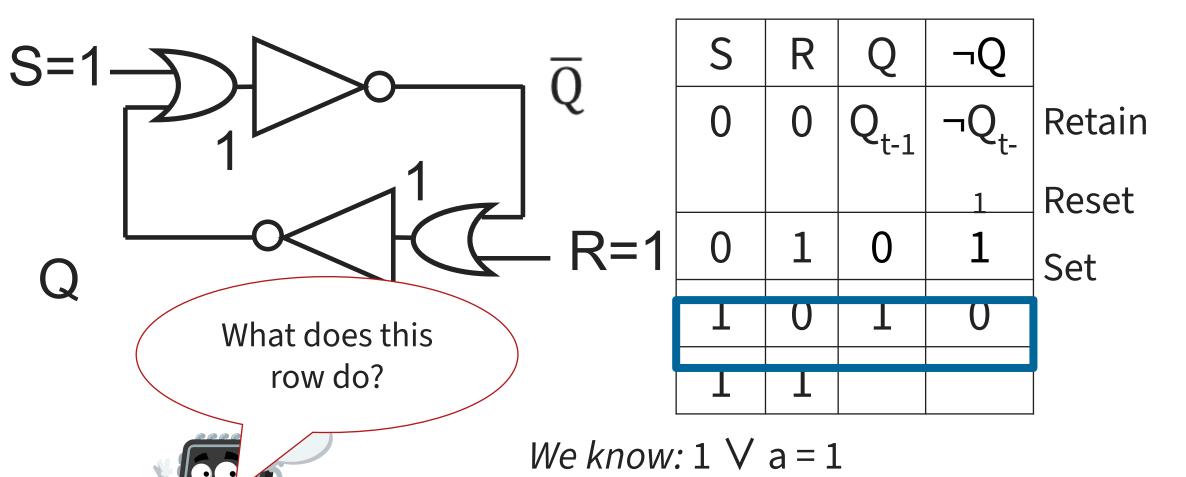
Retain

Reset

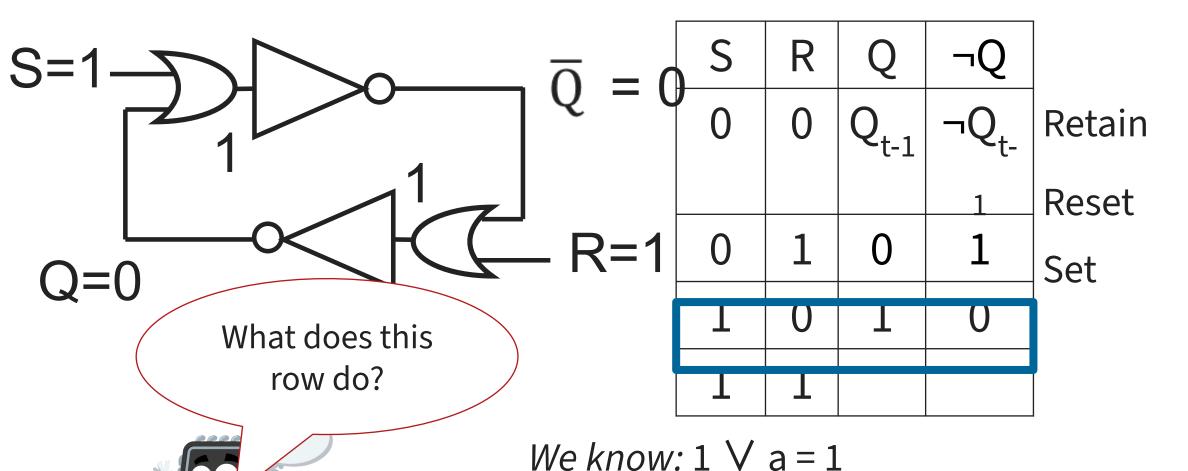
Set



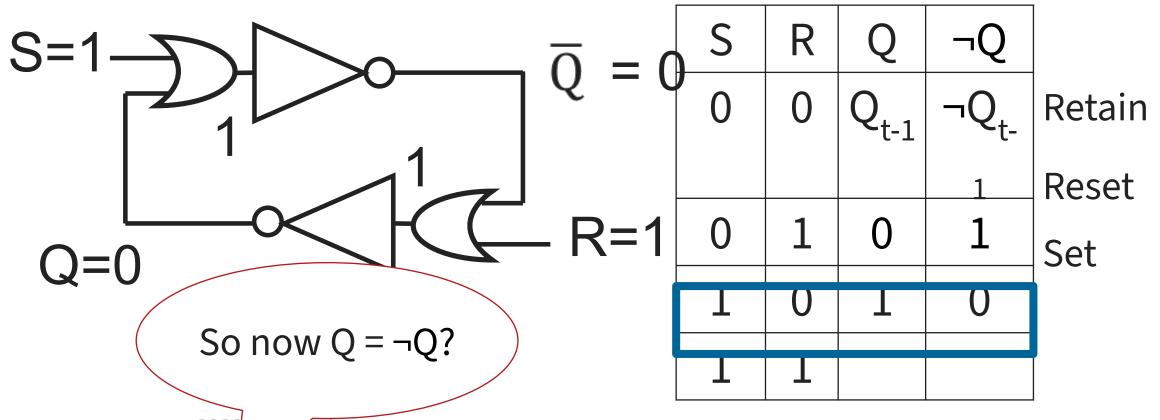
S	R	Q	¬Q	
0	0	Q_{t-1}	$\neg Q_{t-}$	Retain
			1	Reset
0	1	0	1	Set
1	U	1	O	
I	1			



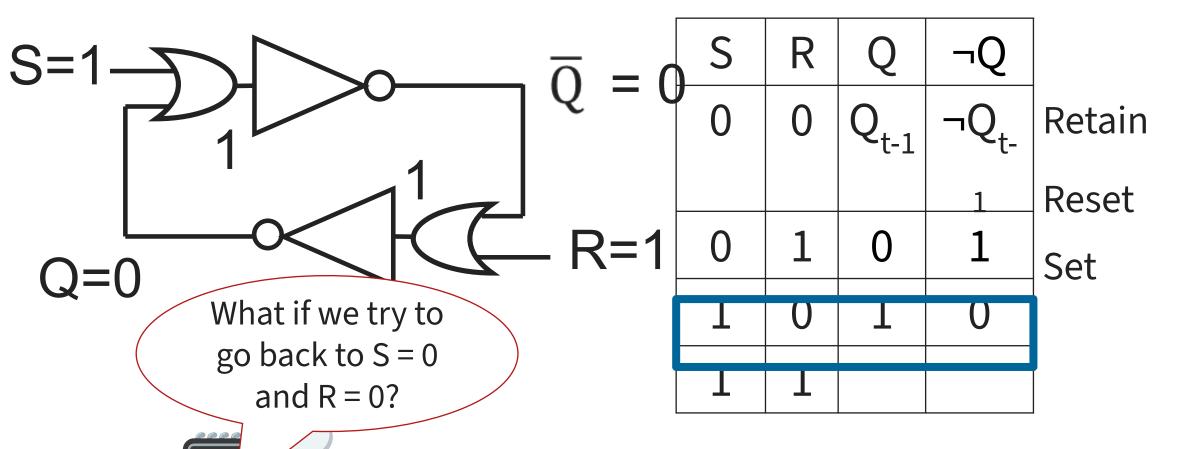


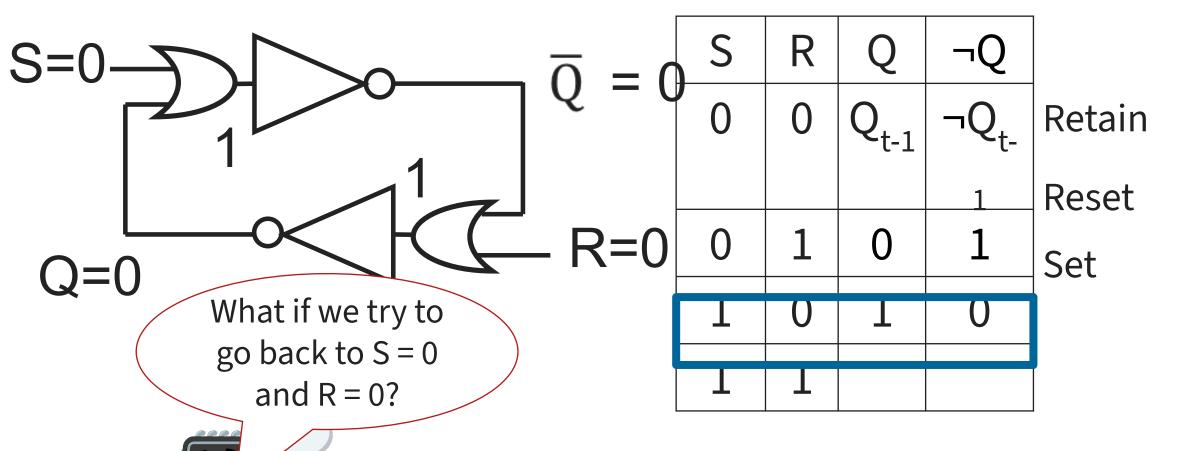




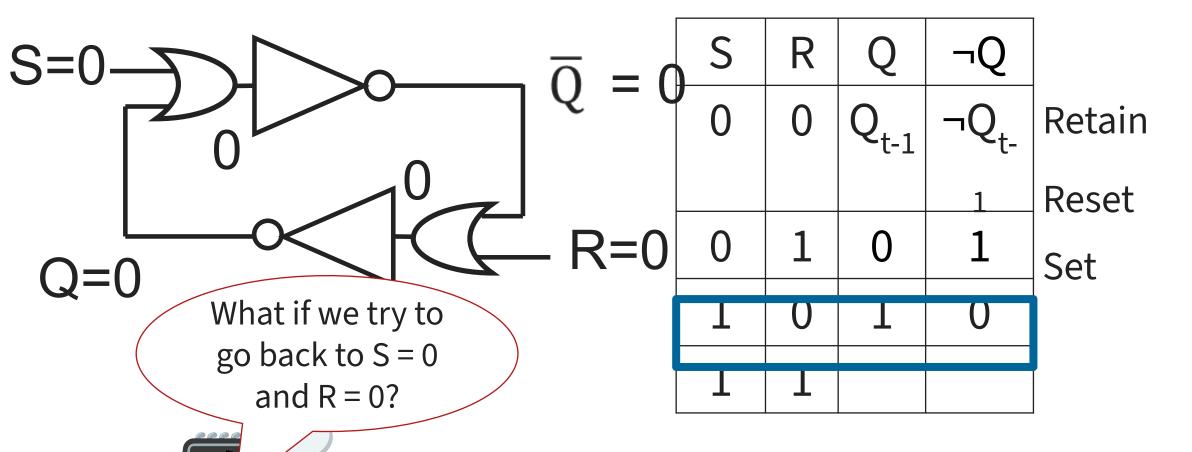


We know: $1 \lor a = 1$

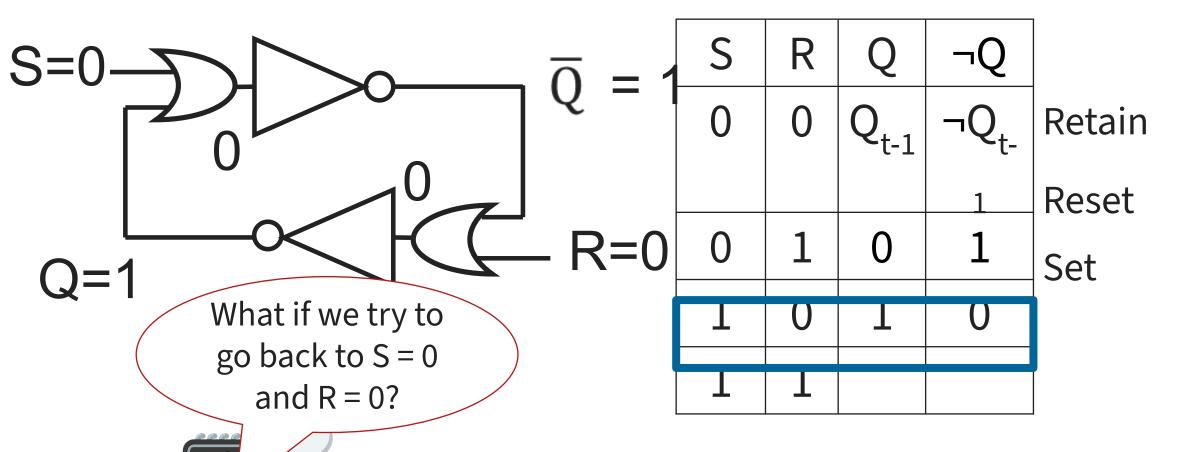


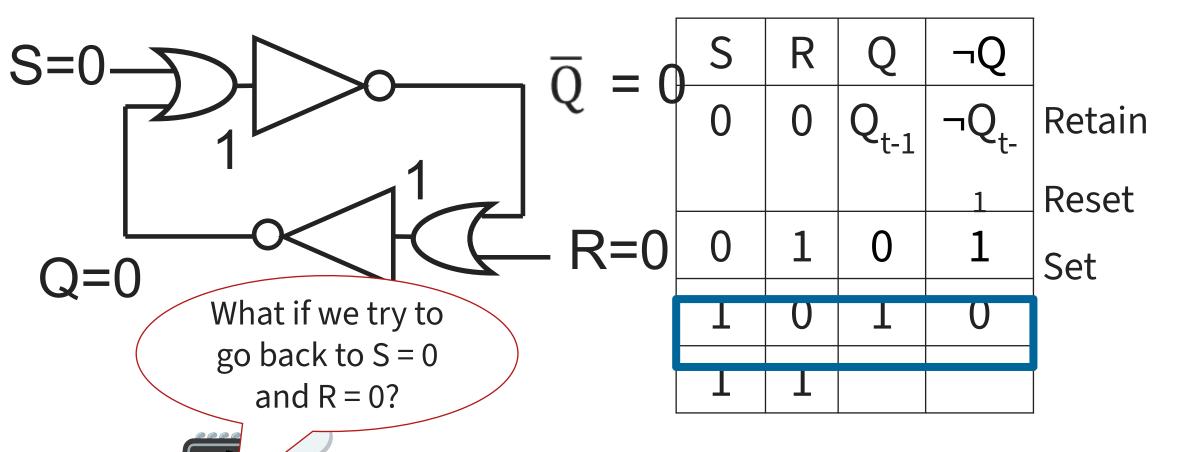




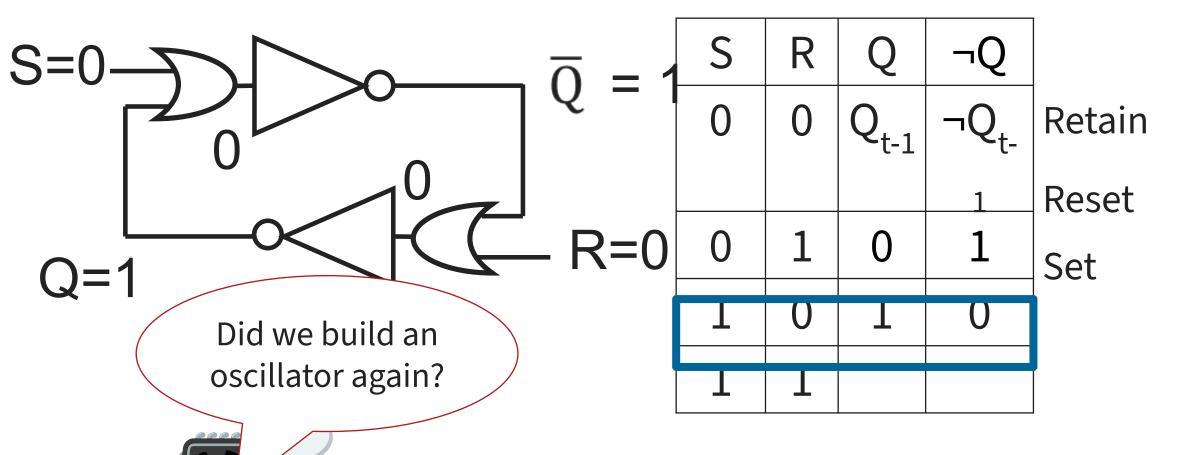




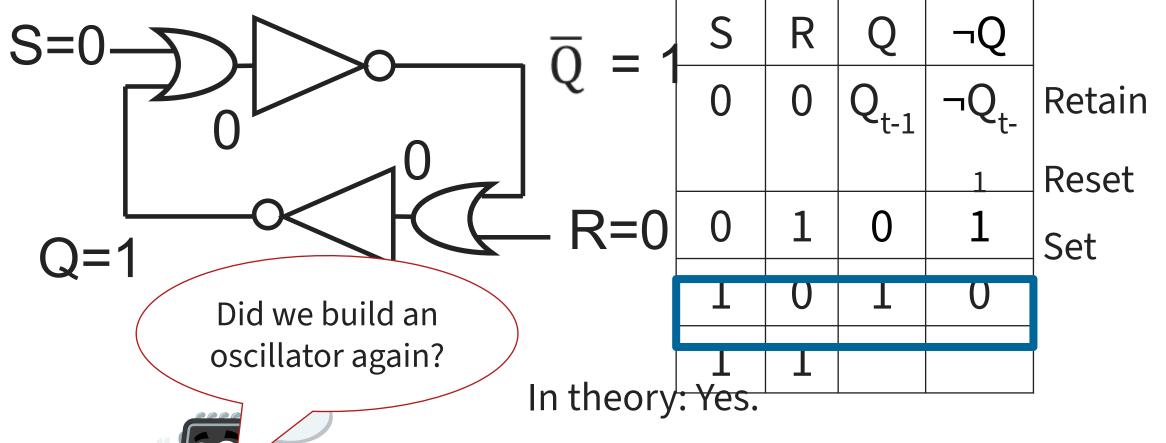






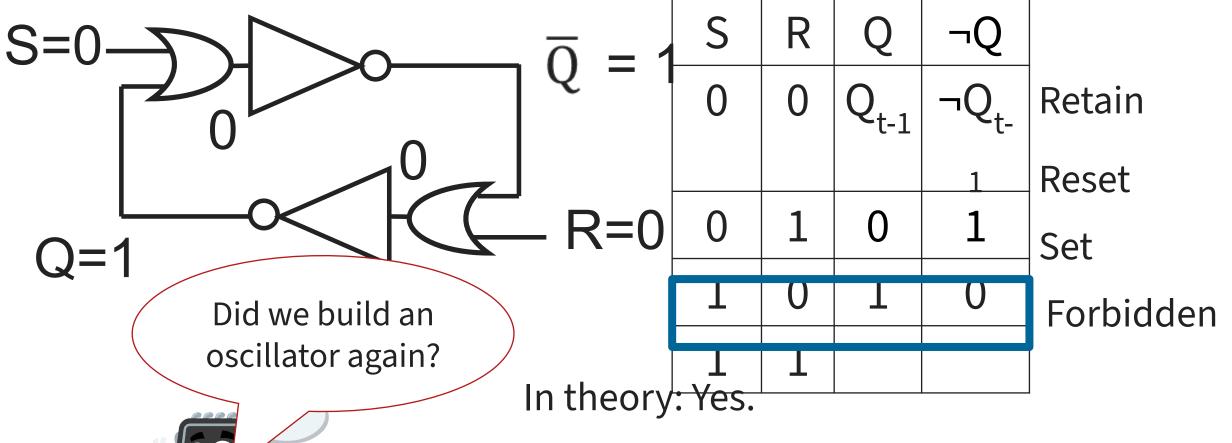








In practice: we will end up with stable feedback, but unpredictable if Q = 0 or Q = 1.

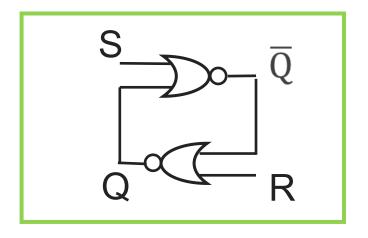




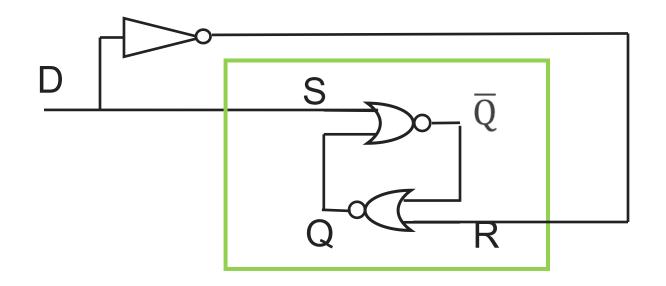
In practice: we will end up with stable feedback, but unpredictable if Q = 0 or Q = 1.

Next Goal

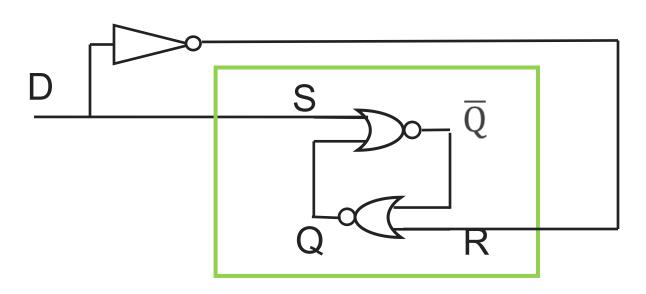
How do we avoid the forbidden state of S-R Latch?



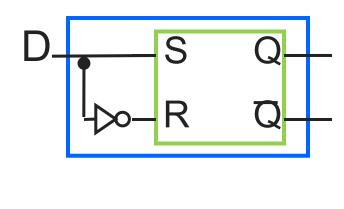




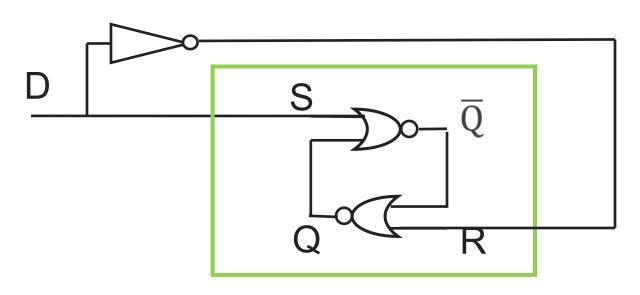




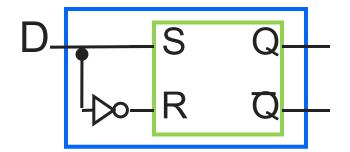
Fill in the truth table?



D	Q	
0		
1		

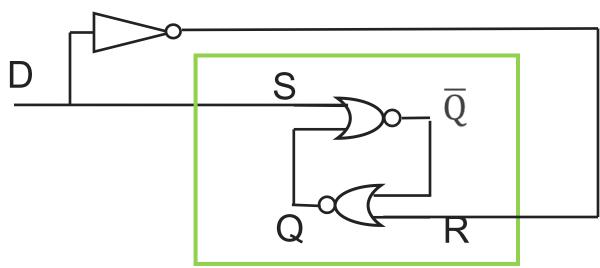


Fill in the truth table?



D	Q	¬Q
0	0	1
1	1	0

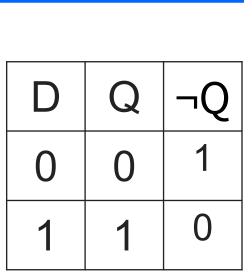
Fourth Attempt: (Unclocked) D Latch



Cannot enter an undefined state

When D changes, Q changes

... immediately (...after a delay of 2 Ors and 2 NOTs)



We aren't really storing anything anymore!



Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

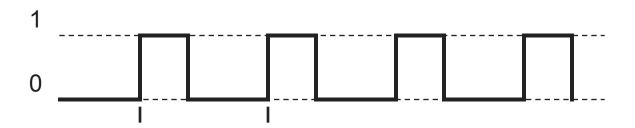
(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding the forbidden state.



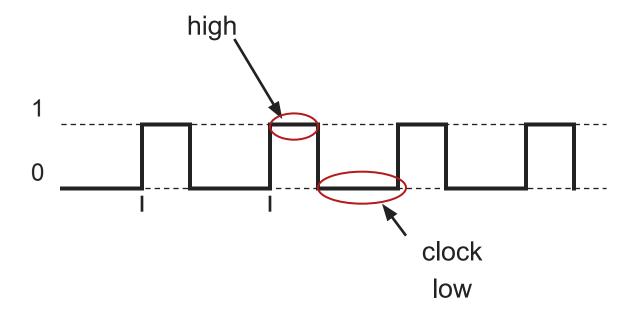
Next Goal

How do we coordinate state changes to a D Latch?

- Usually generated by an oscillating crystal
- Fixed period
- Frequency = 1/period

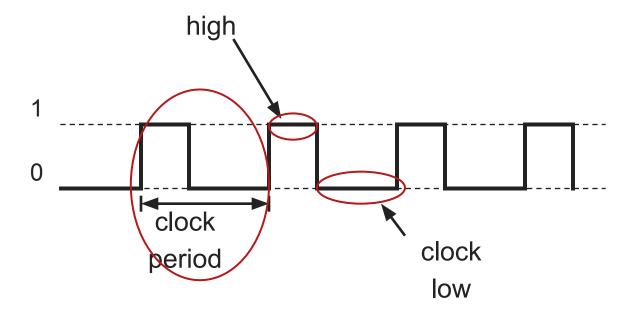


- Usually generated by an oscillating crystal
- Fixed period
- Frequency = 1/period

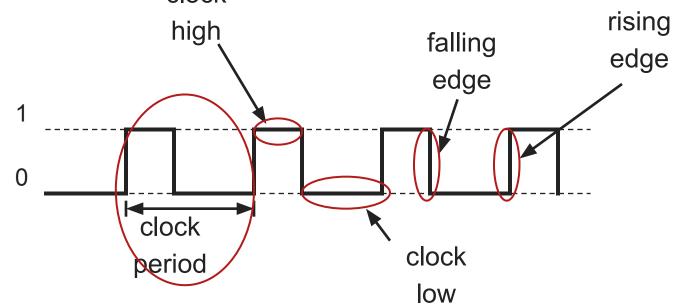




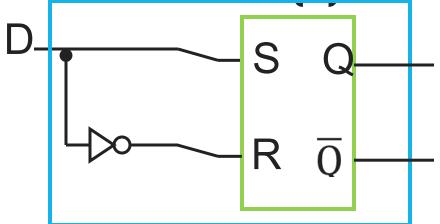
- Usually generated by an oscillating crystal
- Fixed period
- Frequency = 1/period

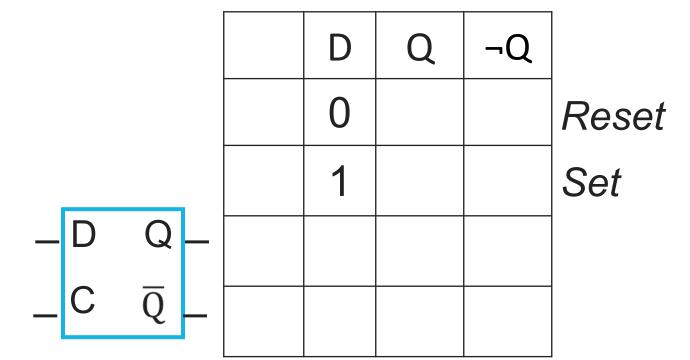


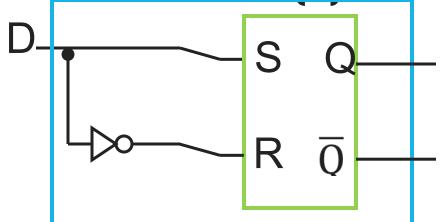
- Usually generated by an oscillating crystal
- Fixed period
- Frequency = 1/period

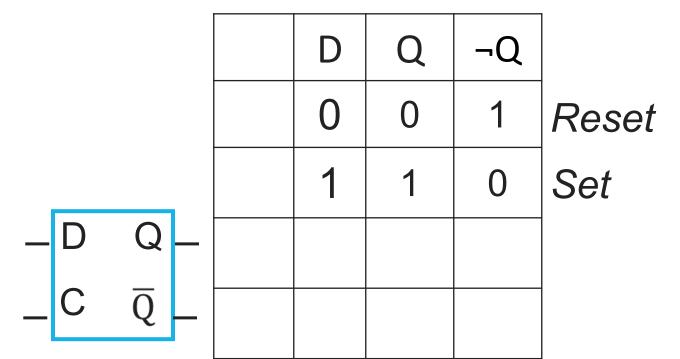


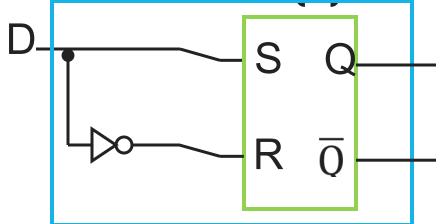


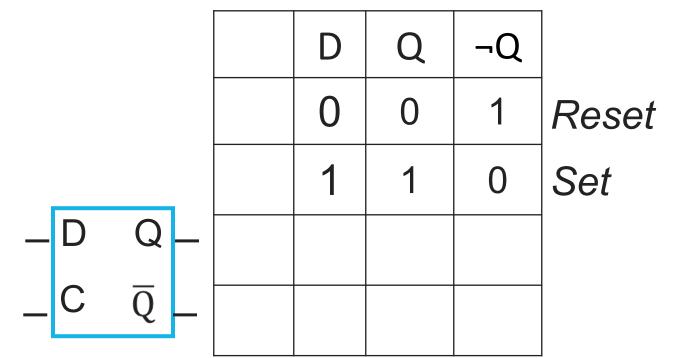


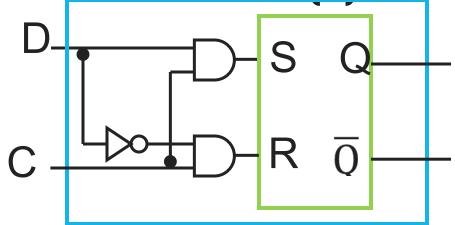




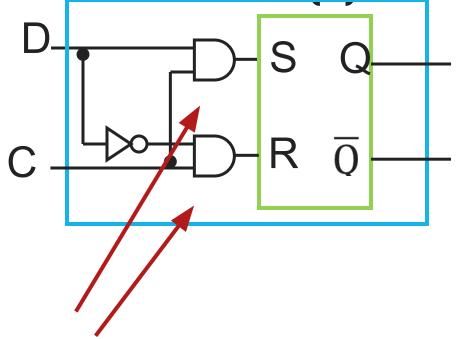






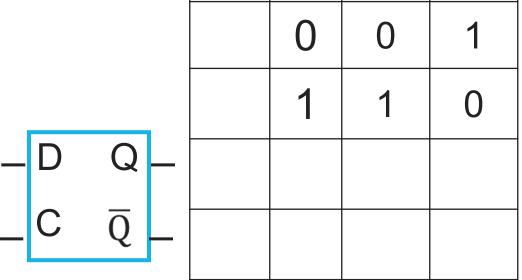


			D	Q	¬Q	
			0	0	1	Reset
			1	1	0	Set
_ D	Q	_				
_ C	$\overline{\mathbb{Q}}$	L				



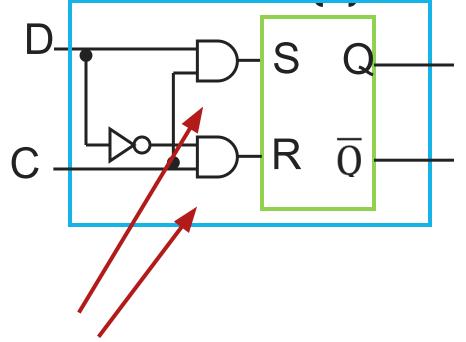
 Inverter prevents SR Latch from entering 1,1 state

AND gate **forces a 0** when C = 0.



Reset Set

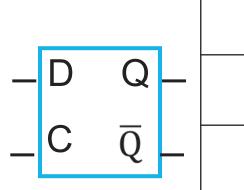




 Inverter prevents SR Latch from entering 1,1 state

AND gate forces a 0 when C = 0.

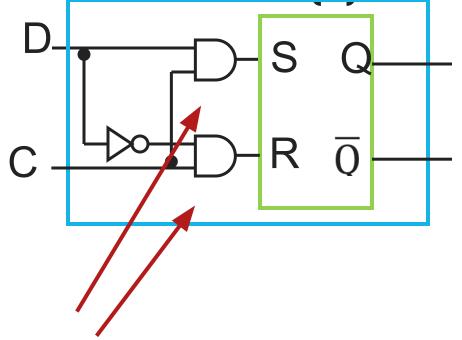
Remember: $0 \land a = 0$



0

Reset

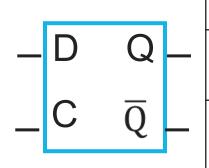
Set



 Inverter prevents SR Latch from entering 1,1 state

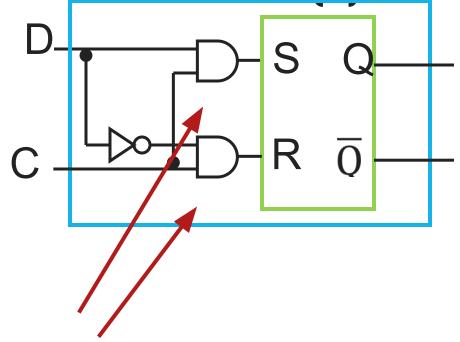
AND gate forces a 0 when C = 0.

Remember: $0 \land a = 0$



C	D	Q	¬Q	
0	0	Q_{t-1}	$\neg Q_{t-1}$	Hold
0	1	Q _{t-1}	¬Q _{t-1}	Hold





 Inverter prevents SR Latch from entering 1,1 state

AND gate **lets D pass** when C = 1.

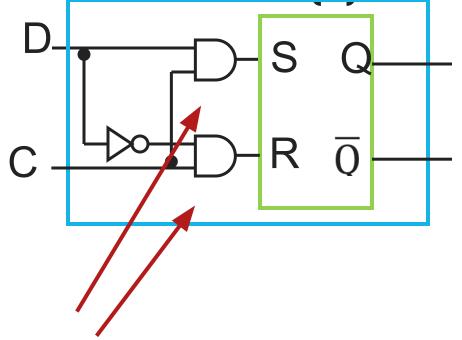
Remember: $1 \land a = a$

_	D	Q	_
_	С	$\overline{\mathbf{Q}}$	

С	D	Q	¬Q
0	0	Q _{t-1}	¬Q _{t-1}
0	1	Q _{t-1}	¬Q _{t-1}

Hold

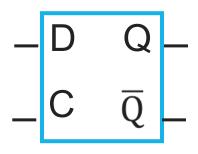
Hold



 Inverter prevents SR Latch from entering 1,1 state

AND gate **lets D pass** when C = 1.

Remember: $1 \land a = a$



С	D	Q	¬Q
0	0	Q _{t-1}	$\neg Q_{t-1}$
0	1	Q _{t-1}	$\neg Q_{t-1}$
1	0	0	1
1	1	1	0

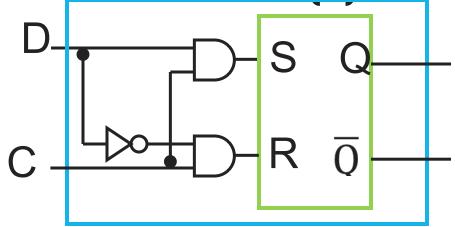
Hold

Hold

Reset

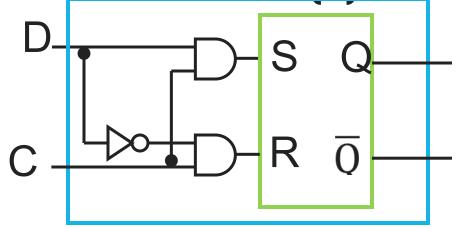
Set





- Level sensitive
- Inverter prevents SR Latch from entering 1,1 state

	С	D	Q	ΓQ	
	0	0	Q _{t-1}	¬Q _{t-1}	Hold
	0	1	Q _{t-1}	¬Q _{t-1}	Hold
	1	0	0	1	Reset
_	1	1	1	0	Set



- Level sensitive
- Inverter prevents SR Latch from entering 1,1 state

C = 1, D Latch *transparent*: set/reset (according to D)

C = 0, D Latch *opaque*: keep state (ignore D)

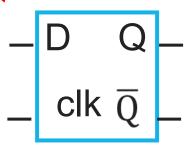
С	D	Q	¬Q
0	0	Q _{t-1}	$\neg Q_{t-1}$
0	1	Q _{t-1}	¬Q _{t-1}
1	0	0	1
1	1	1	0

Hold Reset

Set

Hold



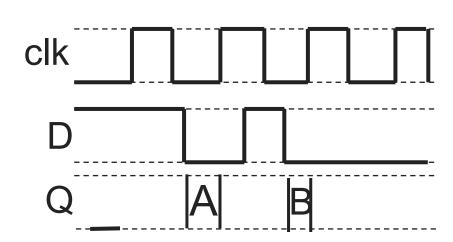


a)
$$A = 0, B = 0$$

b)
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

d)
$$A = 1, B = 1$$



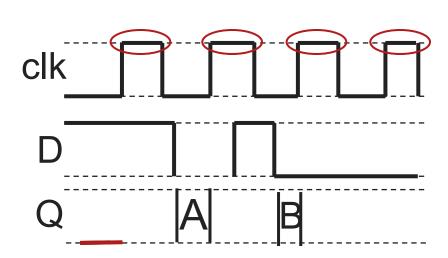
clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0

a)
$$A = 0, B = 0$$

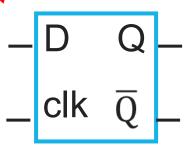
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

d)
$$A = 1, B = 1$$



clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0

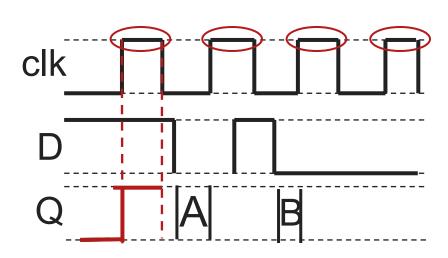


a)
$$A = 0, B = 0$$

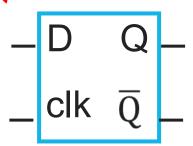
b)
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

$$d) A = 1, B = 1$$



clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0

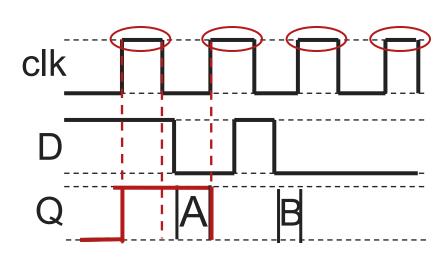


a)
$$A = 0, B = 0$$

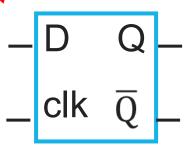
b)
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

d)
$$A = 1, B = 1$$



clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0

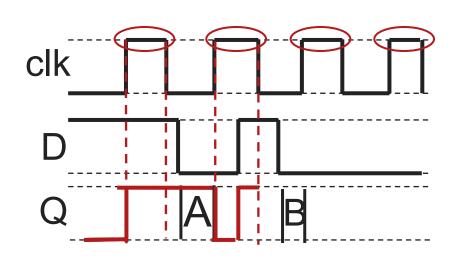


a)
$$A = 0, B = 0$$

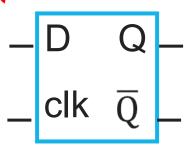
a)
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

d)
$$A = 1, B = 1$$



clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0

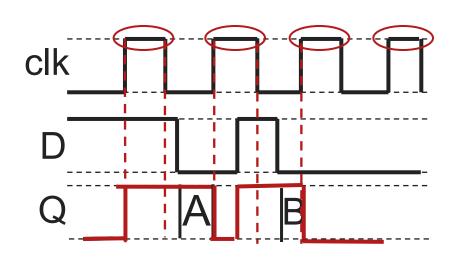


a)
$$A = 0, B = 0$$

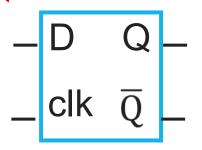
b)
$$A = 0, B = 1$$

c)
$$A = 1, B = 0$$

d)
$$A = 1, B = 1$$



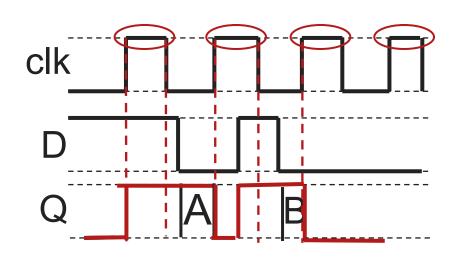
clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0



Level Sensitive D Latch

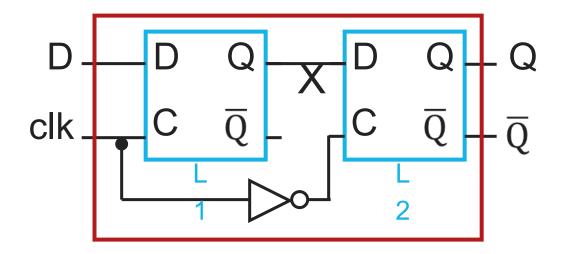
Clock high: set/reset (according to D) Clock low:

keep state (ignore D)

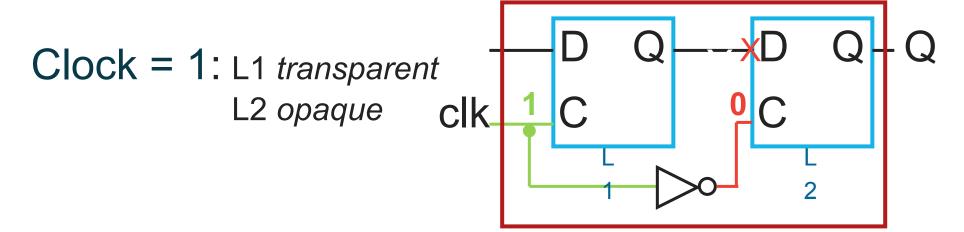


clk	D	Q	
0	0	Q	
0	1	Q	
1	0	0	1
1	1	1	0





- Edge-Triggered
- Data captured when clock high
- Output changes only on falling edges





Clock = 1: L1 transparent
L2 opaque clk 1 C C 2



Clock = 1: L1 transparent
L2 opaque clk

Clock = 1: L1 transparent

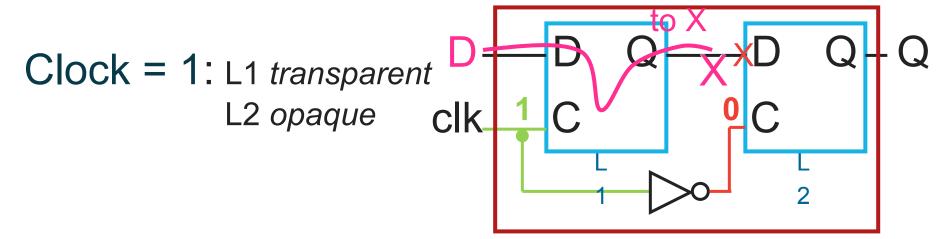
L2 opaque clk

L3 opaque clk

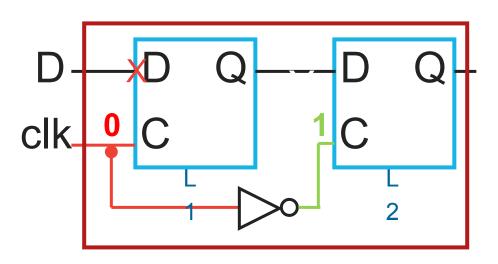
L4 opaque clk

L5 opaque clk





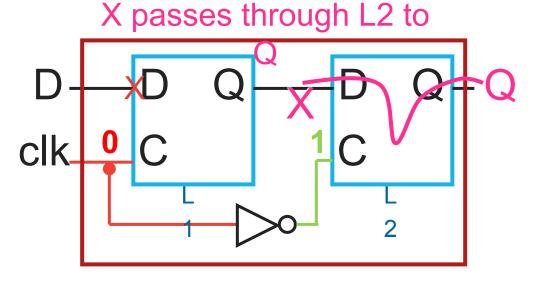
Clock = 0: L1 opaque L2 transparent





Clock = 1: L1 transparent
L2 opaque clk 1 C C

Clock = 0: L1 opaque L2 transparent



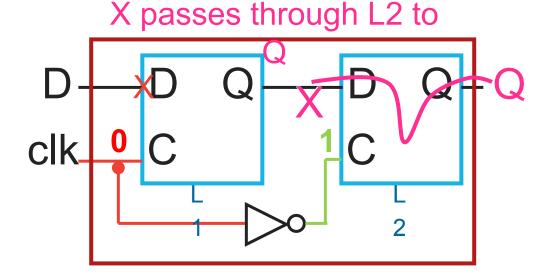


Clock = 1: L1 transparent
L2 opaque clk 1 C

2 C

Clock = 0: L1 opaque L2 transparent

Sample data at the **falling** *CLK* **edge** (1 □ 0)

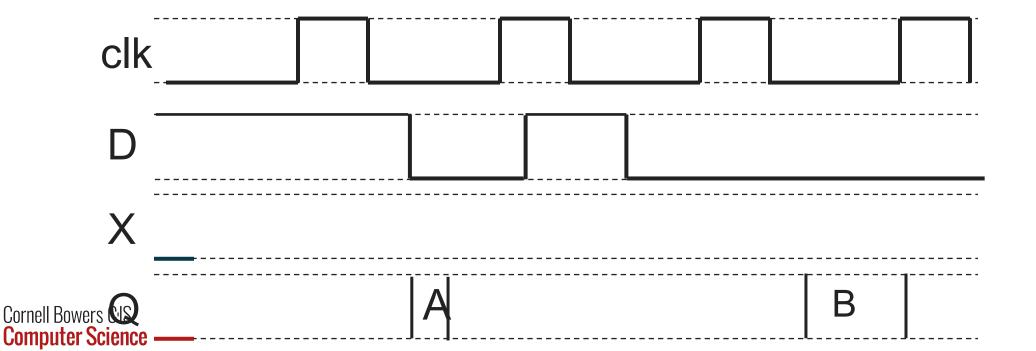




PollEV Question – start here

Clk C C

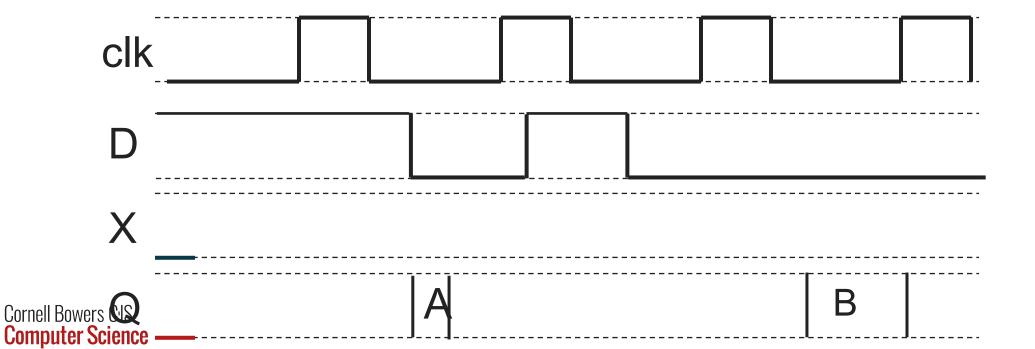
- a) A = 0, B = 0
- b) A = 0, B = 1
- c) A = 1, B = 0
- d) A = 1, B = 1

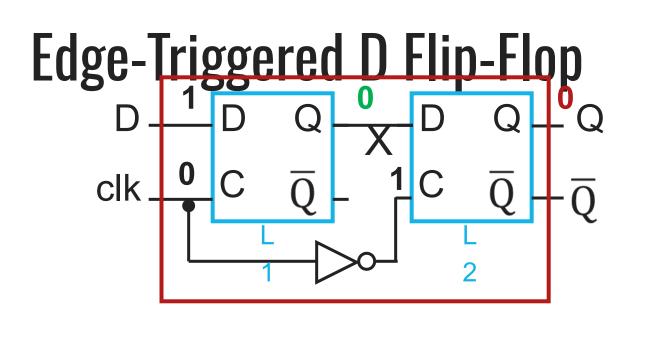


PollEV Question – start here

Clk C C

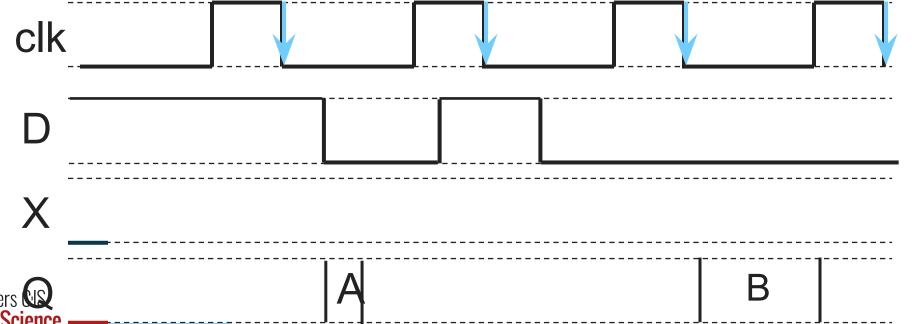
- a) A = 0, B = 0
- b) A = 0, B = 1
- (a) A = 1, B = 0
- A = 1, B = 1

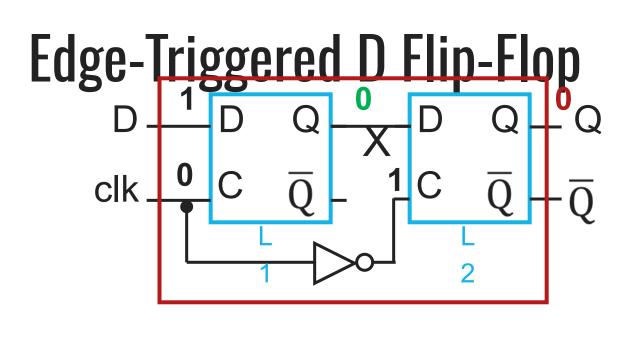




D Flip-Flop

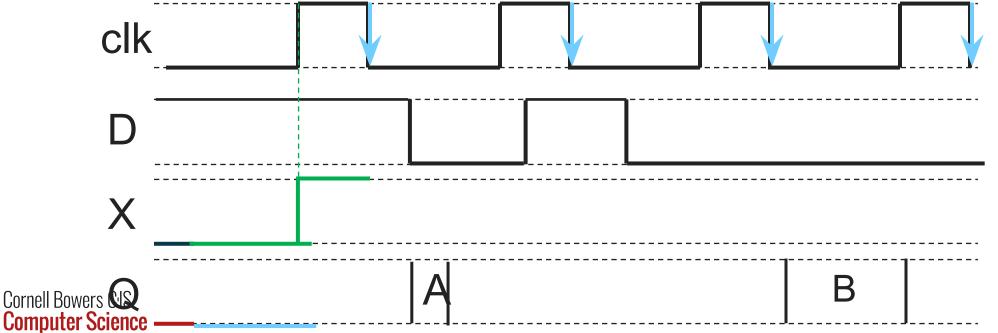
- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

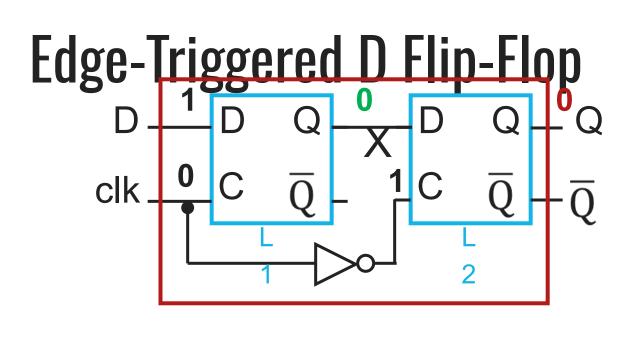




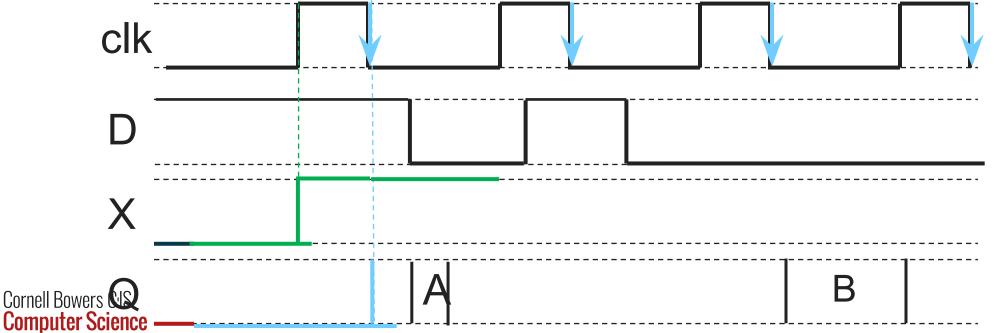
D Flip-Flop

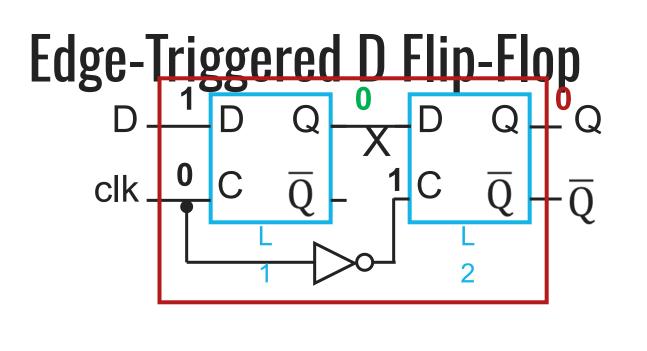
- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges



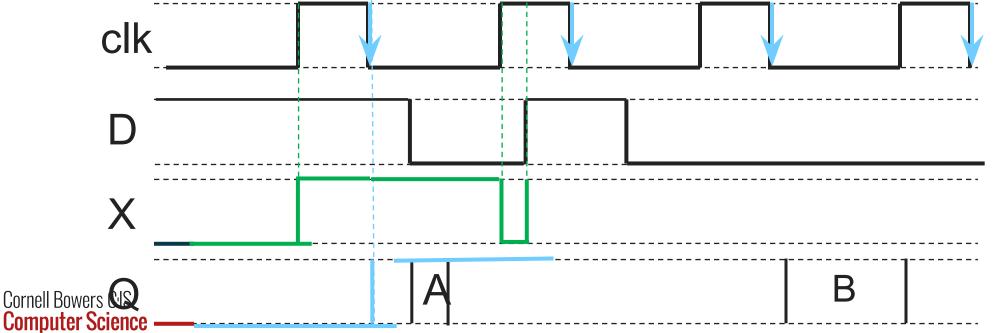


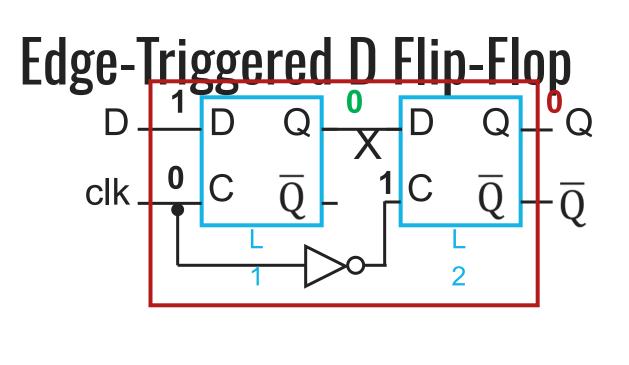
- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges



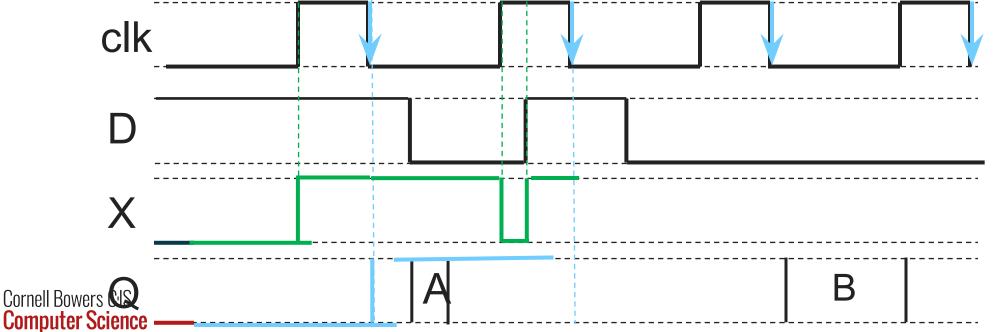


- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

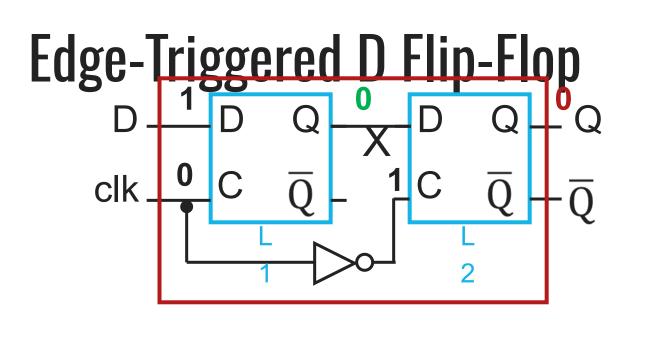




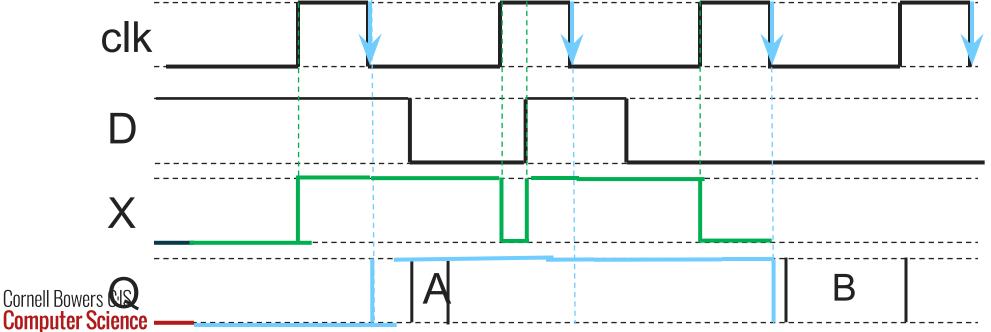
- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

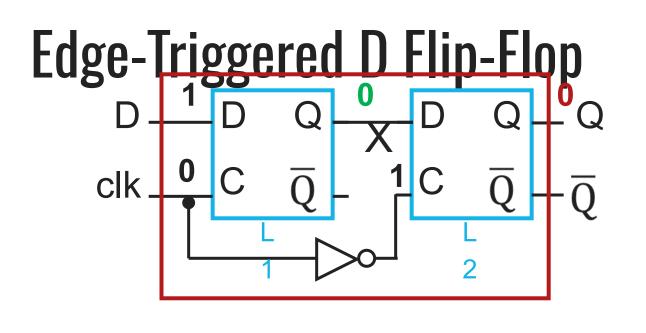




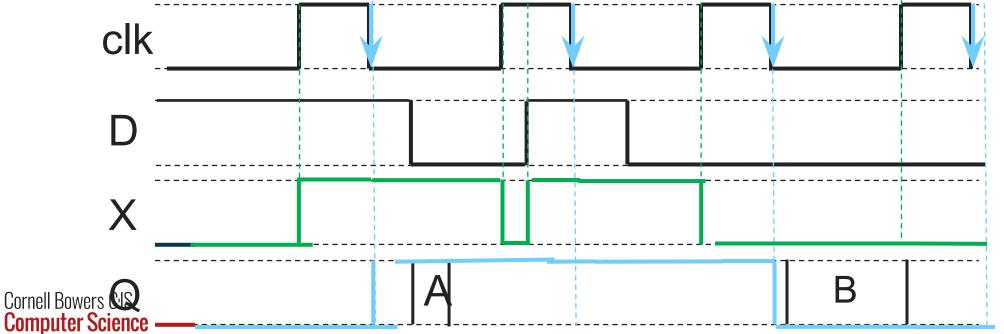


- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

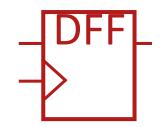




- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

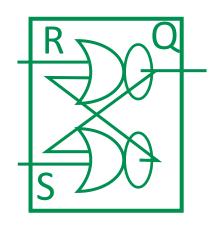


Building a D Flip Flop (DFF)



Step 1: Create an SR Latch

Set	Reset	Q
0	0	Q
0	1	0
1	0	1
1	1	?





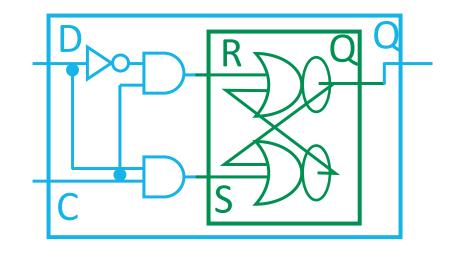
Building a D Flip Flop (DFF)

_ DFF _

Step 1: Create an SR Latch

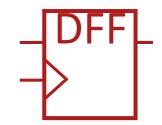
Step 2: Create a D Latch

Clk	Data	Q
0	0	Q
0	1	Q
1	0	0
1	1	1





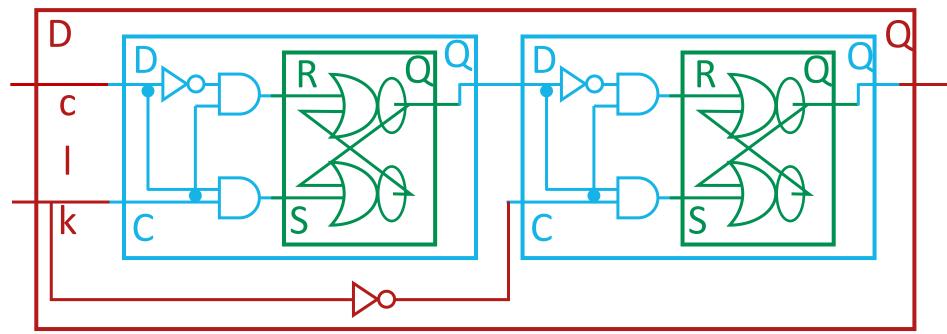
Building a D Flip Flop (DFF)



Step 1: Create an SR Latch

Step 2: Create a D Latch

Step 3: Duplicate the D Latch, chain together





Clock Disciplines

Level sensitive

• State changes when clock is high (or low)



Clock Disciplines

Level sensitive

State changes when clock is high (or low)

Edge triggered

State changes at clock edge
 positive edge-triggered

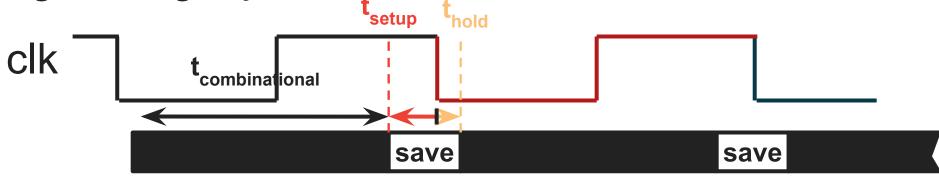
negative edge-triggered



Clock Methodology

Clock Methodology

Negative edge, synchronous



Edge-Triggered □ signals must be stable near falling edge

"near" = before and after

t_{setup} t_{holo}



Round 3: D Flip-Flop

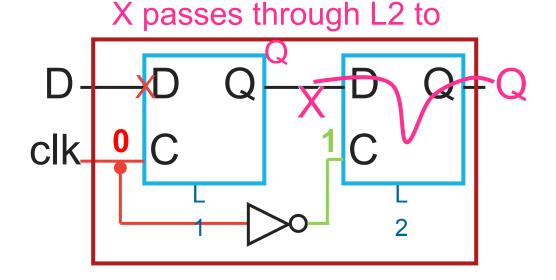
Clock = 1: L1 transparent
L2 opaque clk 1 C

2

2

Clock = 0: L1 opaque L2 transparent

Sample data at the **falling** *CLK* **edge** (1 □ 0)



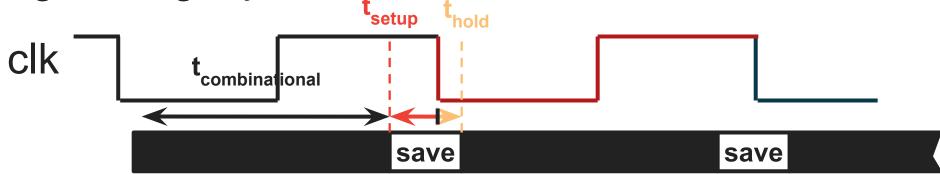
D passes through L1



Clock Methodology

Clock Methodology

Negative edge, synchronous



Edge-Triggered □ signals must be stable near falling edge

"near" = before and after

t_{setup} t_{holo}



Takeaway



Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.



(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.



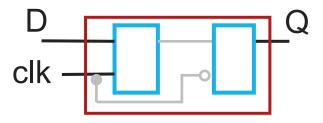
An Edge-Triggered D Flip-Flip stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.



Next Goal

How do we store more than one bit, N bits?

Registers



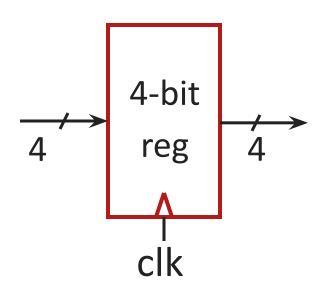
Register

• D flip-flops in parallel

Registers, **D3** clk

Register

- D flip-flops in parallel
- shared clock
- extra clocked inputs: write_enable, reset, ...

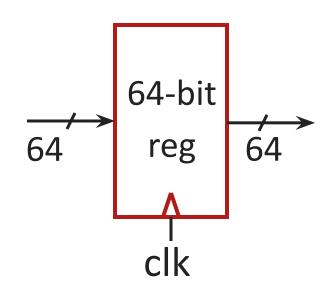




Registers, **D1 D2 D3** clk

Register

- D flip-flops in parallel
- shared clock
- extra clocked inputs: write_enable, reset, ...





Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.

An Edge-Triggered D Flip-Flip stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.

An *N*-bit **register** stores *N*-bits. It is created with *N* D-Flip-Flops in parallel along with a shared clock.

