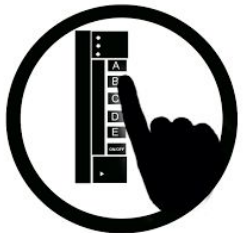
Introduction

CS 3410: Computer System Organization and Programming

[K. Bala, A. Bracy, E. Sirer, Z. Susag, H. Weatherspoon, and K. Laeufer]

Poll Everywhere Test (4 Questions)



PollEv.com/cs3410

Who am I?

Kevin Laeufer (Dr. Laeufer)

- **College:** RTHW Aachen, Germany (B.Sc. in Electrical Engineering)
- **Ph.D.**: Programming Languages and Computer Architecture at the University of California, Berkeley.
- **Research**: tools for digital hardware design (like CPUs!)

Co-Instructor

Giulia Guidi (Professor Guidi)

- **College:** Politecnico di Milano, Italy (B.Sc. and M.Sc. in Biomedical Engineering)
- **Ph.D.**: High Performance Computing (HPC) at the University of California, Berkeley
- **Research**: HPC, sparse linear algebra, scientific computing
- Co-taught CS 3410 in Fall 2024

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PLATFORM 2
South Hayward


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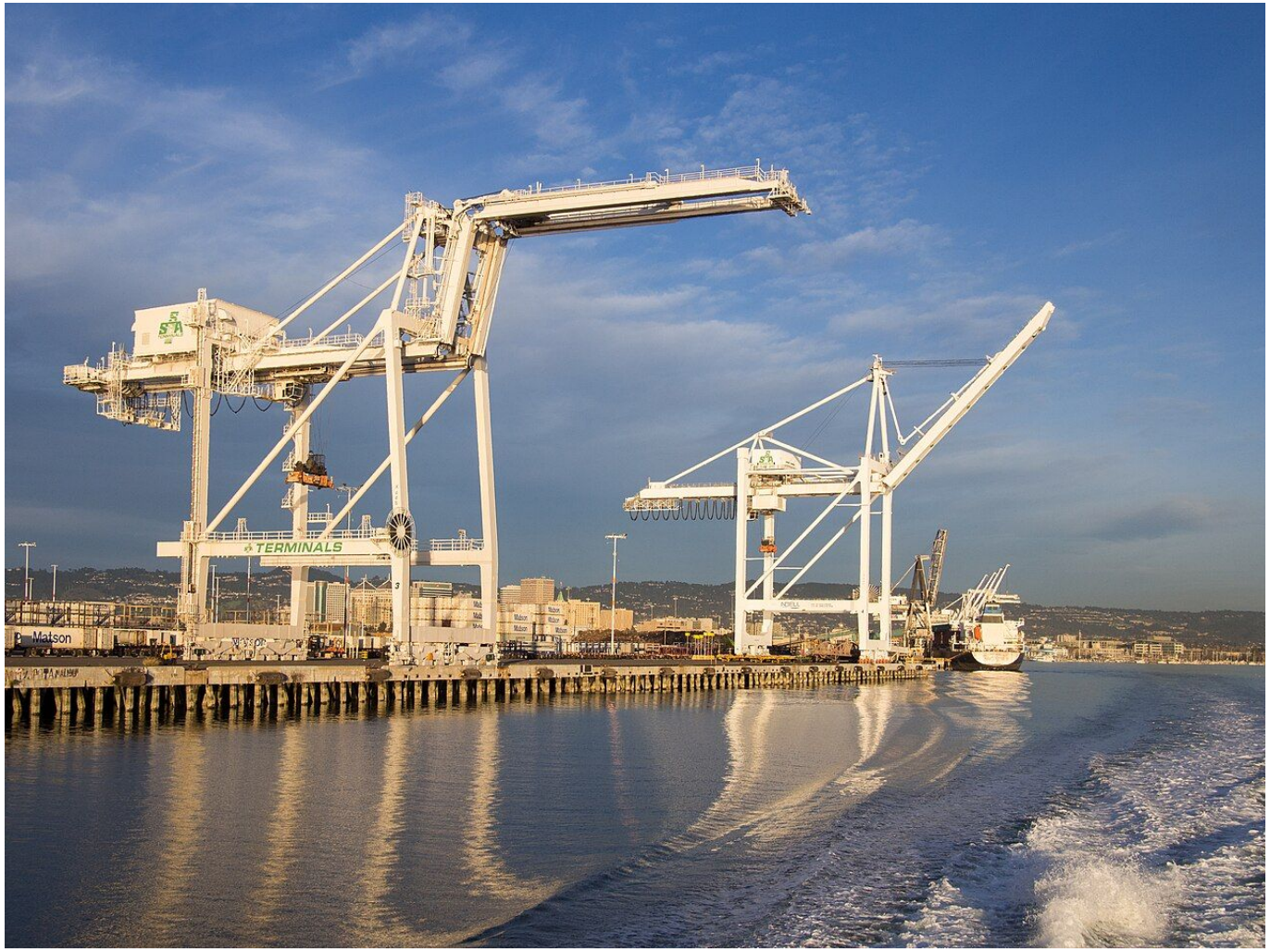
TERMINALS
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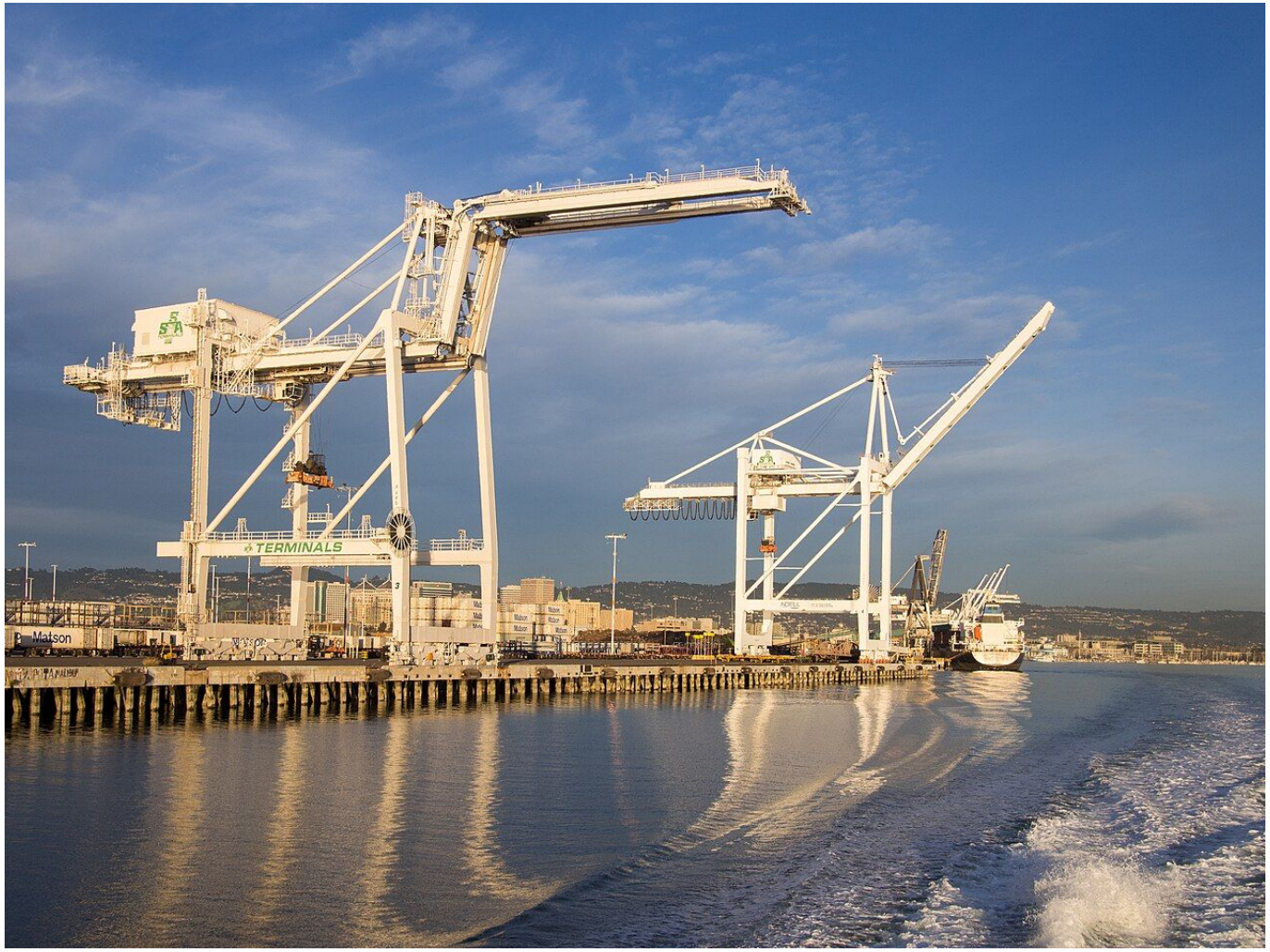

AT- AT Walkers?


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TERMINALS
Matson

Why?

Why?

Why?

- fewer items

Why?

- fewer items
- all the same shape

Why?

- fewer items
- all the same shape
- improved security

an **abstraction** built to manage

complexity

Is this **CS3410** or *Intro to Shipping Containers*?

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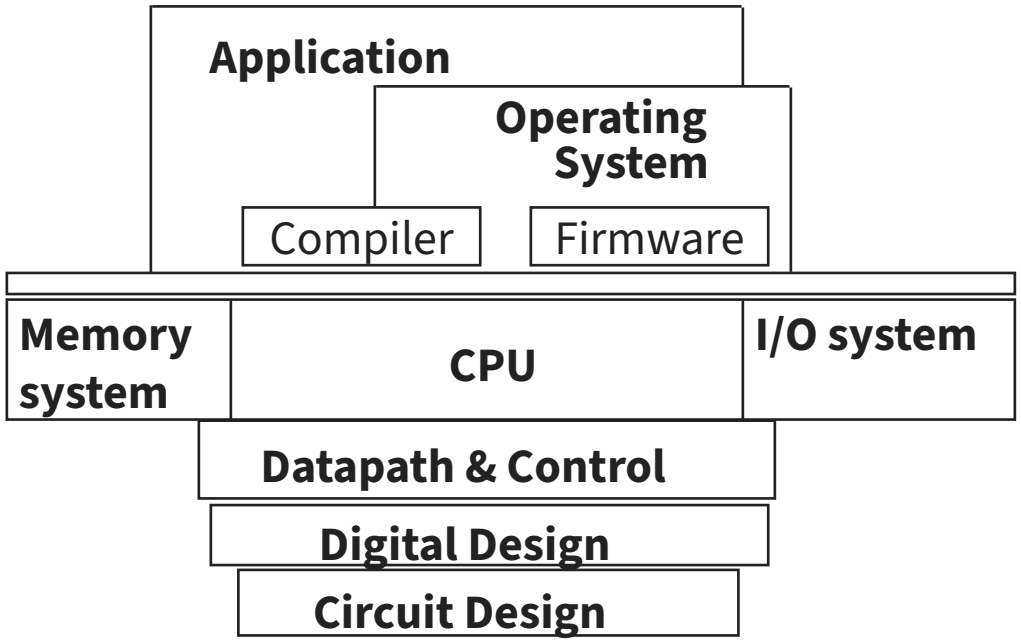

Instruction Set Architecture

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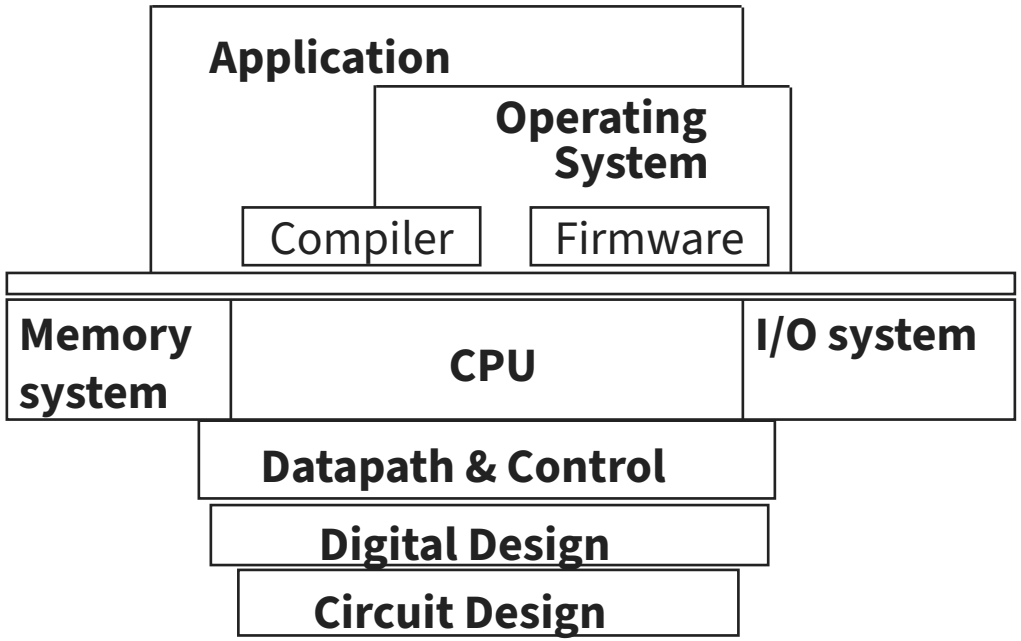
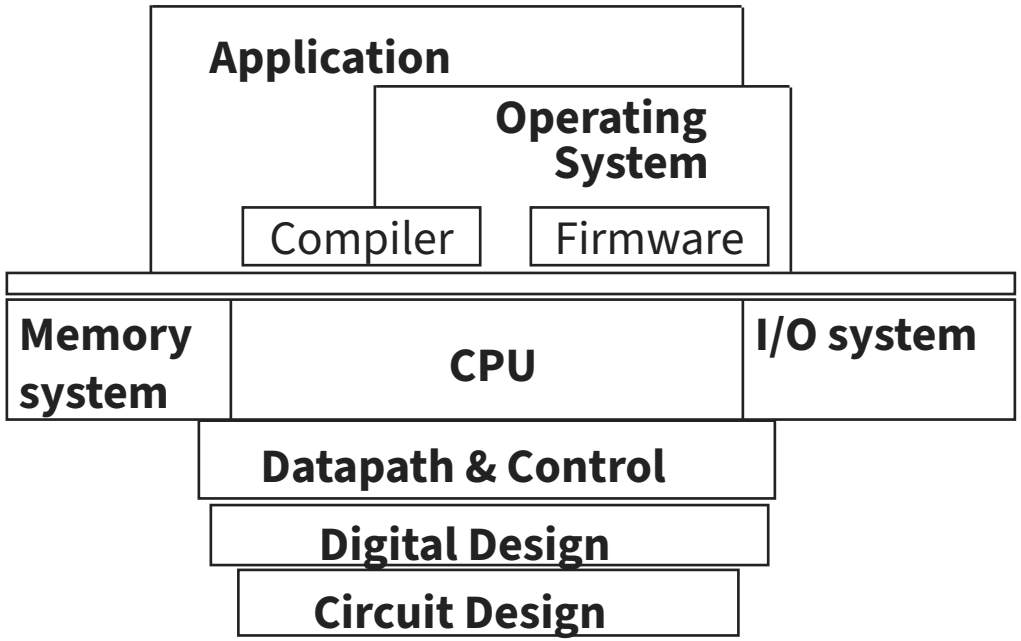


Instruction Set Architecture

How does my **Python program** run?

Is this **CS3410** or *Intro to Shipping Containers*?

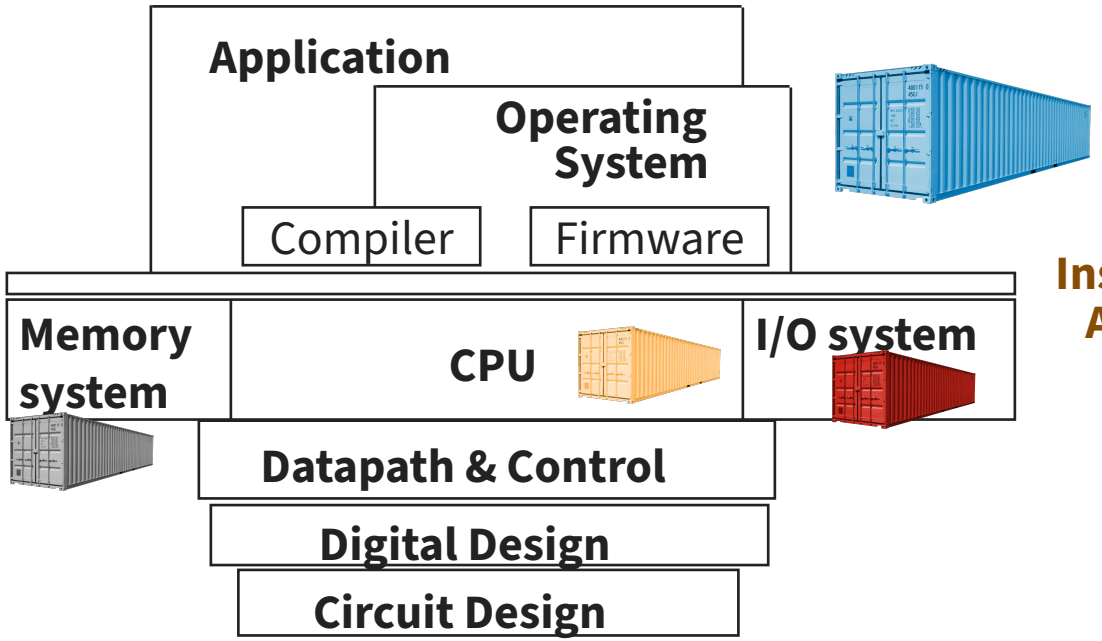


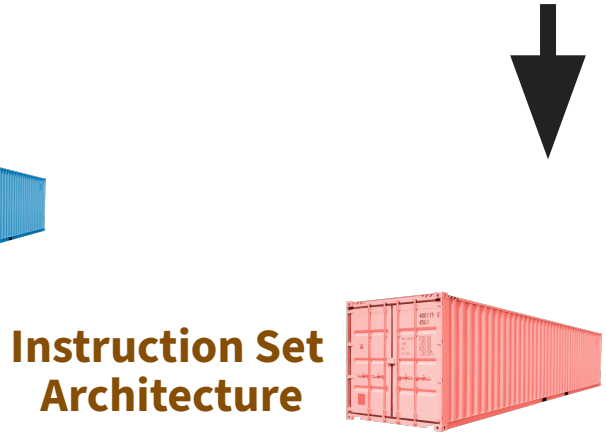
Instruction Set Architecture

How does my **Python program** run?

What do I do with this **switch** to make it compute?

Is this **CS3410** or *Intro to Shipping Containers*?

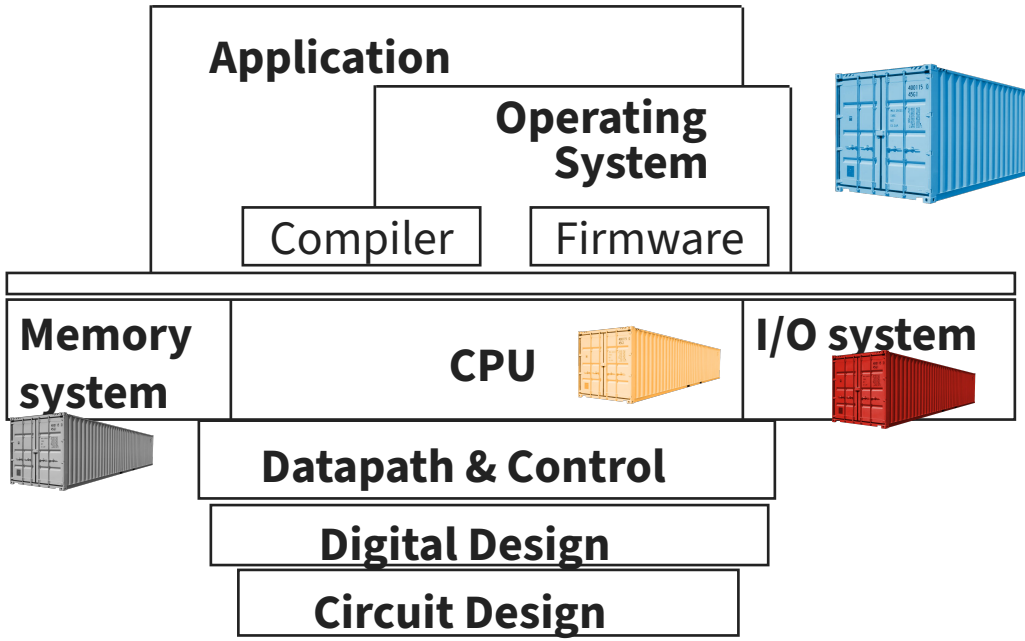




How does my **Python program** run?

What do I do with this **switch** to make it compute?

Is this **CS3410** or *Intro to ~~Shipping Containers~~?* **abstractions** built to manage **complexity**



How does my **Python program** run?

Instruction Set Architecture

What do I do with this **switch** to make it compute?

Is this **CS3410** or *Intro to ~~Shipping Containers~~*? **abstractions** built to manage **complexity**

- The C Programming Language
- Operating System Abstractions
- Parallel Programming (Synchronization)
- The RISC-V Instruction Set Architecture (Assembly)
- Digital Circuits and State Elements

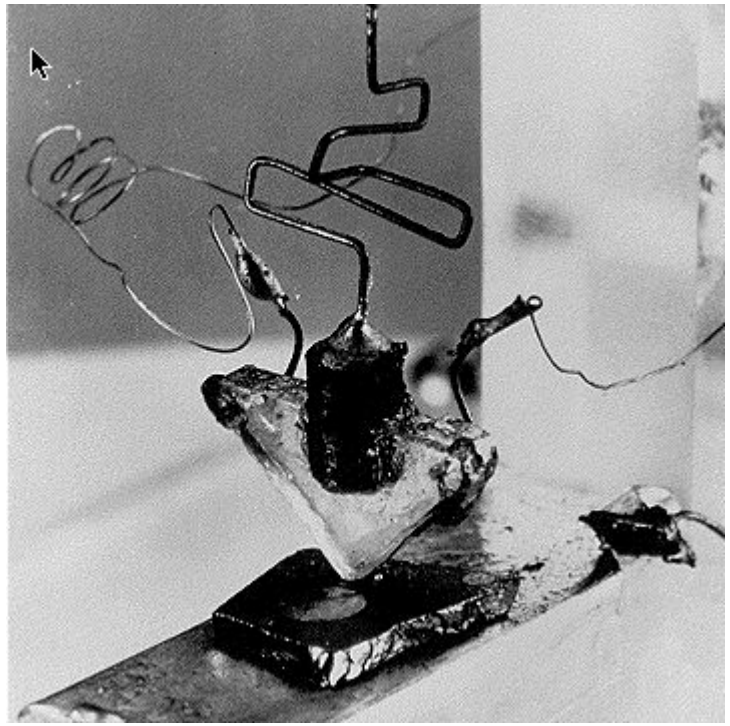
Where did it begin?

Electrical Switch

- On/Off
- Binary

Transistor

- Bardeen, Brattain, and Schokley

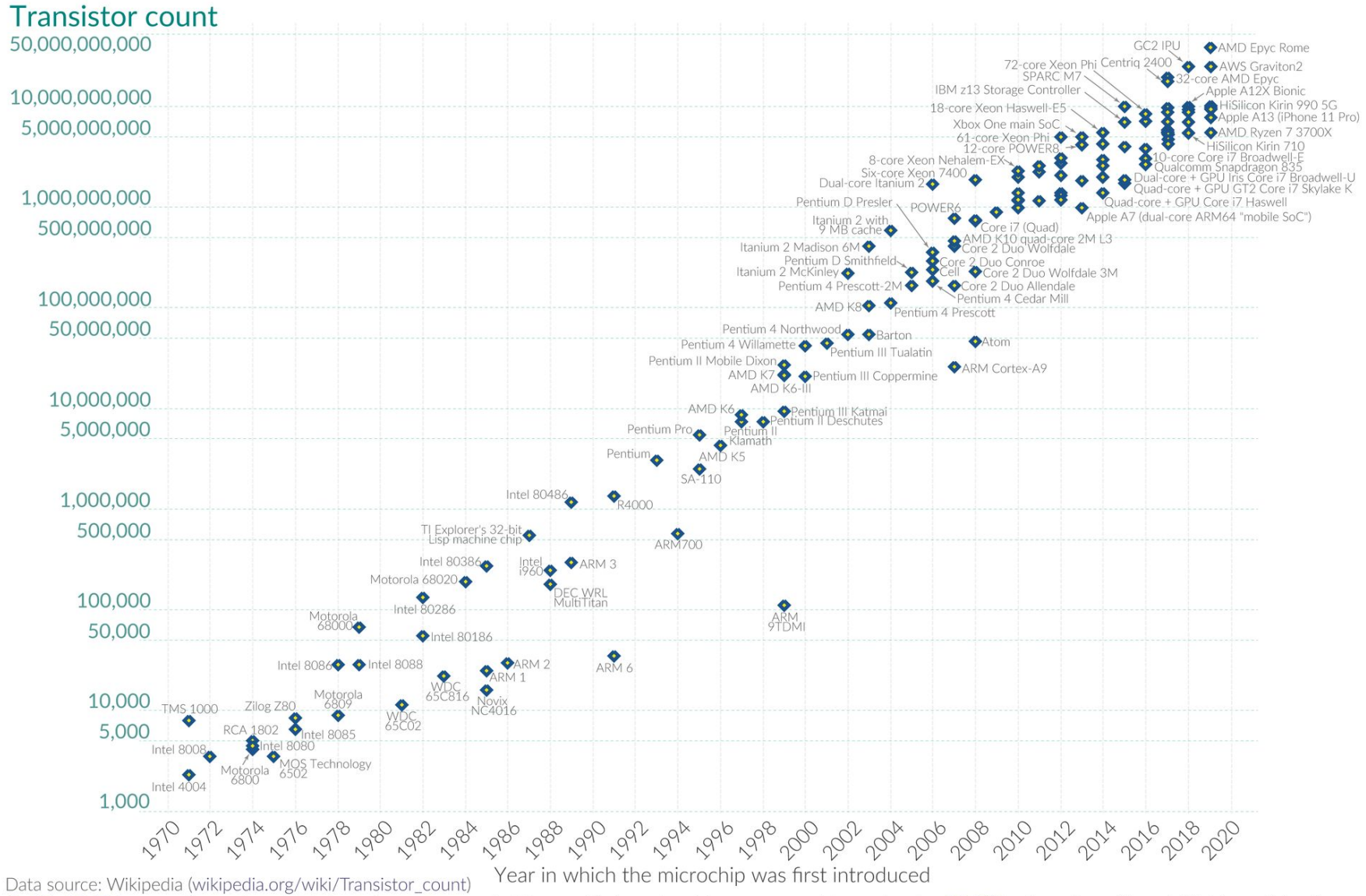


The first transistor on a workbench at AT&T Bell Labs in 1947

Moore's Law: The number of transistors on microchips doubles every two years

Our World in Data

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years.
This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.



Data source: Wikipedia (wikipedia.org/wiki/Transistor_count)
OurWorldinData.org – Research and data to make progress against the world's largest problems.

What to do with all these transistors?

- Apple M4
  - 28 billion transistors, 3nm
  - 177 square millimeters
  - 4x-10x performance, 4x-6x efficiency, 8x-40x GPU, 16x Neural processing cores

https://en.wikipedia.org/wiki/Transistor_count
https://en.wikipedia.org/wiki/Apple_M4

Representing Information

631

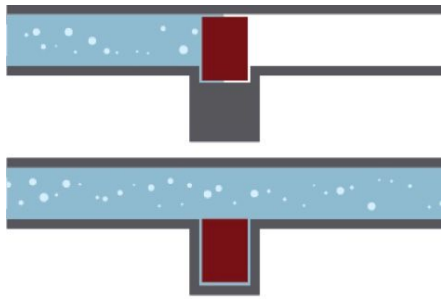
“Hello World”

Representing ~~Information~~ Numbers

631

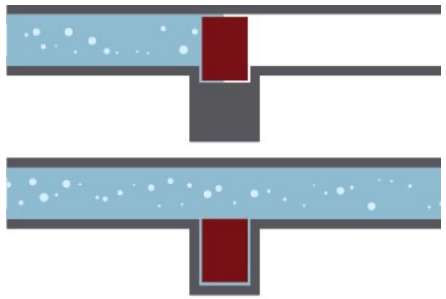
We can store all kinds of information, like images and text, as numbers.

Representing ~~Information~~ Numbers



Water Flow (l/min)

Representing ~~Information~~ Numbers



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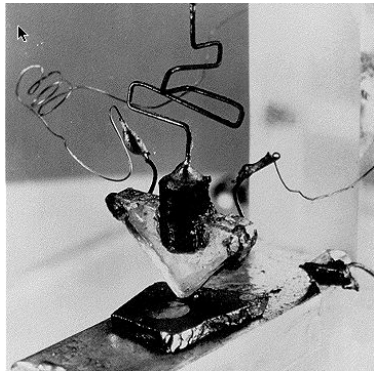
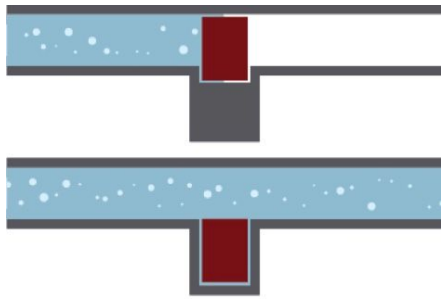


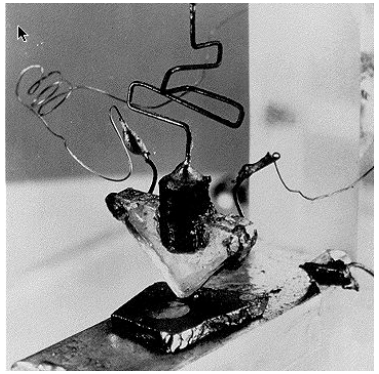
Voltage (V)

Representing ~~Information~~ Numbers



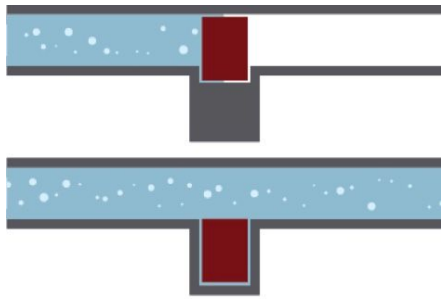
Analog

Water Flow (l/min)



Voltage (V)

Representing ~~Information~~ Numbers

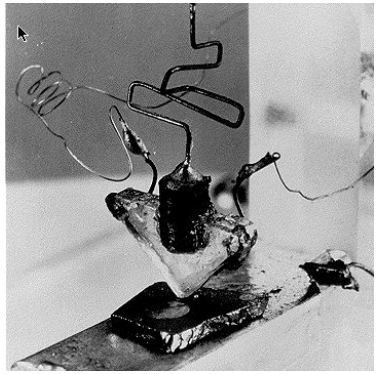


Analog

Water Flow (l/min)

Digital: 0 / 1

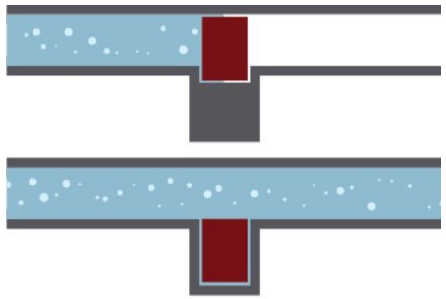
No Water / Water Flowing



Voltage (V)

Low / High Voltage

Representing ~~Information~~ Numbers

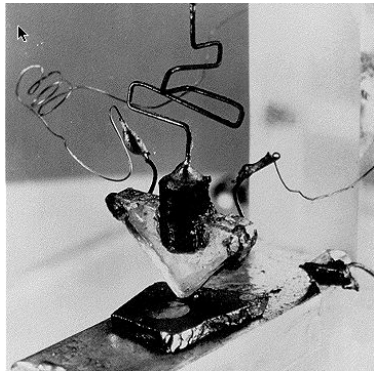


Analog

Water Flow (l/min)

Digital: 0 / 1

No Water / Water Flowing



Voltage (V)

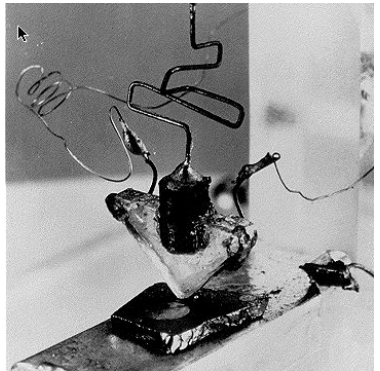
Low / High Voltage

abstraction built to manage **complexity**

Numbers beyond 0 and 1

Digital: 0/1

Analog: Voltage



Numbers beyond 0 and 1

Digital: 0/1

Numbers beyond 0 and 1

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

637_{10}

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

637_{10}

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

$$101_{2}$$

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

$$101_{2} = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= \quad 4_{10} + \quad 0_{10} + \quad 1_{10}$$

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 4_{10} + 0_{10} + 1_{10}$$
$$= 5_{10}$$

Number: 637

?

Digital: 0/1

Numbers beyond 0 and 1

$$637_{10} = 6 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$$

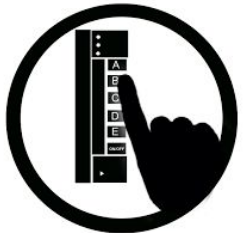
$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 4_{10} + 0_{10} + 1_{10}$$
$$= 5_{10}$$

Number: 637

?

Digital: 0/1

Poll: How many numbers can we represent with 4 digital switches?



a) 4
b) 40
c) 15
d) 16
e) 31

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Poll: What is 2^{10}?

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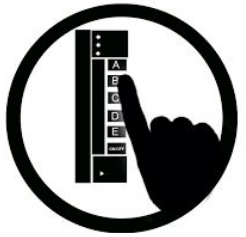

a) 1024
b) $2^5 \times 2^2$
c) $2^5 \times 2^5$
d) The number of distinct binary numbers you can express in 10 digits.
e) The number of distinct decimal numbers you can express in 10 digits.

Pick all correct answers.

Let's count!

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0		
1	1		
2	10		
3	11		
4	100		
5	101		
6	110		
7	111		
8	1000		
9	1001		
10	1010		
11	1011		
12	1100		
13	1101		
14	1110		
15	1111		
16	1 0000		

Let's count!

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0		0
1	1		1
2	10		2
3	11		3
4	100		4
5	101		5
6	110		6
7	111		7
8	1000		8
9	1001		9
10	1010		
11	1011		
12	1100		
13	1101		
14	1110		
15	1111		
16	1 0000		

Let's count!

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0		0
1	1		1
2	10		2
3	11		3
4	100		4
5	101		5
6	110		6
7	111		7
8	1000		8
9	1001		9
10	1010		a
11	1011		b
12	1100		c
13	1101		d
14	1110		e
15	1111		f
16	1 0000		

Let's count!

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0		0
1	1		1
2	10		2
3	11		3
4	100		4
5	101		5
6	110		6
7	111		7
8	1000		8
9	1001		9
10	1010		a
11	1011		b
12	1100		c
13	1101		d
14	1110		e
15	1111		f
16	1 0000		10

Let's count!

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000		8
9	1001		9
10	1010		a
11	1011		b
12	1100		c
13	1101		d
14	1110		e
15	1111		f
16	1 0000		10

Let's count!

A note on notation

$4_{10} =$ 4

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	a
11	1011	13	b
12	1100	14	c
13	1101	15	d
14	1110	15	e
15	1111	17	f
16	1 0000	20	10

Let's count!

A note on notation

$4_{10} =$ 4

$100_2 =$ 0b100

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Let's count!

A note on notation

4_{10} = 4

100_2 = 0b100

4_8 = 0o4

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16	1 0000	20	10

Let's count!

A note on notation

$4_{10} =$ 4

$100_2 =$ 0b100

$4_8 =$ 0o4

$4_{16} =$ 0x4

Decimal	Binary	Octal	Hexadecimal
base 10	base 2	base 8	base 16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	a
11	1011	13	b
12	1100	14	c
13	1101	15	d
14	1110	15	e
15	1111	17	f
16	1 0000	20	10

Converting Between Bases

Strategy #1: Left to Right

637_{10}

Converting Between Bases

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637_{10}

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637_{10}

8^3	8^2	8^1	8^0
512	64	8	1

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

8^3	8^2	8^1	8^0
512	64	8	1
1			

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 64 x 2 = -3

8^3	8^2	8^1	8^0
512	64	8	1
1	2?		

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 64 x 1 = 61

8^3	8^2	8^1	8^0
512	64	8	1
1	1		

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

637 / 8 = 79 R **5**

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

637 / 8 = 79 R **5**

79 / 8 = 9 R **7**

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

637 / 8 = 79 R **5**

79 / 8 = 9 R **7**

9 / 8 = 1 R **1**

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

637 / 8 = 79 R **5**

79 / 8 = 9 R **7**

9 / 8 = **1** R **1**

Converting Between Bases

Strategy #1: Left to Right

637_{10}

637 - 512 = 125

125 - 8 x 7 = 5

8^3	8^2	8^1	8^0
512	64	8	1
1	1	7	5

Strategy #2: Right to Left

637 / 8 = 79 R **5**

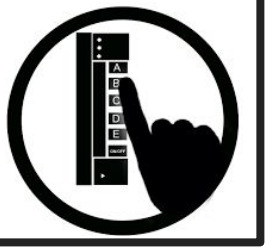
79 / 8 = 9 R **7**

9 / 8 = **1** R **1**

Convert the number 657_{10} to base 16
What is the least significant digit of this number?

a) D
b) F
c) 0
d) 1
e) 11

PollEv.com/cs3410



Convert from Binary to other powers of 2

- Binary to Octal
  - 3 bits (000—111) have values 0…7 = 1 octal digit

    example: `0b 1 001 111 101`

    `1 1 7 5` → `0o1175`

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    example: `0b 1 001 111 101`

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- Binary to Hexadecimal
  - 4 bits (0000—1111) have values 0…15 = 1 hex digit

    example: `0b 10 0111 1101`

    `2 7 d` → `0x27d`

Convert from Binary to other powers of 2

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  - 3 bits (000—111) have values 0…7 = 1 octal digit

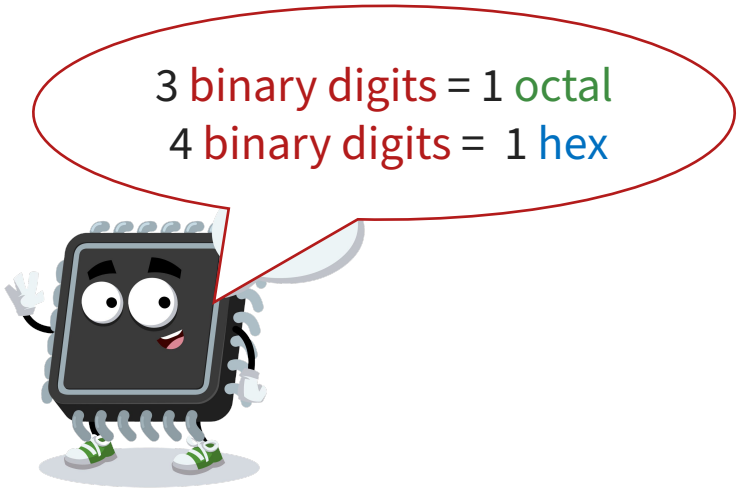
    example: `0b 1 001 111 101`

    `1 1 7 5` □ `0o1175`

- Binary to Hexadecimal
  - 4 bits (0000—1111) have values 0…15 = 1 hex digit

    example: `0b 10 0111 1101`

    `2 7 d` □ `0x27d`



Numbers beyond 0 and 1

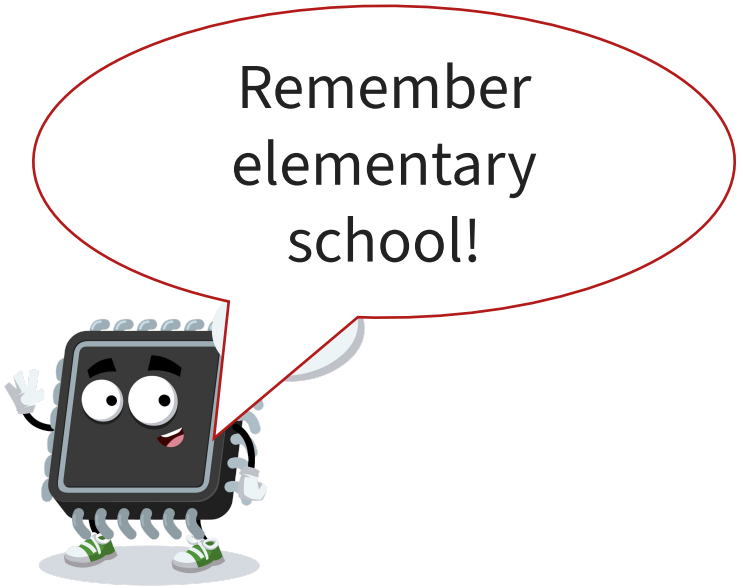
- We use binary to represent numbers on top of our digital abstraction.
- We learned how to convert between different bases.

Number: 637

Digital: 0/1

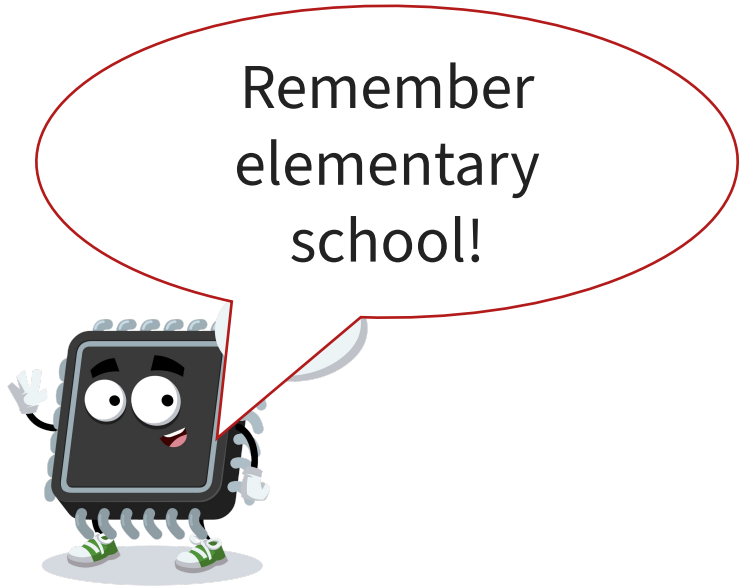
Adding Binary Numbers

$$\begin{array}{r} 24_{10} \\ +\ 17_{10} \\ \hline \quad\quad \end{array}$$



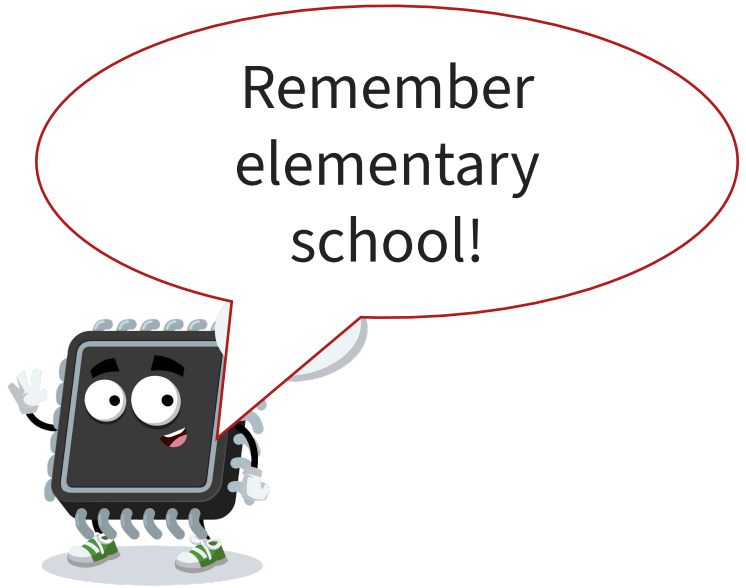
Adding Binary Numbers

$$
\begin{array}{r}
24_{10} \\
+\ 17_{10} \\
\quad (+1) \\
\hline
\\
1_{10}
\end{array}
$$



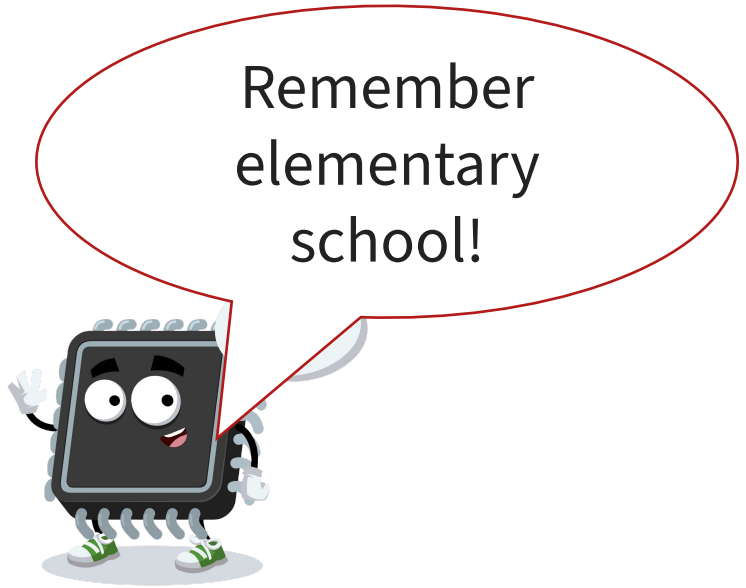
Adding Binary Numbers

$$
\begin{array}{r}
24_{10} \\
+\ 17_{10} \\
\quad (+1) \\
\hline
\\
41_{10}
\end{array}
$$



Adding Binary Numbers

$$\begin{array}{r} 24_{10} \\ +\ 17_{10} \\ \quad (+1) \\ \hline 41_{10} \end{array} \qquad \begin{array}{r} 11000_{2} \\ +\ 10001_{2} \\ \hline \end{array}$$



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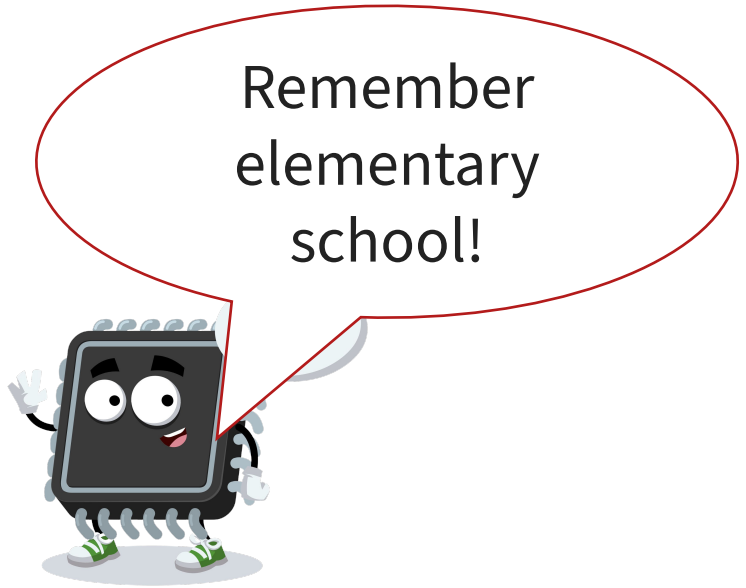

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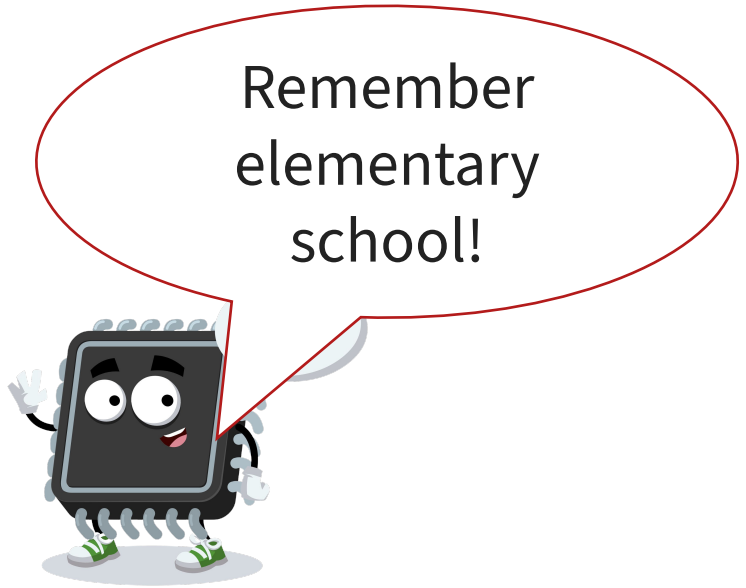

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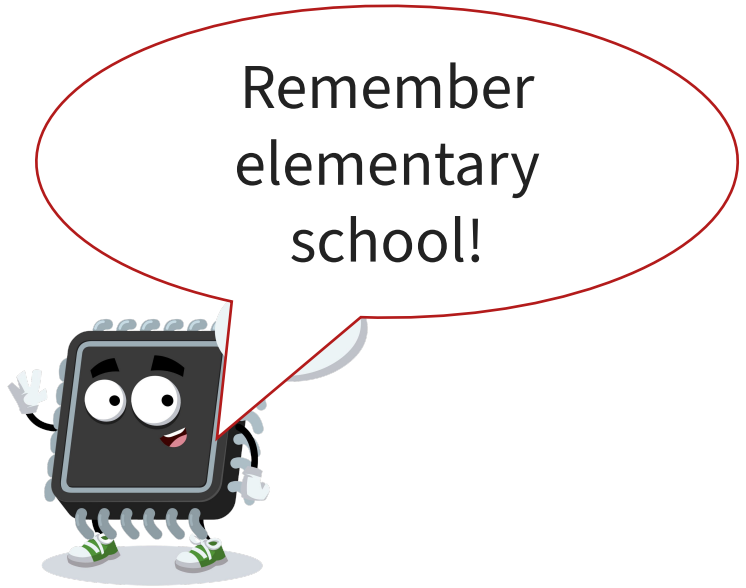

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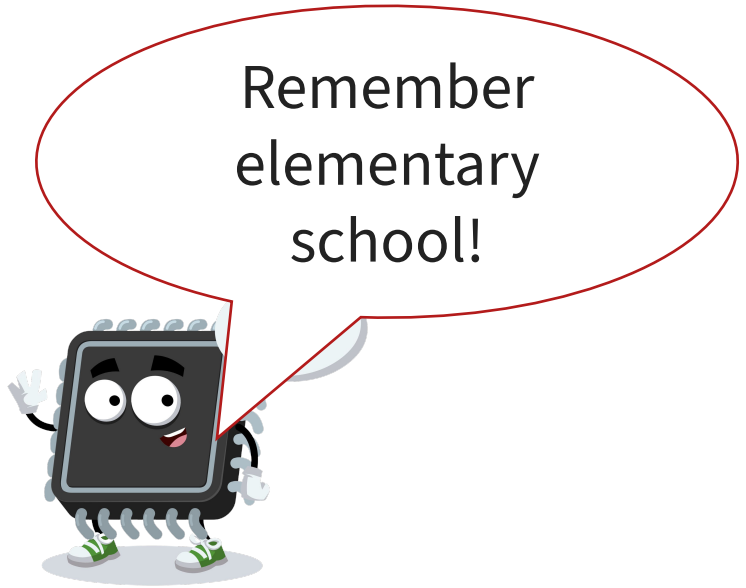

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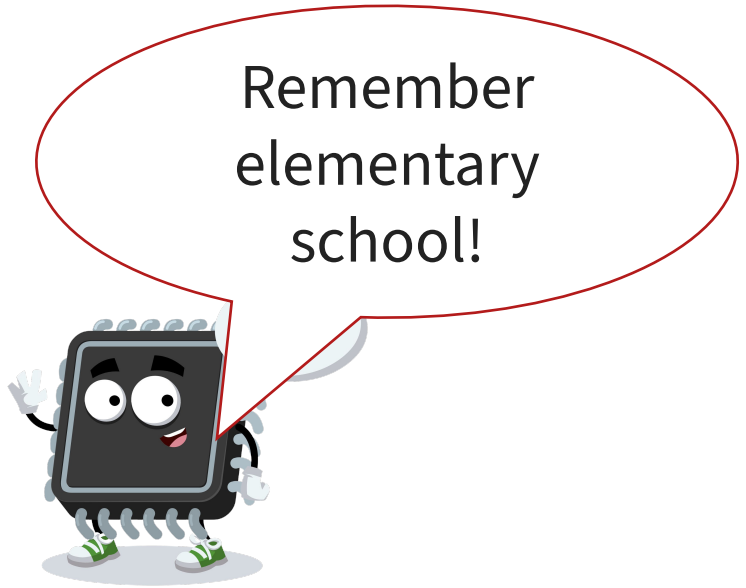

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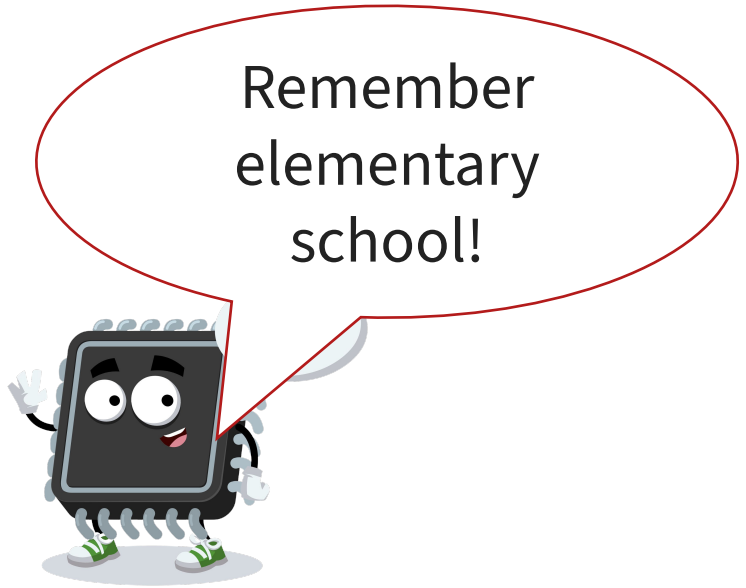

How about negative numbers?

Old & Dusty Option: Sign/Magnitude

$$-\ 7_{10}$$
$$+\ 7_{10}$$

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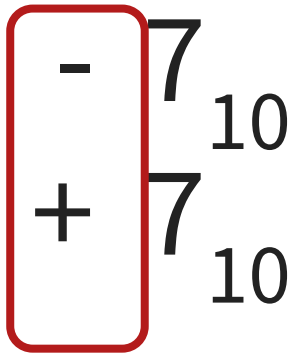

1 bit of information

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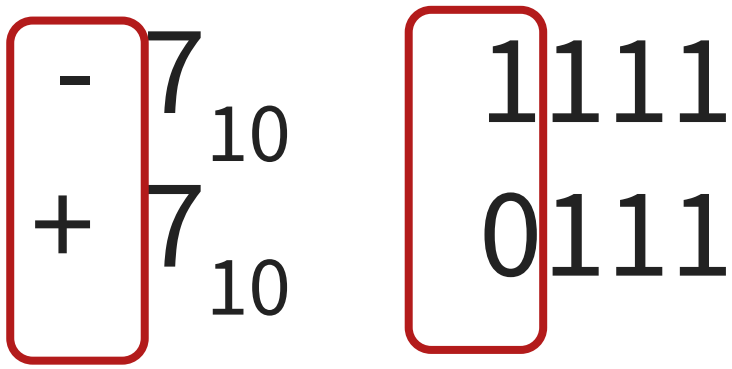
Old & Dusty Option: Sign/Magnitude



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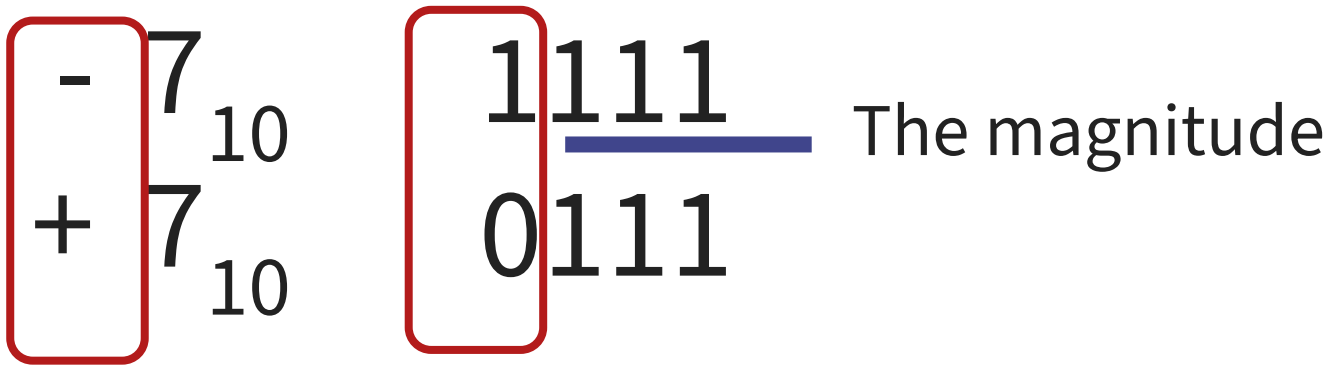

1 bit of information

How about negative numbers?

Old & Dusty Option: Sign/Magnitude

-	7_{10}	1111
+	7_{10}	0111

The magnitude

1 bit of information

IBM 7090, 1959: “a second-generation transistorized version of the earlier IBM 709 vacuum tube mainframe computers”

How about negative numbers?

Old & Dusty Option: Sign/Magnitude

$$\begin{array}{|c|l|} \hline - & 7_{10} \\ + & 7_{10} \\ \hline \end{array} \qquad \begin{array}{|l|} \hline 1111 \\ \hline 0111 \\ \hline \end{array}$$

The magnitude

1 bit of information

Problems:

- +0 and -0
- complicated circuits

IBM 7090, 1959: “a second-generation transistorized version of the earlier IBM 709 vacuum tube mainframe computers”

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New, Awesome Option: Two's Complement

Positive numbers are represented as usual

- 0 = 0000, 1 = 0001, 3 = 0011, 7 = 0111

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- complement *all* the bits (i.e. flip all the bits)
- then add 1

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- -1: 1 ⇒ 0001 ⇒ 1110 ⇒ 1111

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- complement *all* the bits (i.e. flip all the bits)
- then add 1
- -1: 1 ⇒ 0001 ⇒ 1110 ⇒ 1111
- -3: 3 ⇒ 0011 ⇒ 1100 ⇒ 1101
- -0: 0 ⇒ 0000 ⇒ 1111 ⇒ 0000 (this is good, -0 = +0)

Two's Complement

Non-negatives unchanged:

- +0 = 0000
- +1 = 0001
- +2 = 0010
- +3 = 0011
- +4 = 0100
- +5 = 0101
- +6 = 0110
- +7 = 0111
- +8 = 1000

Two's Complement

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How do you express this magnitude in the negative?

Two's Complement

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- +6 = 0110
- +7 = 0111
- +8 = 1000

How do you express this magnitude in the negative?

	flip	Negatives then add 1
$\overline{0}$	= 1111	-0 = 0000
$\overline{1}$	= 1110	-1 = 1111
$\overline{2}$	= 1101	-2 = 1110
$\overline{3}$	= 1100	-3 = 1101
$\overline{4}$	= 1011	-4 = 1100
$\overline{5}$	= 1010	-5 = 1011
$\overline{6}$	= 1001	-6 = 1010
$\overline{7}$	= 1000	-7 = 1001
$\overline{8}$	= 0111	-8 = 1000

(8 no longer expressible in 4 bits when moving to signed)

Two's Complement

Non-negatives unchanged:

- +0 = 0000
- +1 = 0001
- +2 = 0010
- +3 = 0011
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- +5 = 0101
- +6 = 0110
- +7 = 0111
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How do you express this magnitude in the negative?

Cornell Bowers CIS Computer Science

(8 no longer expressible in 4 bits when moving to signed)

Two's Complement vs. Unsigned

4 bit
Two's
Complement
-8 … 7

-1 =	1111	= 15
-2 =	1110	= 14
-3 =	1101	= 13
-4 =	1100	= 12
-5 =	1011	= 11
-6 =	1010	= 10
-7 =	1001	= 9
-8 =	1000	= 8
+7 =	0111	= 7
+6 =	0110	= 6
+5 =	0101	= 5
+4 =	0100	= 4
+3 =	0011	= 3
+2 =	0010	= 2
+1 =	0001	= 1
0 =	0000	= 0

4 bit
Unsigned
Binary
0 … 15

Two's Complement vs. Unsigned

Another way to look at it: the MSB is a negative column (here -8)

-1 =	1111	= 15
-2 =	1110	= 14
-3 =	1101	= 13
-4 =	1100	= 12
-5 =	1011	= 11
-6 =	1010	= 10
-7 =	1001	= 9
-8 =	1000	= 8
+7 =	0111	= 7
+6 =	0110	= 6
+5 =	0101	= 5
+4 =	0100	= 4
+3 =	0011	= 3
+2 =	0010	= 2
+1 =	0001	= 1
0 =	0000	= 0

4 bit Two's Complement -8 … 7

4 bit Unsigned Binary 0 … 15

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Another way to look at it: the MSB is a negative column (here -8)

-1 =	1111	= 15
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-5 =	1011	= 11
-6 =	1010	= 10
-7 =	1001	= 9
-8 =	1000	= 8
+7 =	0111	= 7
+6 =	0110	= 6
+5 =	0101	= 5
+4 =	0100	= 4
+3 =	0011	= 3
+2 =	0010	= 2
+1 =	0001	= 1
0 =	0000	= 0

-8 + 0 + 2 + 1

4 bit Two's Complement -8 … 7

4 bit Unsigned Binary 0 … 15

CS3410 Introduction

- Learn how a computer is engineered from switches to executing high-level code.
- Focusing on impactful abstractions that allow us to do something useful with 20+ billion transistors.
- Digital + Binary Numbers: base conversion, addition, negative numbers
- **Read the syllabus** on the course website.
- Fill out the **Introductory Survey on Gradescope**.
- **Next time:**
  - What happens if my number does not fit? (Overflow)
  - The C programming language.