



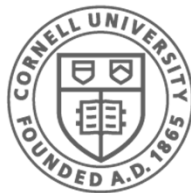
# State

**Prof. Hakim Weatherspoon**

**CS 3410**

Computer Science

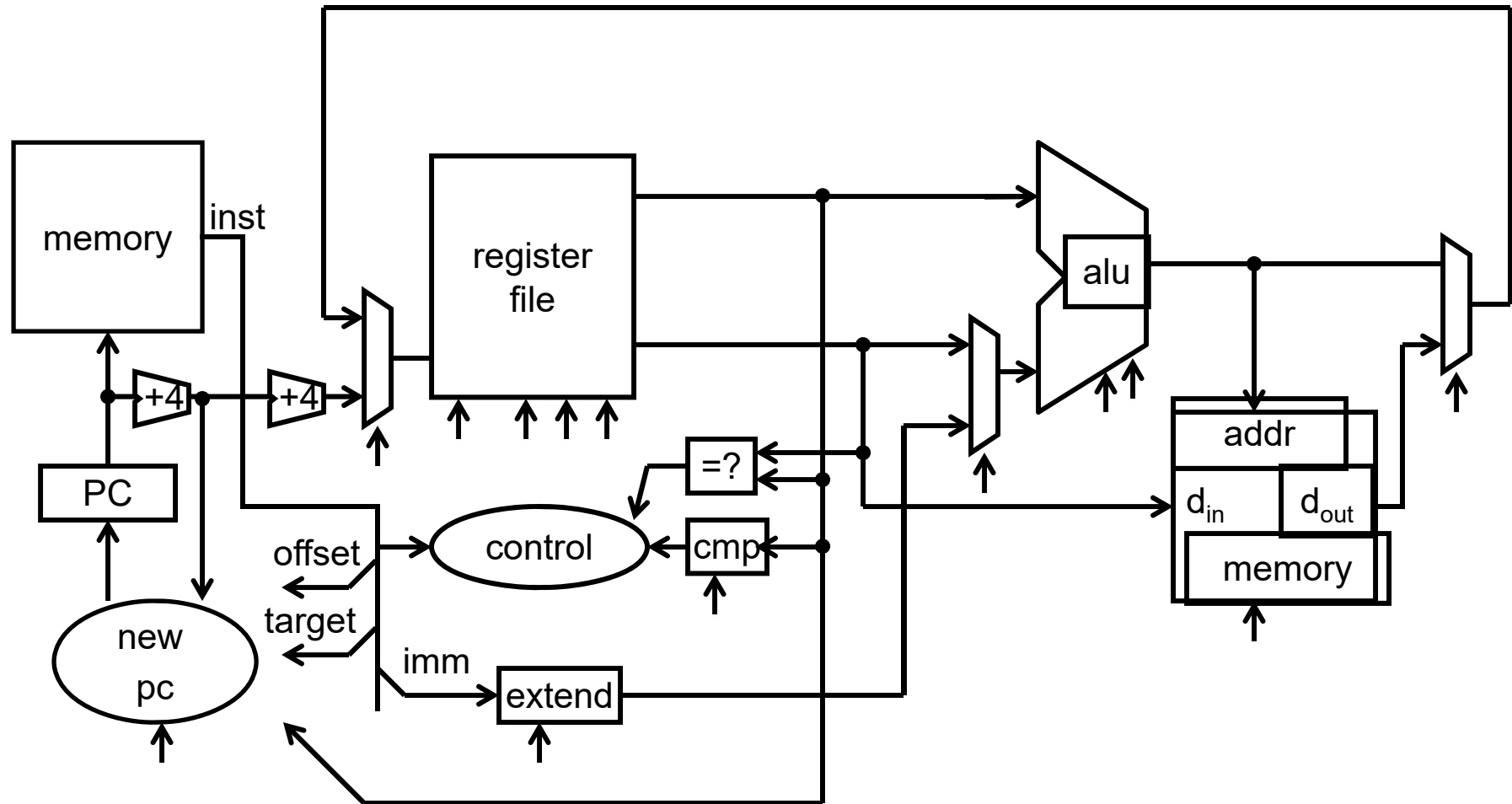
Cornell University



**Cornell CIS**  
COMPUTING AND INFORMATION SCIENCE

[Weatherspoon, Bala, Bracy, and Sirer]

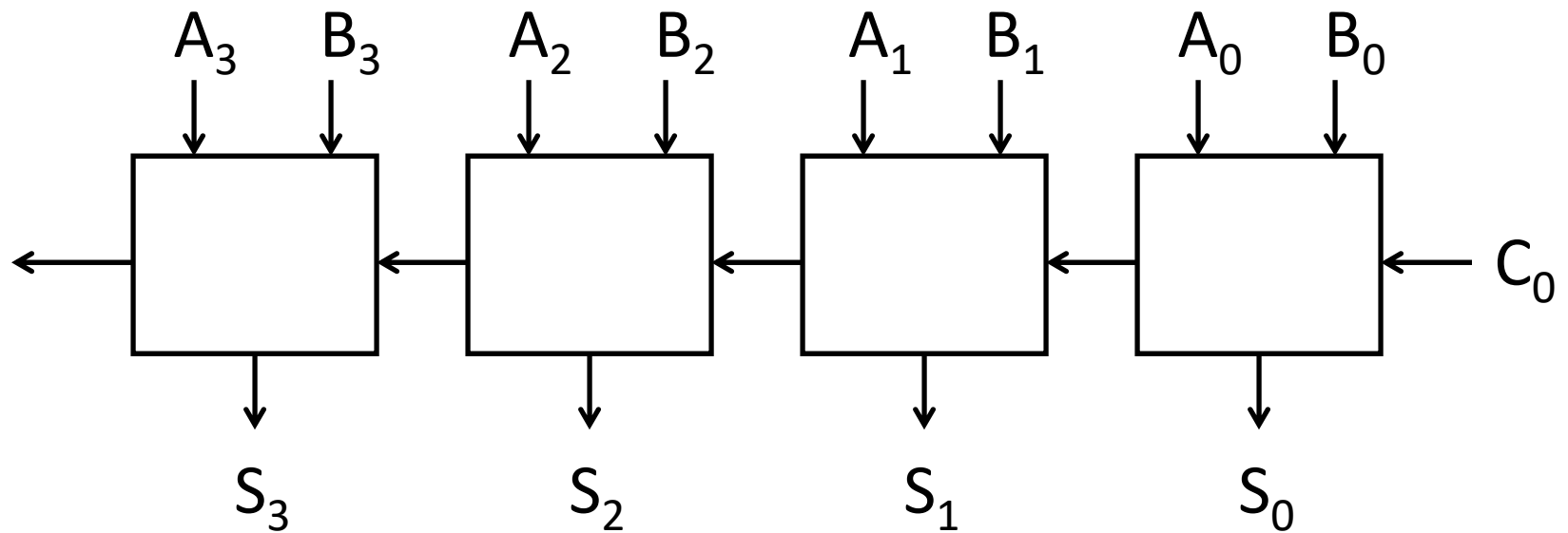
# Big Picture: Building a Processor



A single cycle processor

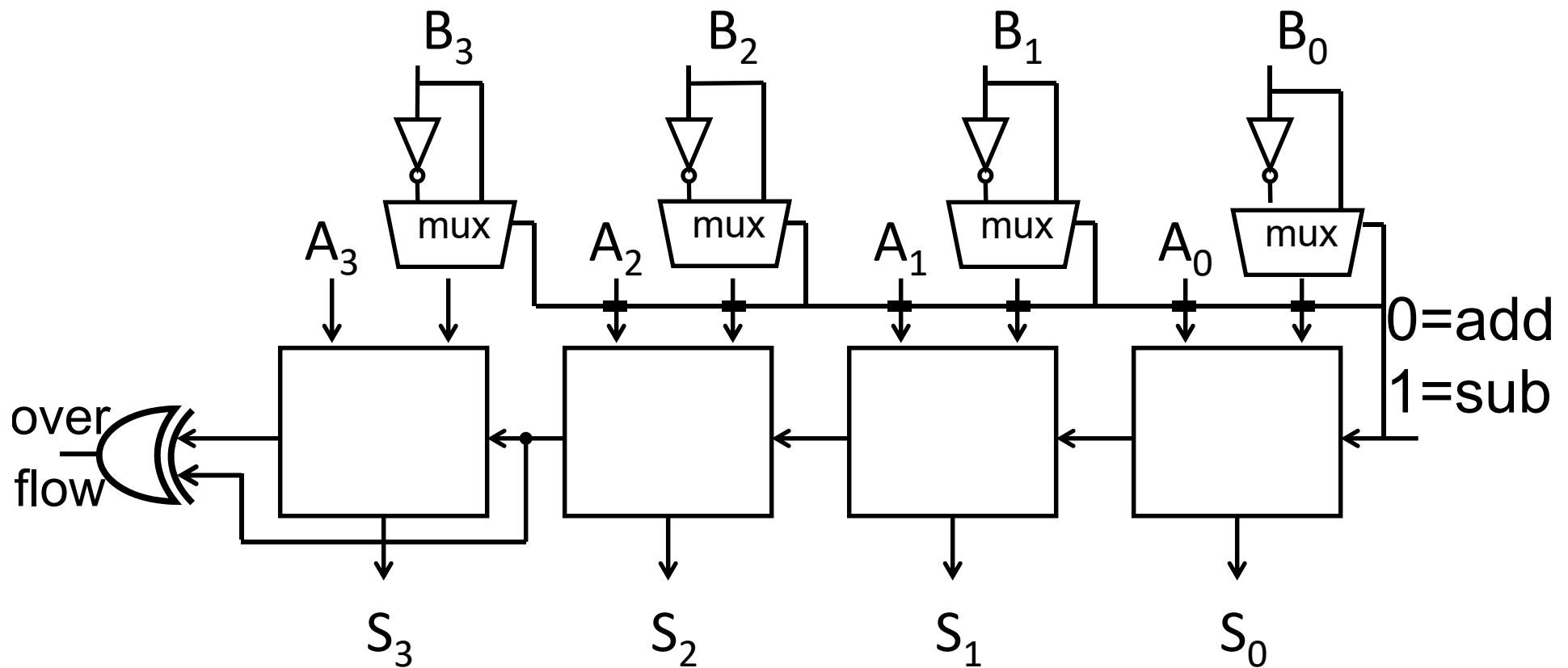
# Review

- We can generalize 1-bit Full Adders to 32 bits, 64 bits ...

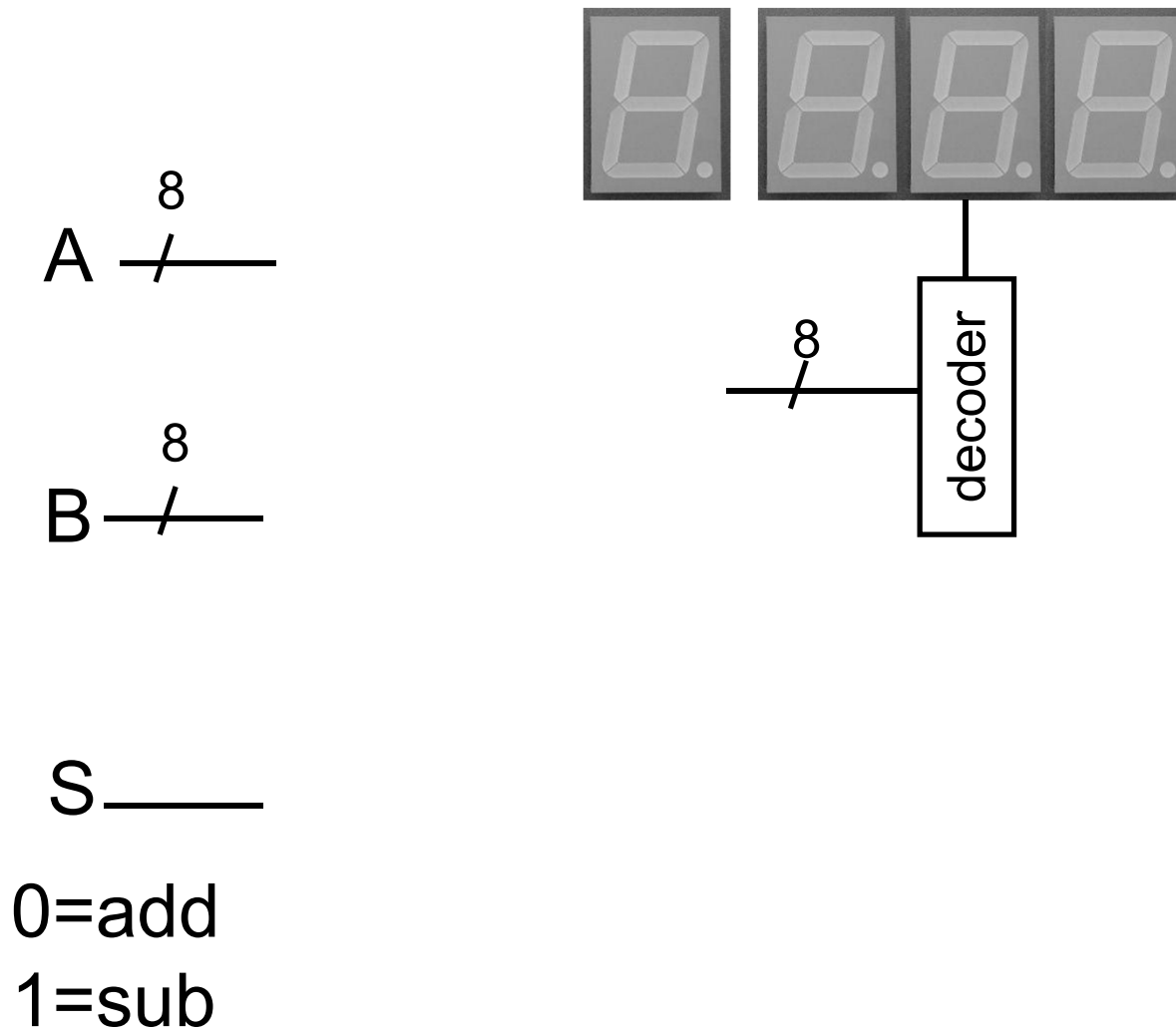


# Review

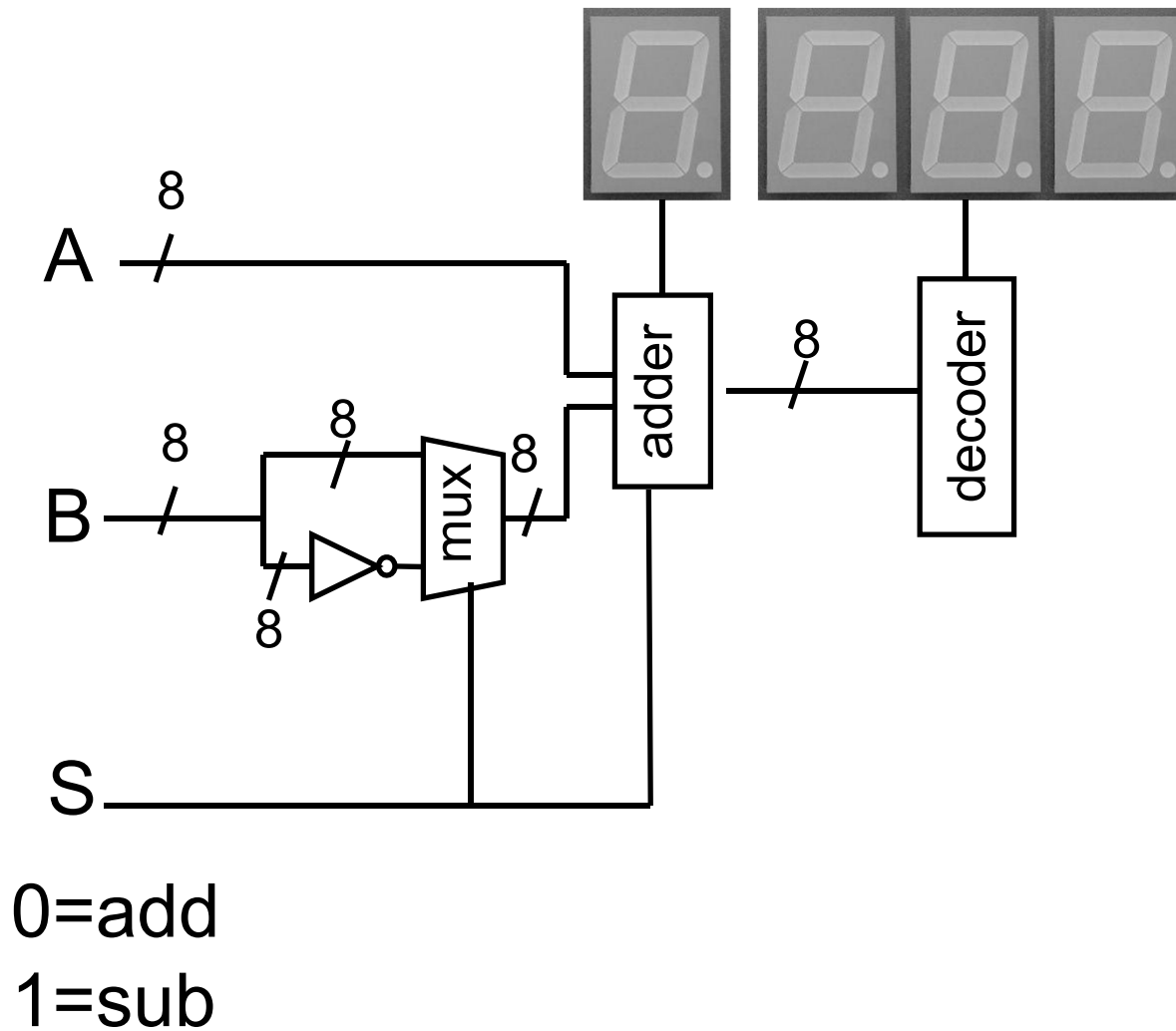
- We can generalize 1-bit Full Adders to 32 bits, 64 bits ...



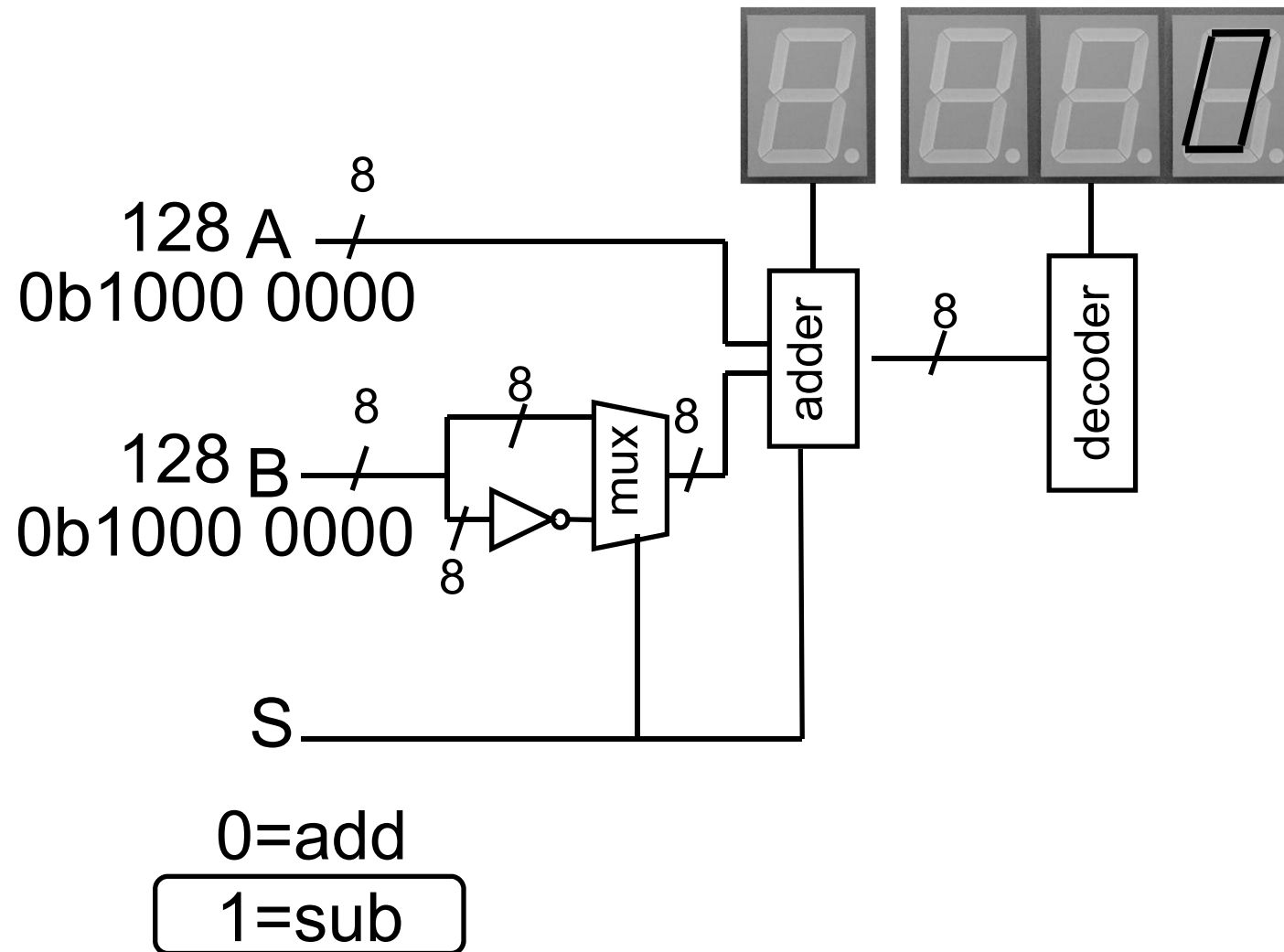
# Example: A Calculator



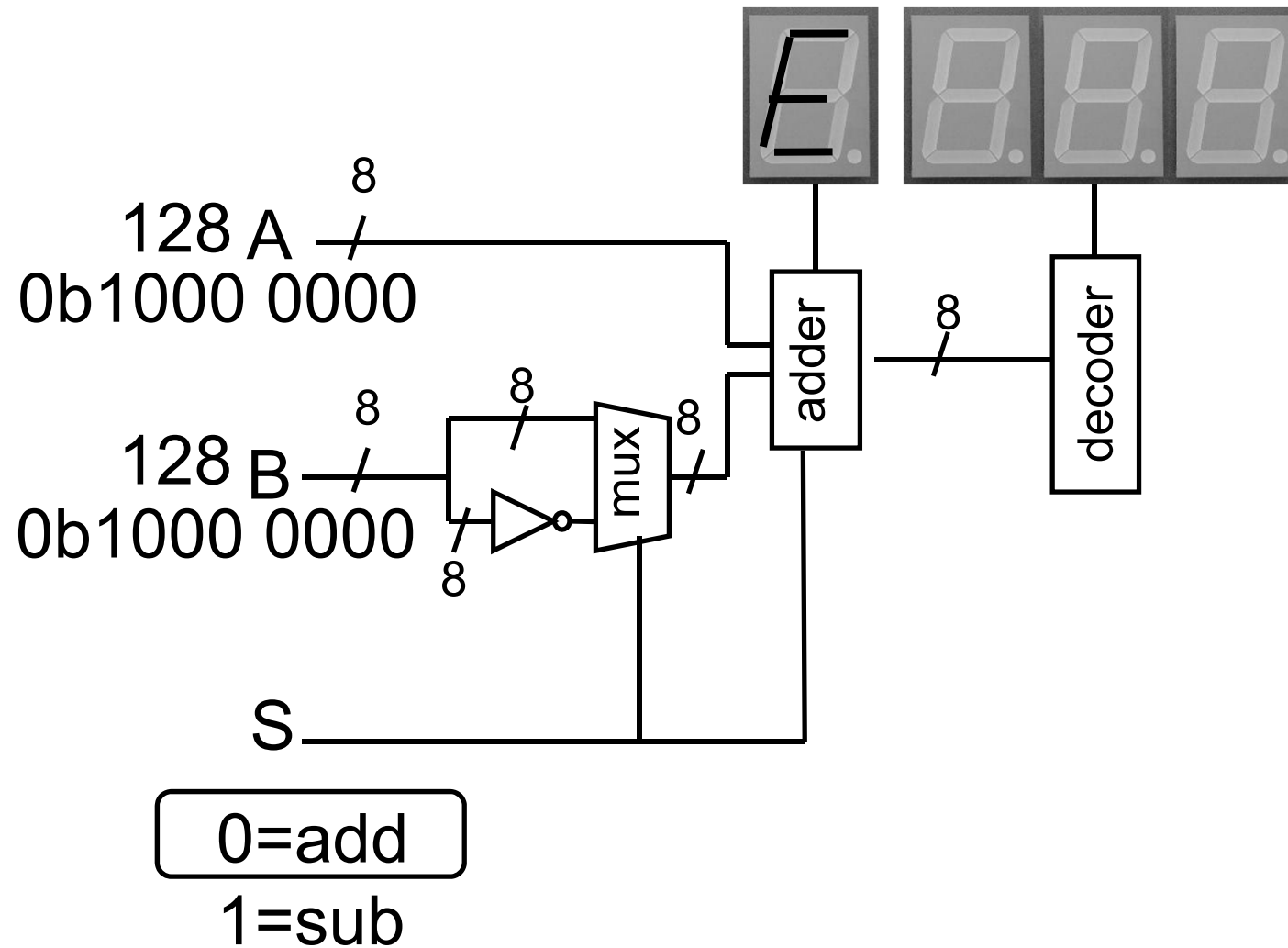
# Example: A Calculator



# Example: A Calculator



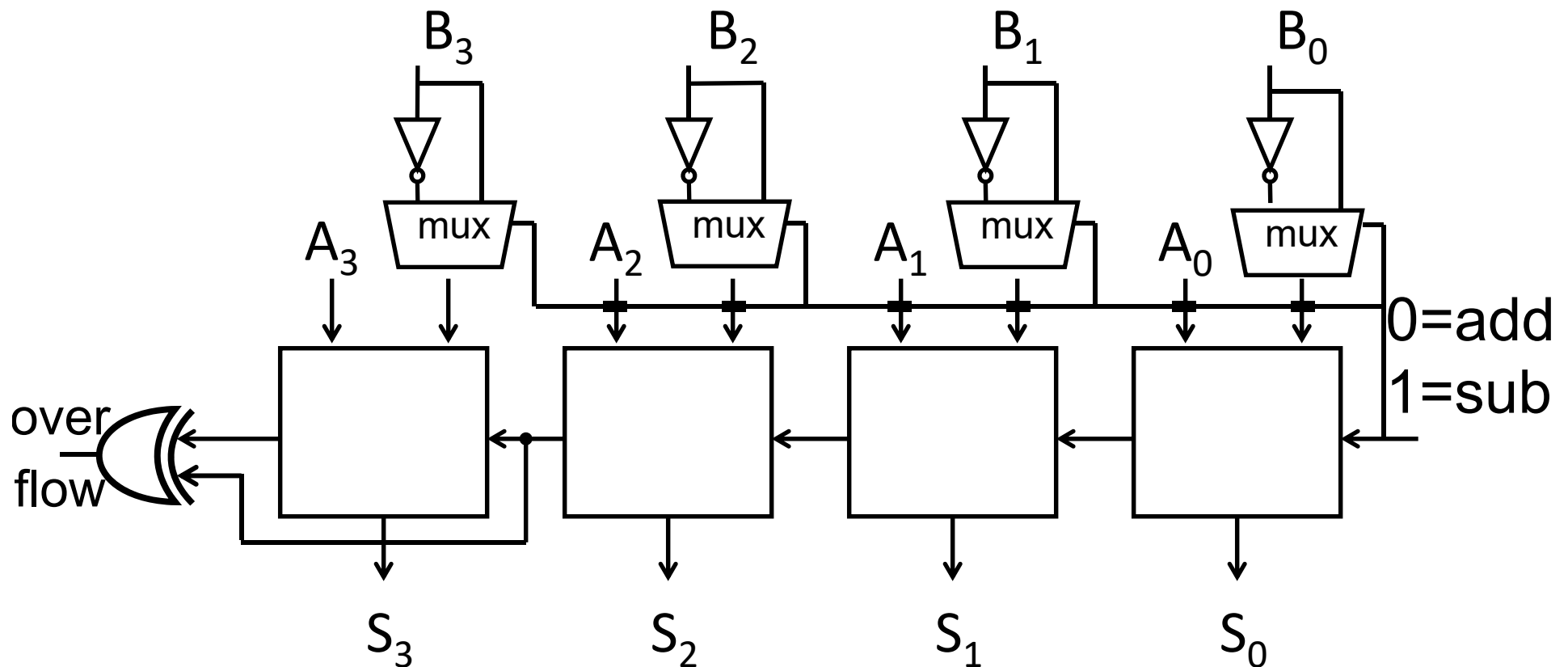
# Example: A Calculator





# Review: Efficiency and Generality

- We can generalize 1-bit Full Adders to 32 bits, 64 bits
- ...
- How long does it take to compute a result?

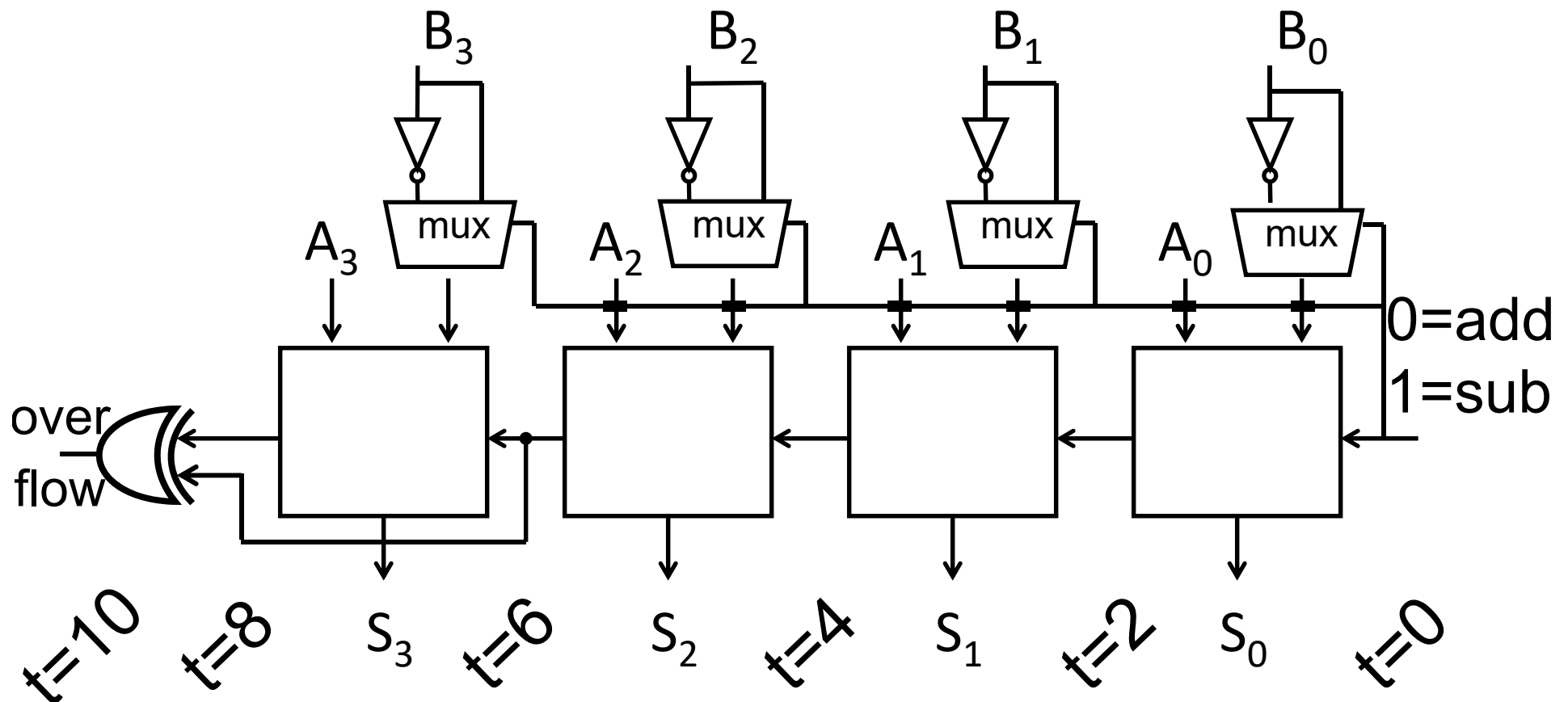


# Review: Efficiency and Generality

- We can generalize 1-bit Full Adders to 32 bits, 64 bits
- ...
- How long does it take to compute a result?
  - A) 2 ns
  - B) 2 gate delays
  - C) 10 ns
  - D) 10 gate delays
  - E) 8 gate delays

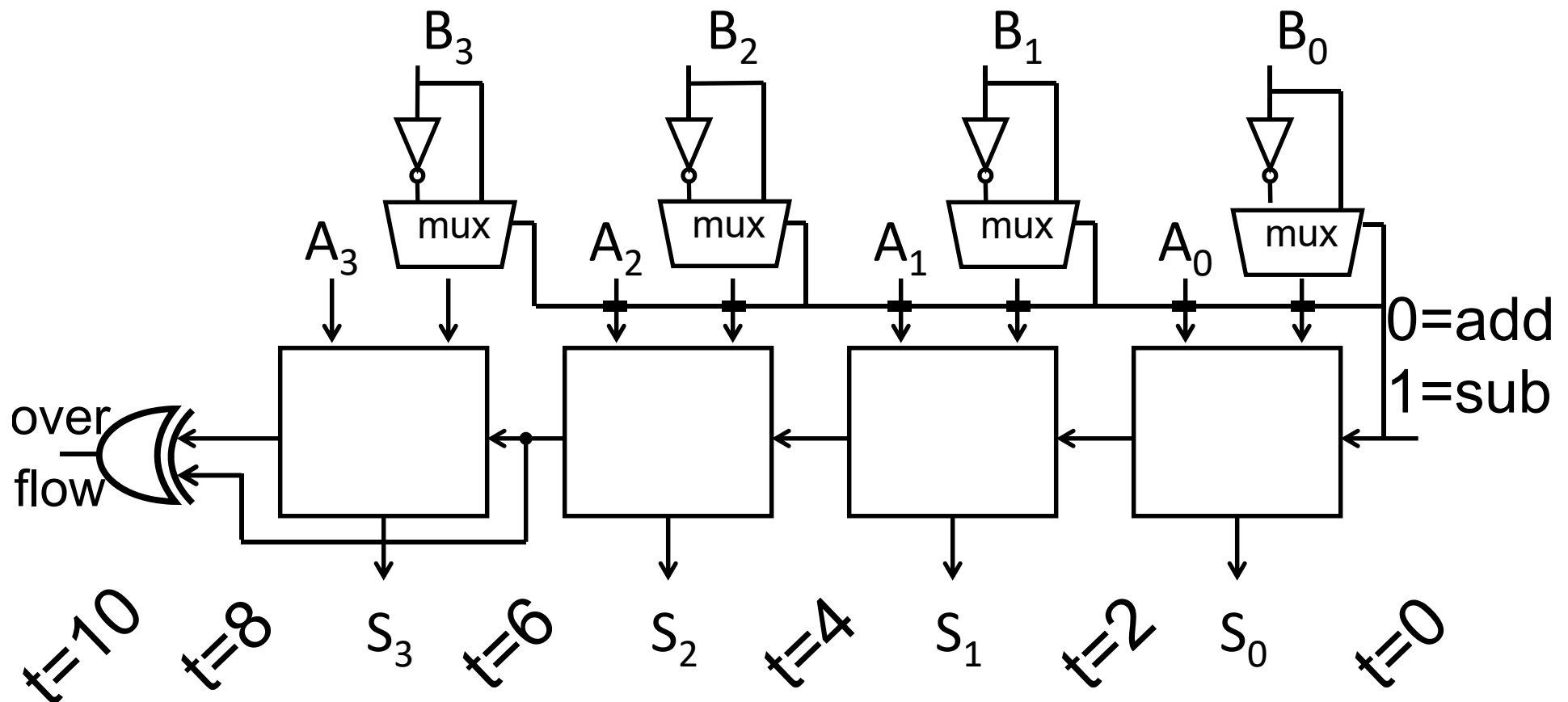
# Review: Efficiency and Generality

- We can generalize 1-bit Full Adders to 32 bits, 64 bits
- ...
- How long does it take to compute a result?



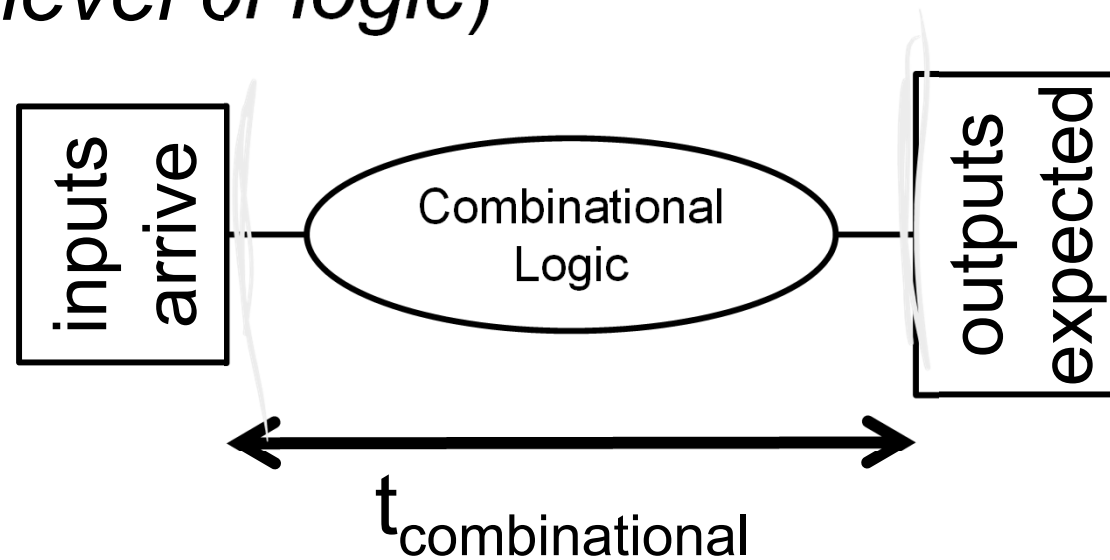
# Review: Efficiency and Generality

- We can generalize 1-bit Full Adders to 32 bits, 64 bits ...
- How long does it take to compute a result?
- Can we **store** the result?

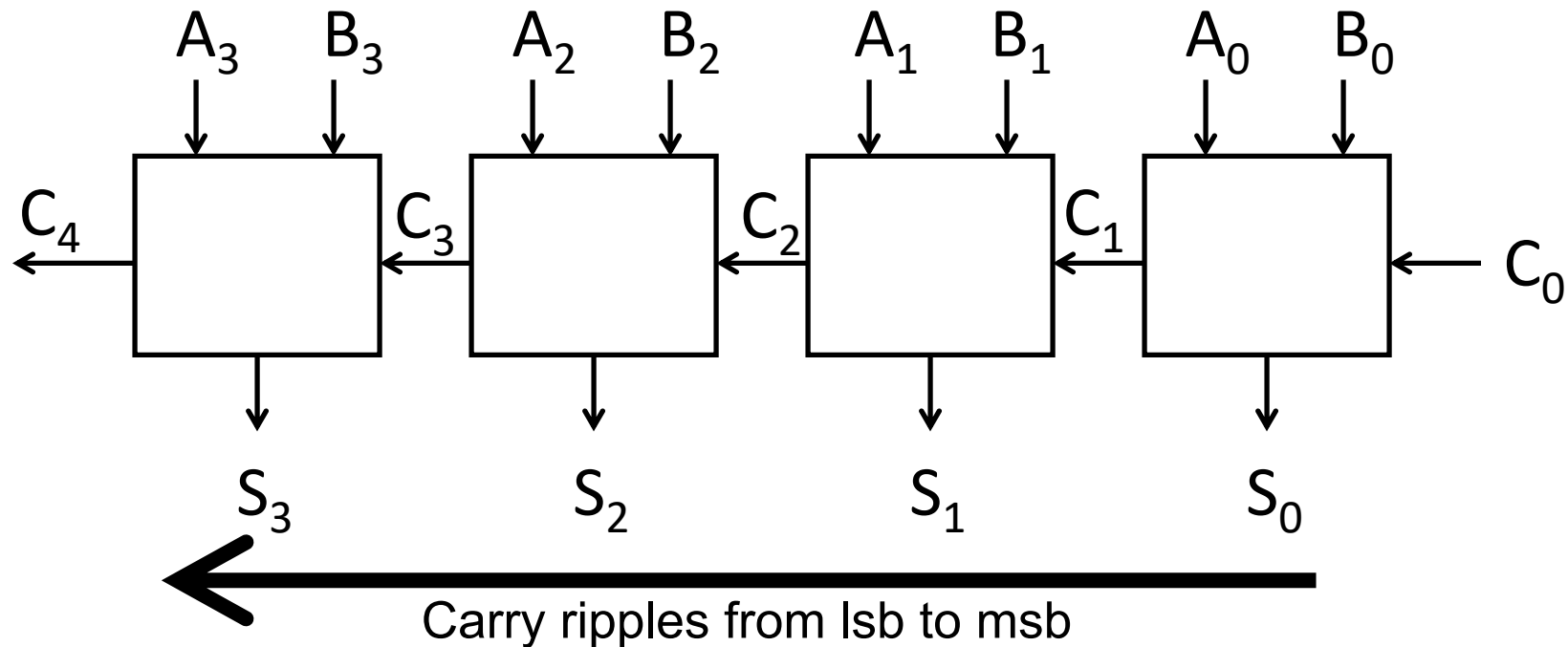


# Performance

Speed of a circuit is affected by the number of gates in series (on the *critical path* or the *deepest level of logic*)



# 4-bit Ripple Carry Adder

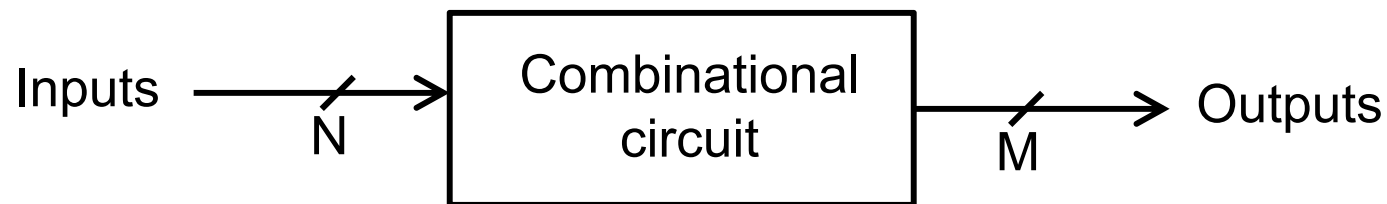


- First full adder, 2 gate delay
- Second full adder, 2 gate delay
- ...

# Stateful Components

Until now is combinational logic

- Output is computed when inputs are present
- System has no internal state
- Nothing computed in the present can depend on what happened in the past!



Need a way to record data

Need a way to build stateful circuits

Need a state-holding device

# Goals for Today

## State

- How do we store **one** bit?
- Attempts at storing (and changing) one bit
  - Set-Reset Latch
  - D Latch
  - D Flip-Flops
  - Master-Slave Flip-Flops
- Register: storing more than one bit, N-bits

## Basic Building Blocks

- Decoders and Encoders



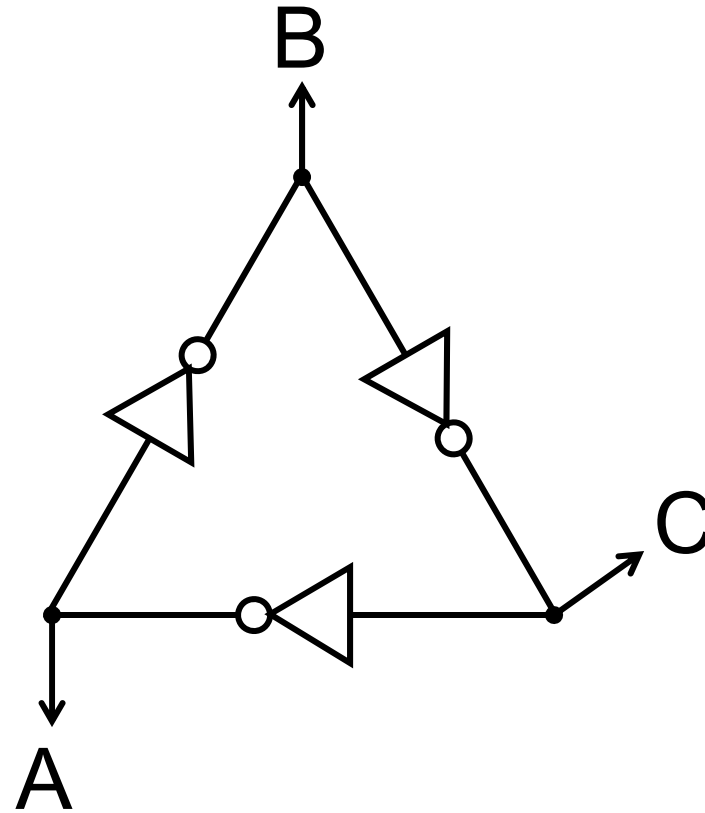


# Goal

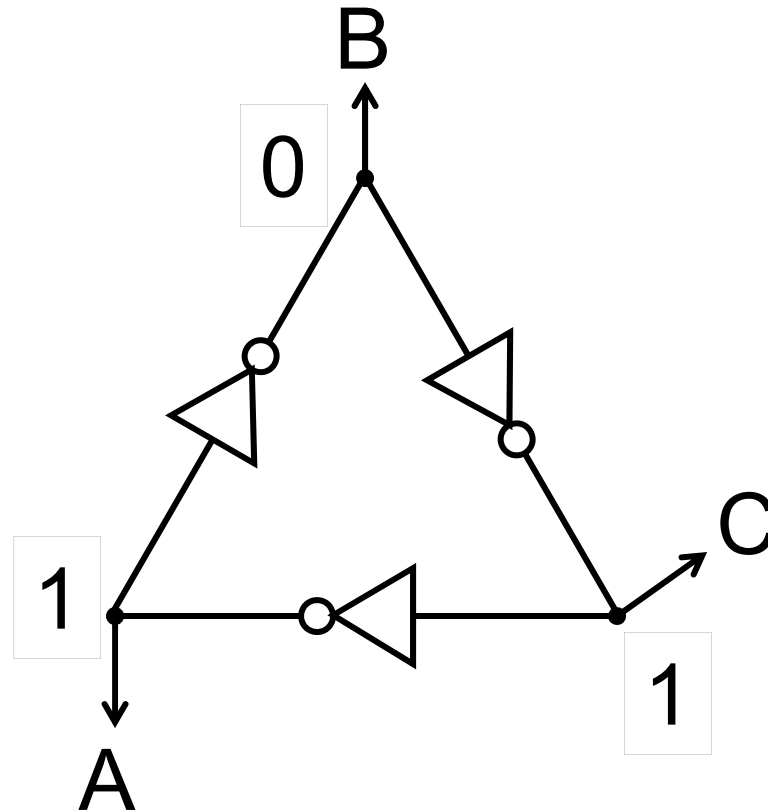
How do we store store ***one*** bit?



# First Attempt: Unstable Devices



# First Attempt: Unstable Devices

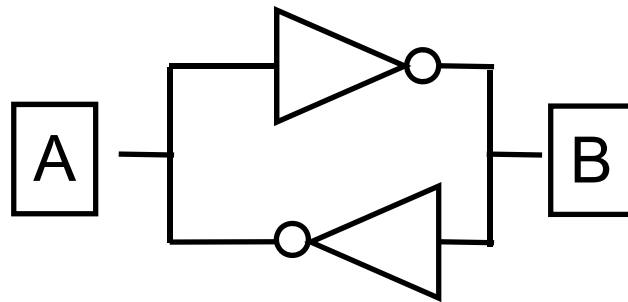


Does not work!

- Unstable
- Oscillates wildly!

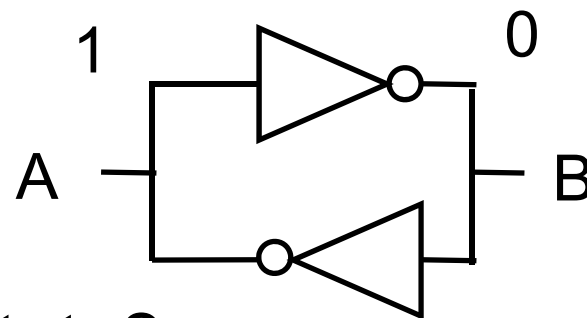
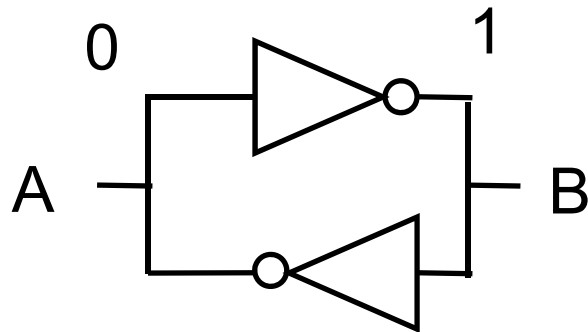
# Second Attempt: Bistable Devices

- Stable and unstable equilibria?



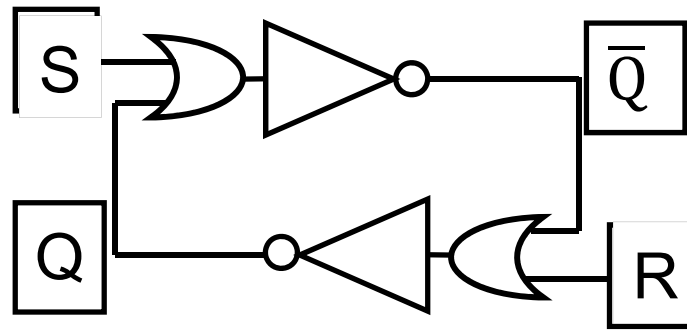
A Simple Device

In stable state,  $\bar{A} = B$

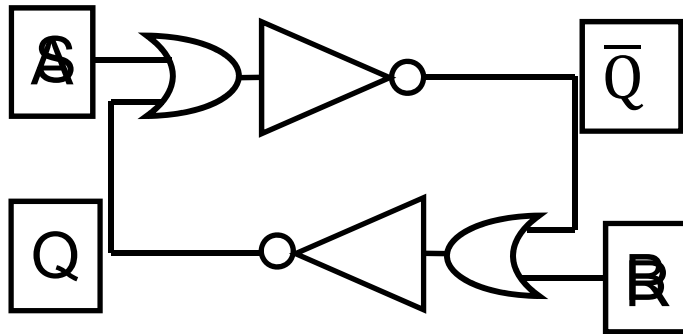


How do we change the state?

# Third Attempt: Set-Reset Latch



# Third Attempt: Set-Reset Latch

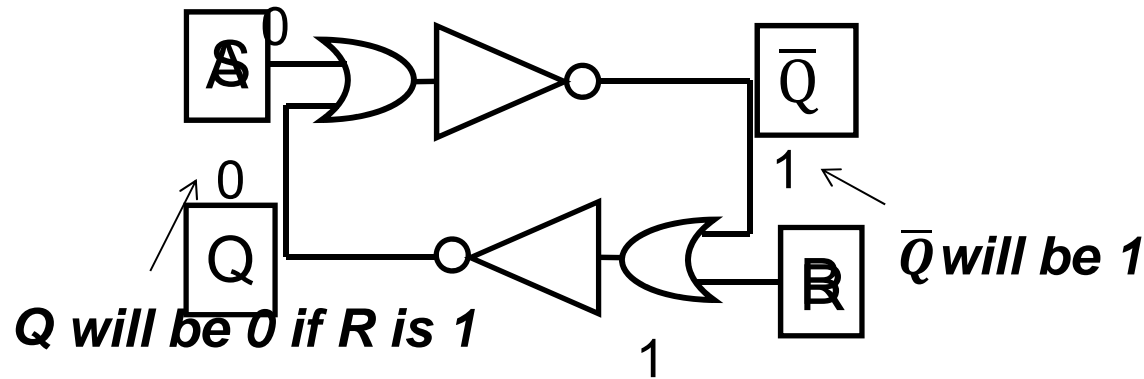


A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	Q-bar
0	0		
0	1		
1	0		
1	1		

Set-Reset (S-R) Latch  
Stores a value Q and its complement

# Third Attempt: Set-Reset Latch

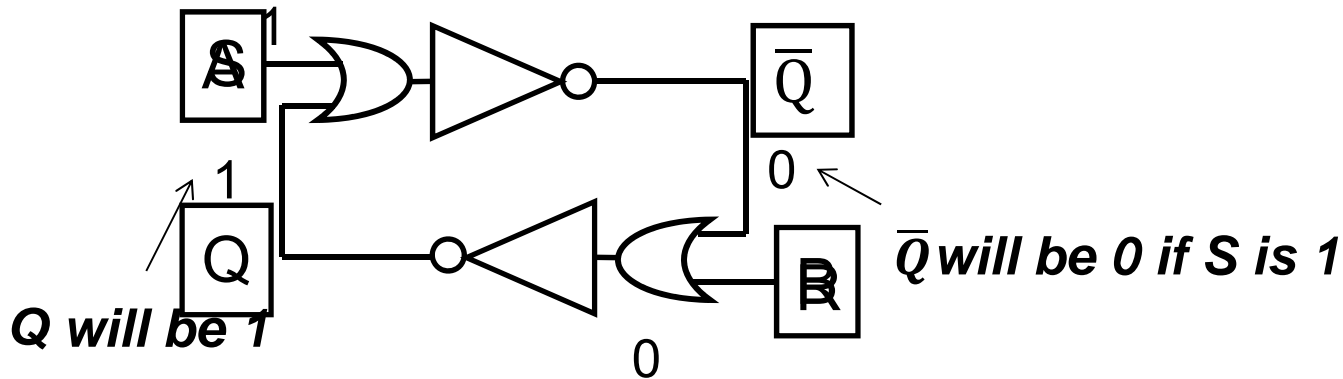


A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	$\bar{Q}$
0	0		
0	1	0	1
1	0		
1	1		

Set-Reset (S-R) Latch  
Stores a value Q and its complement

# Third Attempt: Set-Reset Latch



Set-Reset (S-R) Latch  
Stores a value Q and its  
complement

A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	$\bar{Q}$
0	0		
0	1	0	1
1	0	1	0
1	1		

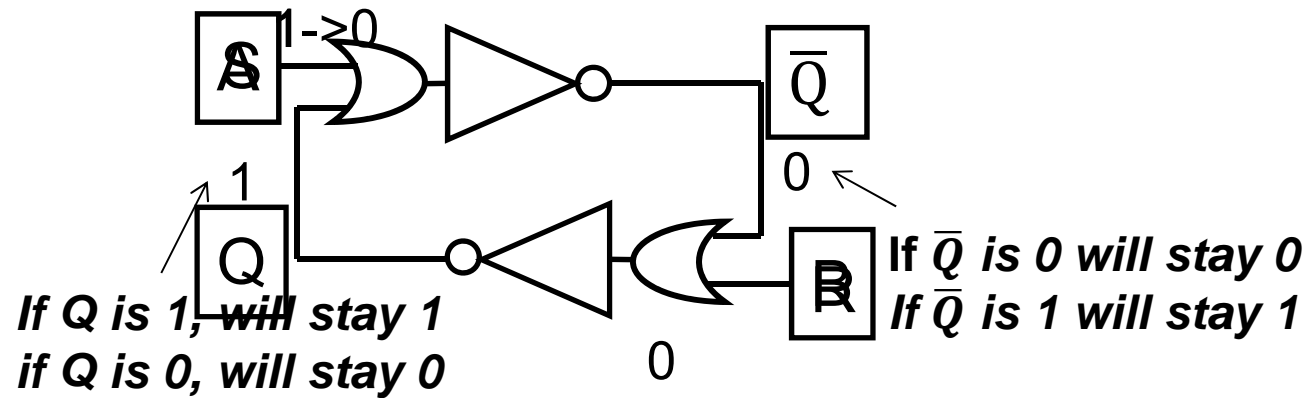
What are the values for Q and  $\bar{Q}$ ?

- a) 0 and 0
- b) 0 and 1
- c) 1 and 0
- d) 1 and 1

iClicker Question



# Third Attempt: Set-Reset Latch

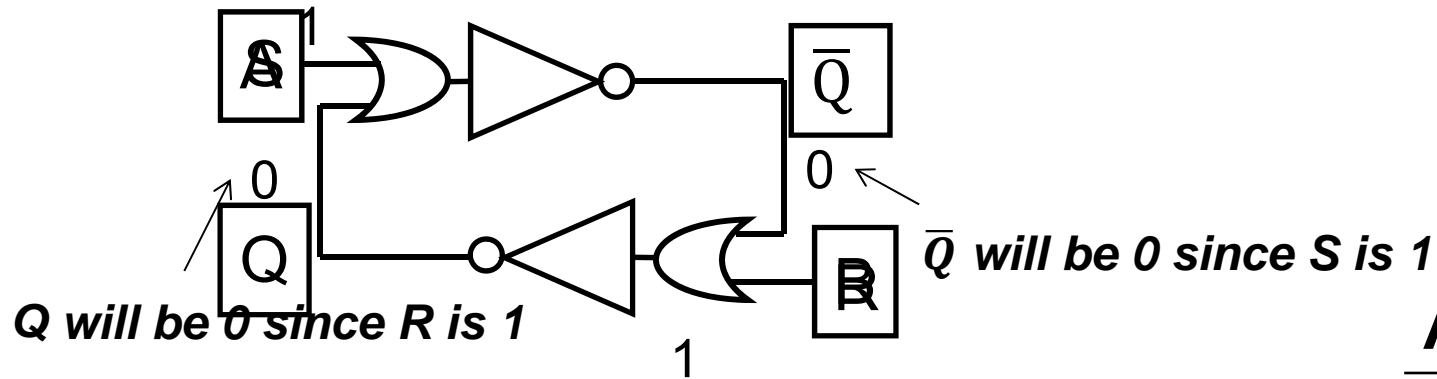


A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	0	1
1	0	1	0
1	1		

Set-Reset (S-R) Latch  
Stores a value Q and its complement

# Third Attempt: Set-Reset Latch



A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	0	1
1	0	1	0
1	1	?	?

Set-Reset (S-R) Latch  
Stores a value Q and its complement

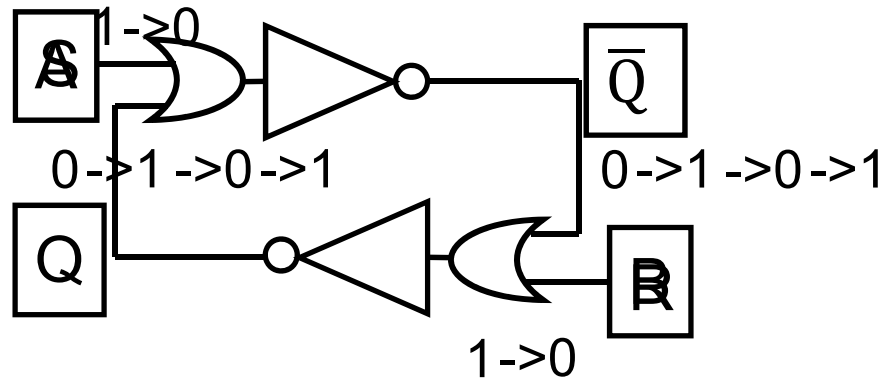
What happens when S,R changes from 1,1 to 0,0?

# iClicker Question

What's wrong with the SR Latch?

- A. Q is undefined when  $S=1$  and  $R=1$   
(That's why this is called the forbidden state.)
- B. Q oscillates between 0 and 1 when the inputs transition from  $1,1 \rightarrow 0,0$
- C. The SR Latch is problematic b/c it has two outputs to store a single bit.
- D. There is nothing wrong with the SR Latch!

# Third Attempt: Set-Reset Latch



A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

S	R	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	0	1
1	0	1	0
1	1	forbidden	

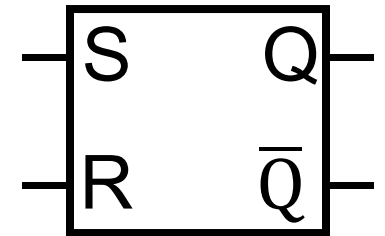
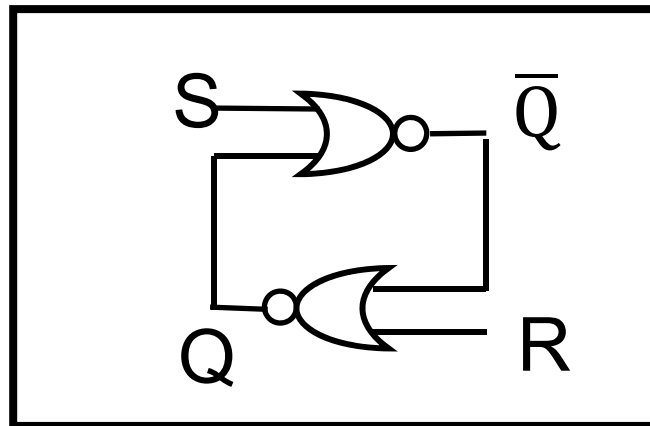
Set-Reset (S-R) Latch

Stores a value Q and its complement

What happens when S,R changes from 1,1 to 0,0?

Q and  $\bar{Q}$  become unstable and will oscillate wildly between values 0,0 to 1,1 to 0,0 to 1,1 ...

# Third Attempt: Set-Reset Latch



S	R	Q	$\bar{Q}$	
0	0	Q	$\bar{Q}$	hold
0	1	0	1	reset
1	0	1	0	set
1	1	forbidden		

Set-Reset (S-R) Latch  
Stores a value Q and its complement

# Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

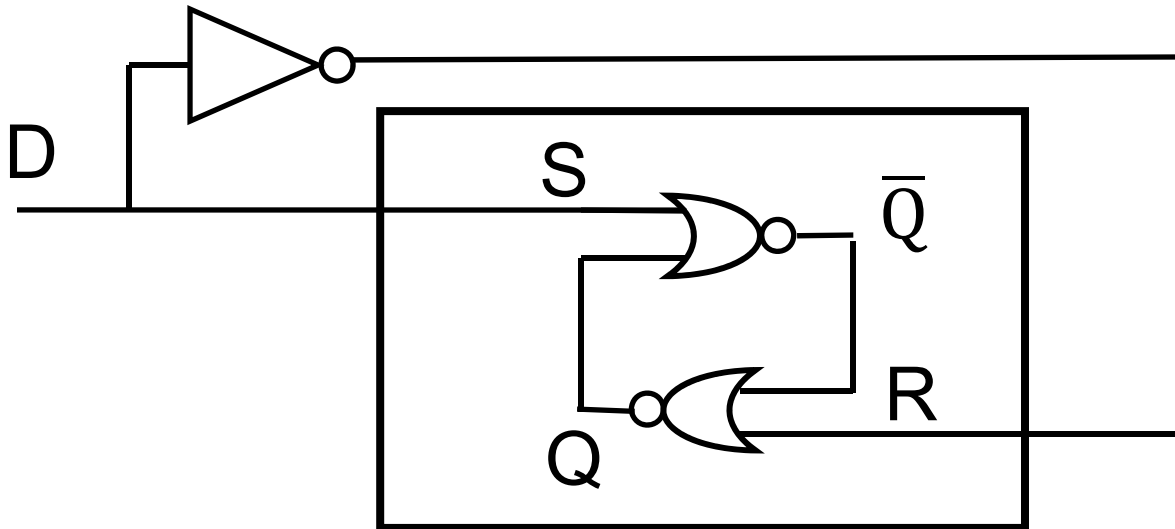


# Next Goal

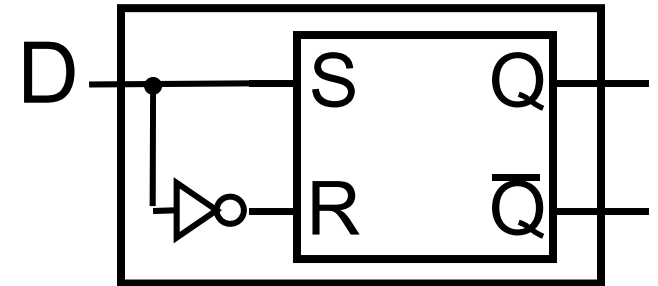
How do we avoid the forbidden state of S-R Latch?



# Fourth Attempt: (Unclocked) D Latch



Fill in the truth table?

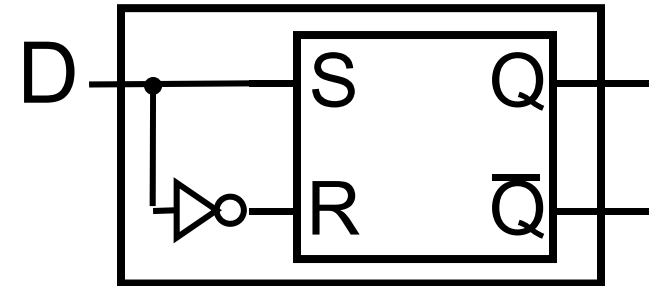
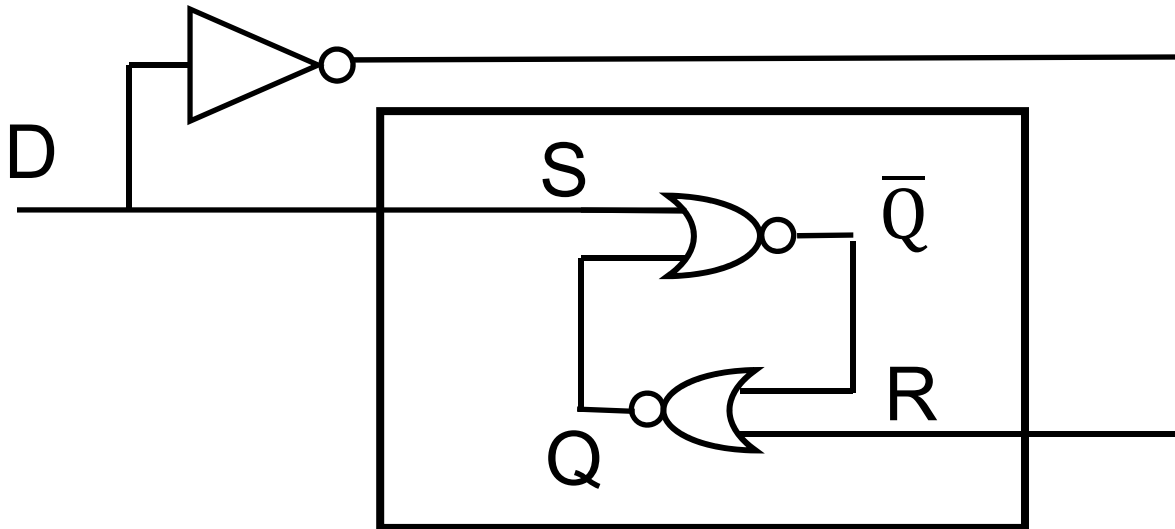


D	Q	$\bar{Q}$
0		
1		

A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0



# Fourth Attempt: (Unclocked) D Latch



Fill in the truth table?

Data (D) Latch

- Easier to use than an SR latch
- No possibility of entering an undefined state

When D changes, Q changes

– ... immediately (...after a delay of 2 Ors and 2 NOTs)

Need to control when the output changes

D	Q	$\bar{Q}$
0	0	1
1	1	0

A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

# Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding the forbidden state.



# Next Goal

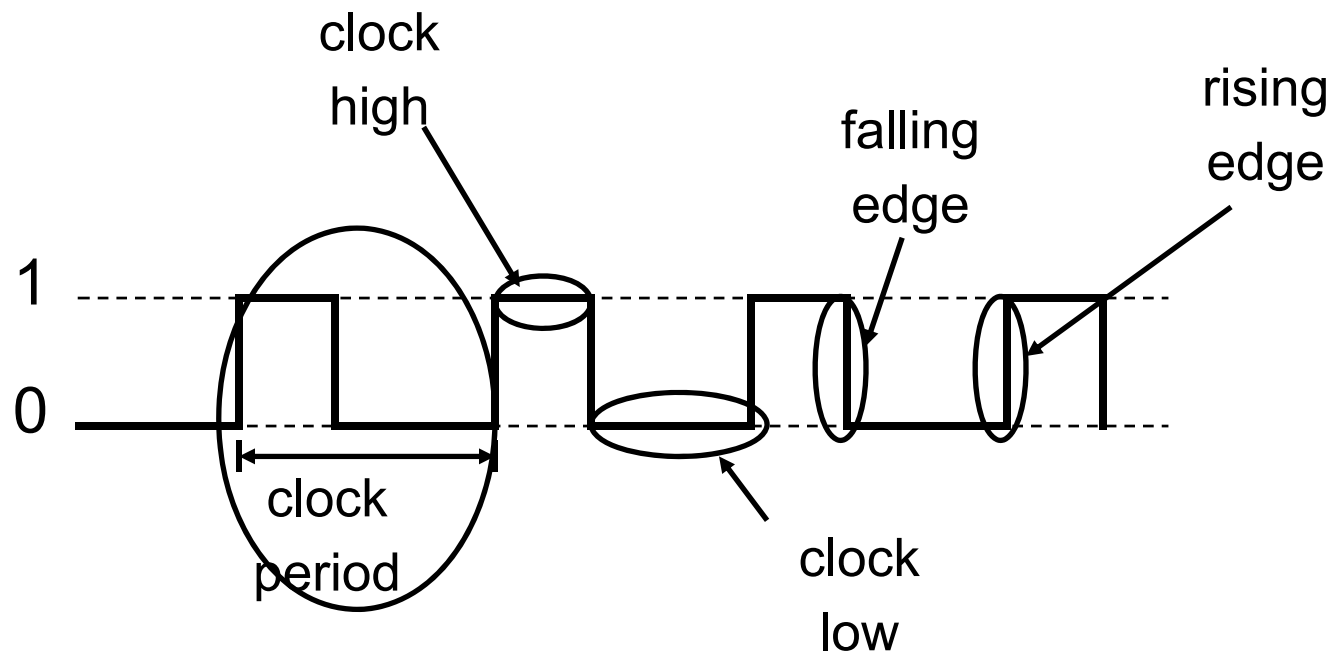
How do we coordinate state changes to a D Latch?



# Aside: Clocks

Clock helps coordinate state changes

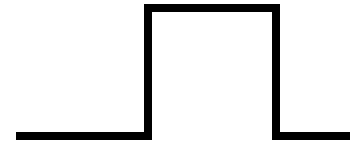
- Usually generated by an oscillating crystal
- Fixed period
- Frequency =  $1/\text{period}$



# Clock Disciplines

## Level sensitive

- State changes when clock is high (or low)



## Edge triggered

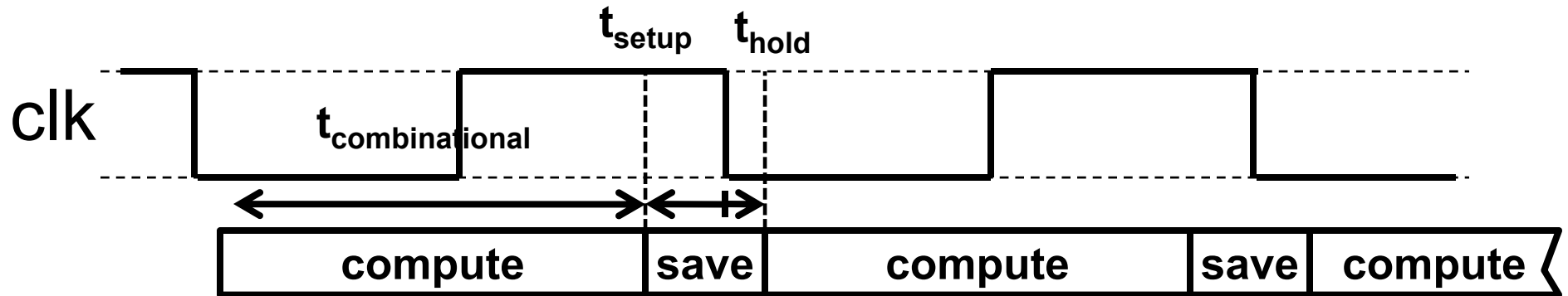
- State changes at clock edge



# Clock Methodology

## Clock Methodology

- Negative edge, synchronous

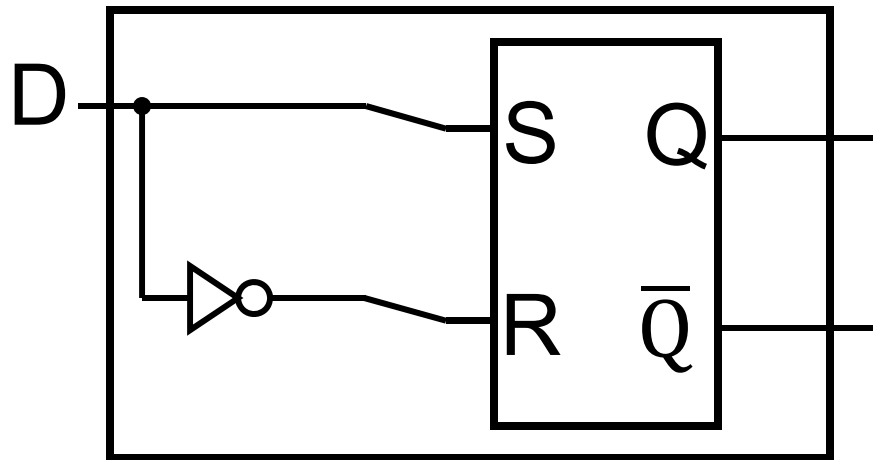


Edge-Triggered → signals must be stable near falling edge

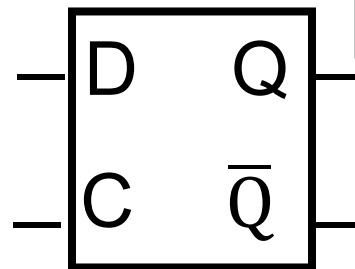
“near” = before and after

$t_{\text{setup}}$   $t_{\text{hold}}$

# Round 2: D Latch (1)



- Inverter prevents SR Latch from entering 1,1 state

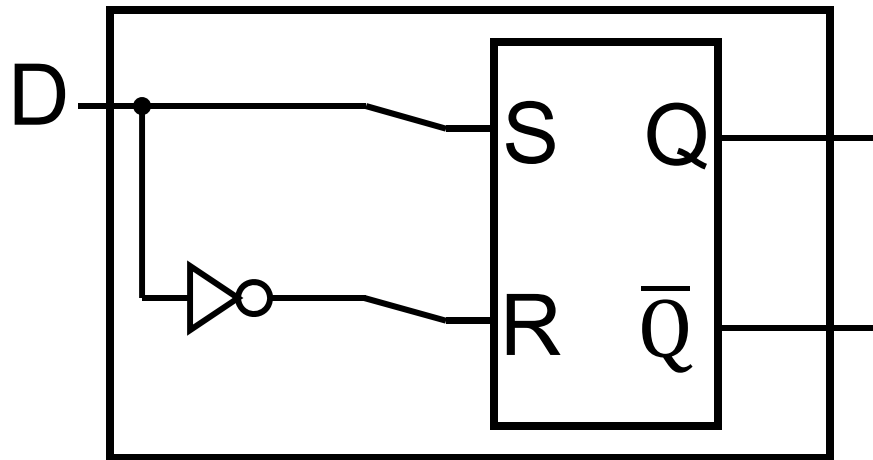


	D	Q	$\bar{Q}$
0	0		
1	1		

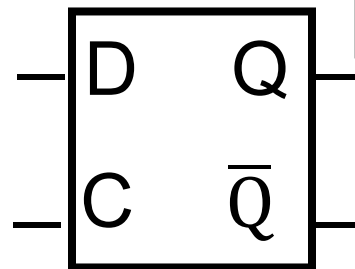
*Reset*

*Set*

# Round 2: D Latch (1)



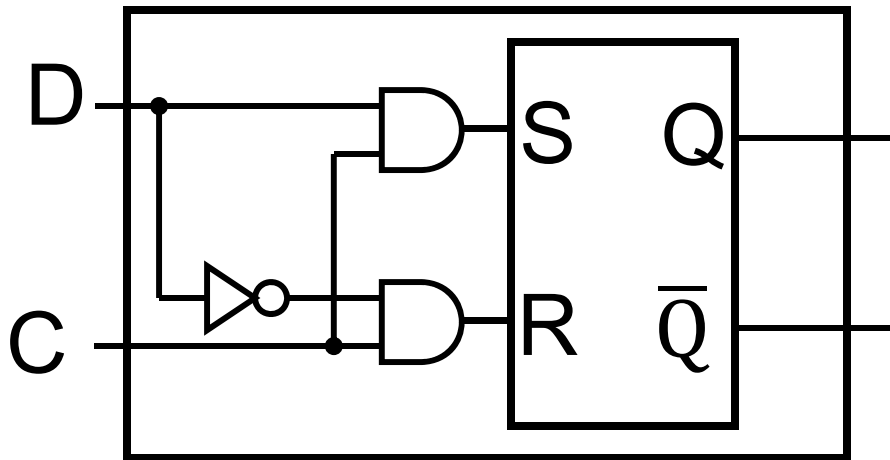
- Inverter prevents SR Latch from entering 1,1 state



	D	Q	$\bar{Q}$	
	0	0	1	<i>Reset</i>
	1	1	0	<i>Set</i>



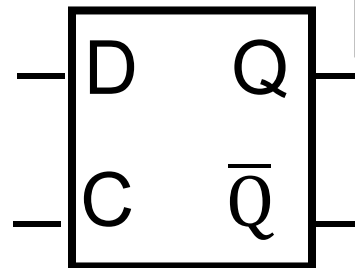
# Round 2: D Latch (1)



- Level sensitive
- Inverter prevents SR Latch from entering 1,1 state
- C enables changes

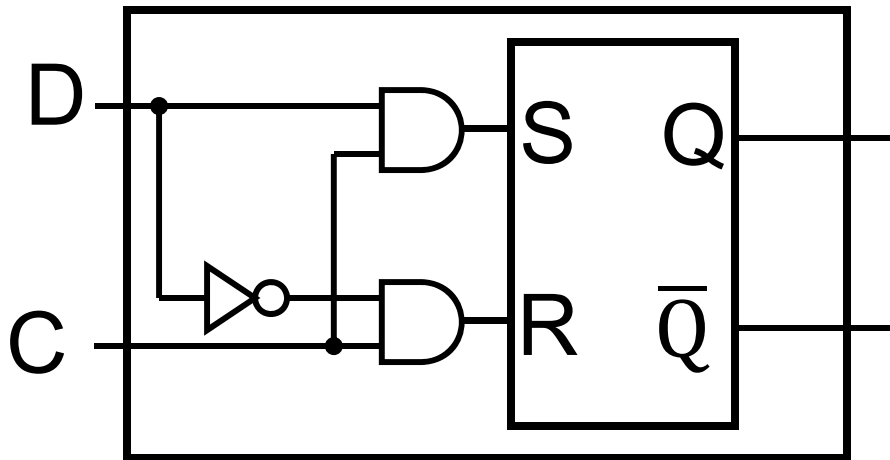
$C = 1$ , D Latch *transparent*:  
set/reset (according to D)

$C = 0$ , D Latch *opaque*:  
keep state (ignore D)



C	D	Q	$\bar{Q}$	
0	0			No Change
0	1			
1	0			Reset
1	1			Set

# Round 2: D Latch (1)

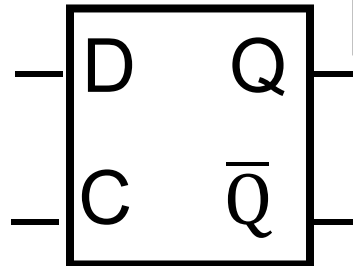


- Level sensitive
- Inverter prevents SR Latch from entering 1,1 state
- C enables changes

$C = 1$ , D Latch *transparent*:  
set/reset (according to D)

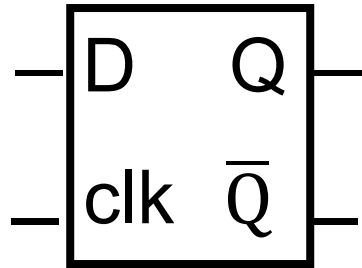
$C = 0$ , D Latch *opaque*:  
keep state (ignore D)

S	R	Q	$\bar{Q}$	
0	0	Q	$\bar{Q}$	hold
0	1	0	1	reset
1	0	1	0	set
1	1	forbidden		



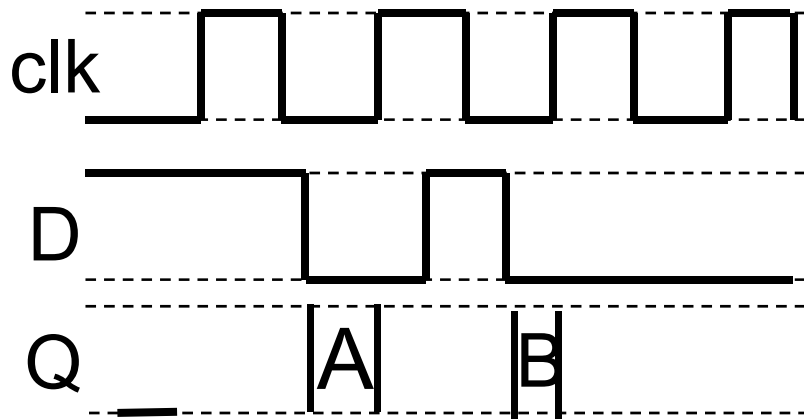
C	D	Q	$\bar{Q}$	
0	0	Q	$\bar{Q}$	No Change
0	1	Q	$\bar{Q}$	
1	0	0	1	Reset
1	1	1	0	Set

# iClicker Question



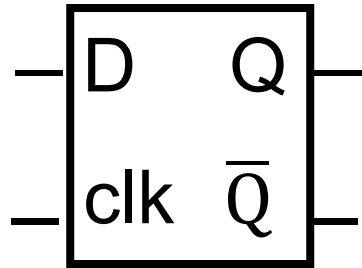
What is the value of Q at A & B?

- a)  $A = 0, B = 0$
- b)  $A = 0, B = 1$
- c)  $A = 1, B = 0$
- d)  $A = 1, B = 1$



clk	D	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	Q	$\bar{Q}$
1	0	0	1
1	1	1	0

# iClicker Question



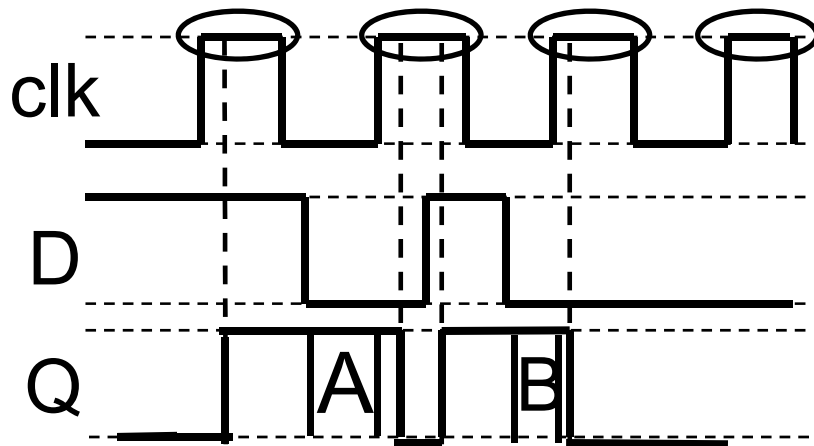
What is the value of Q at A & B?

a)  $A = 0, B = 0$

b)  $A = 0, B = 1$

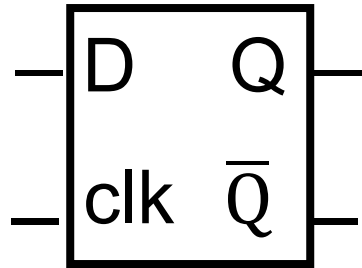
c)  $A = 1, B = 0$

d)  $A = 1, B = 1$



clk	D	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	Q	$\bar{Q}$
1	0	0	1
1	1	1	0

# iClicker Question



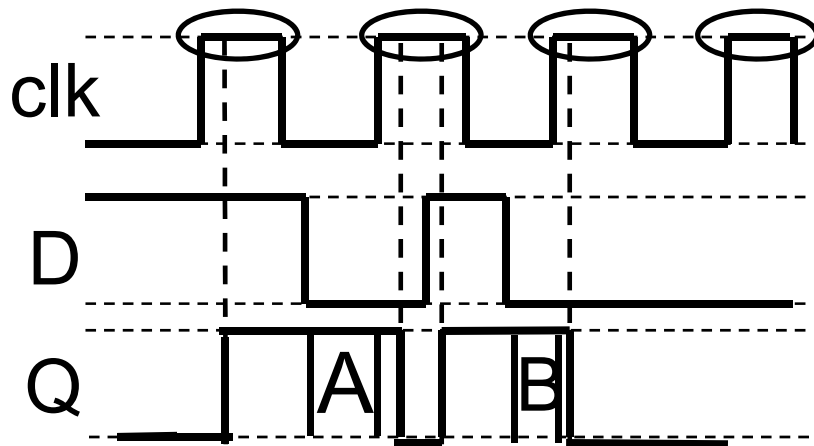
Level Sensitive D Latch

Clock high:

set/reset (according to D)

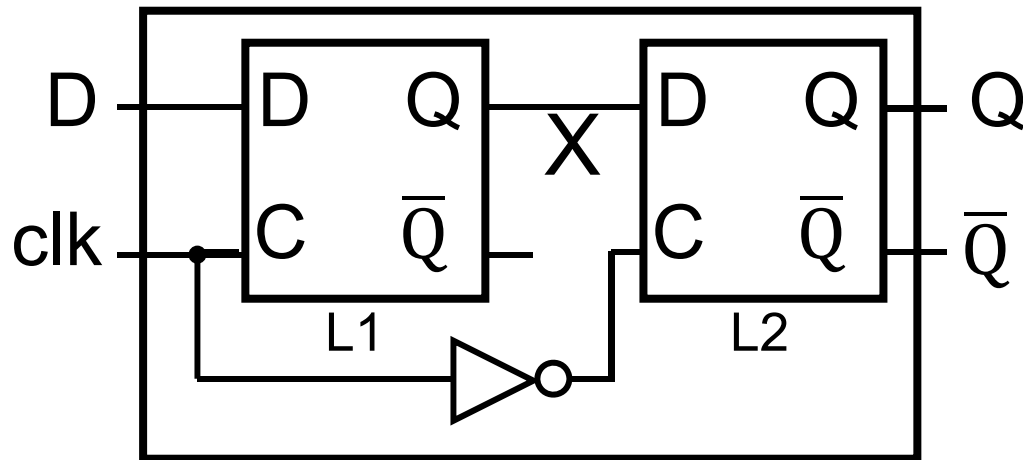
Clock low:

keep state (ignore D)



clk	D	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	Q	$\bar{Q}$
1	0	0	1
1	1	1	0

# Round 3: D Flip-Flop

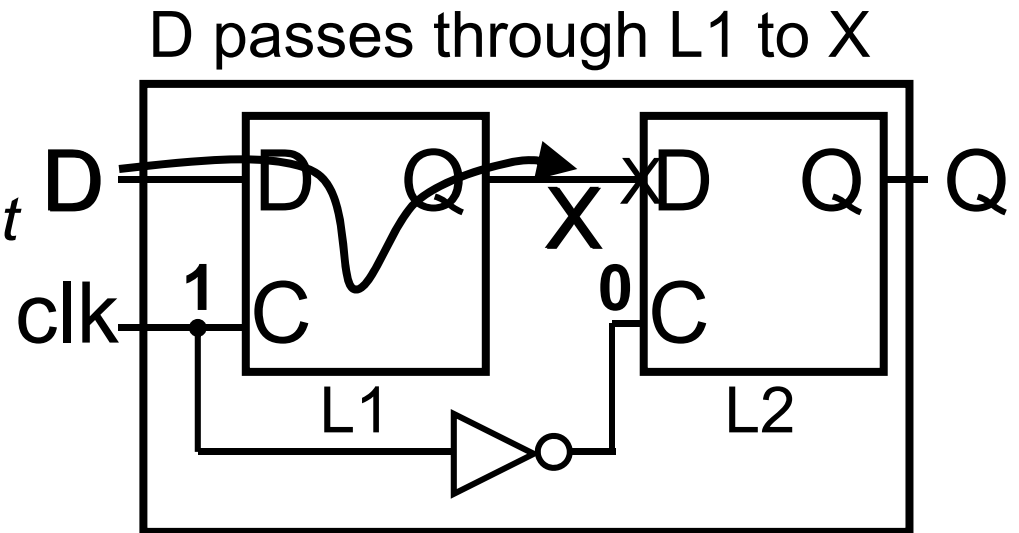


- Edge-Triggered
- Data captured when clock high
- Output changes only on falling edges

# Round 3: D Flip-Flop

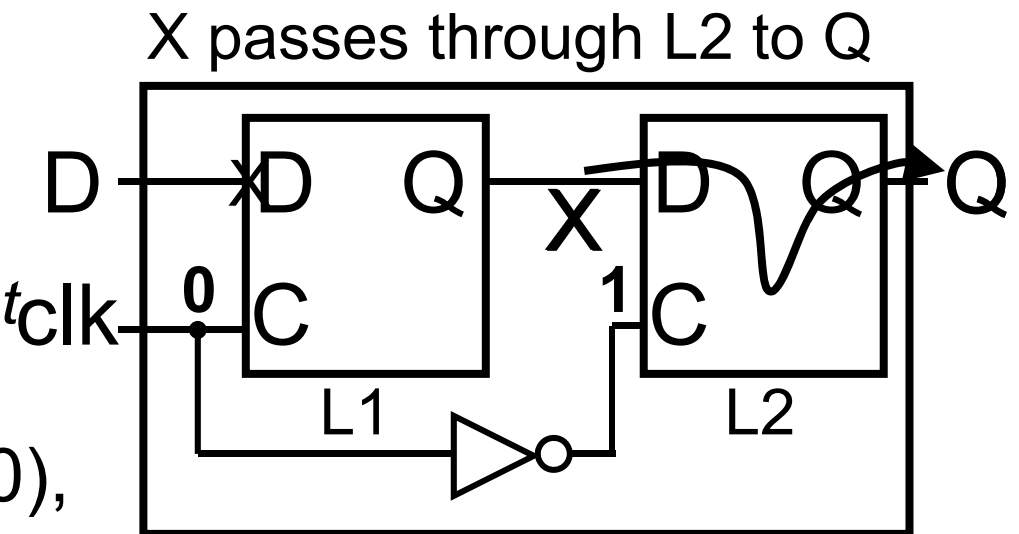
Clock = 1: L1 *transparent*  
L2 *opaque*

When *CLK* rises ( $0 \rightarrow 1$ ),  
now *X* can change,  
*Q* does not change

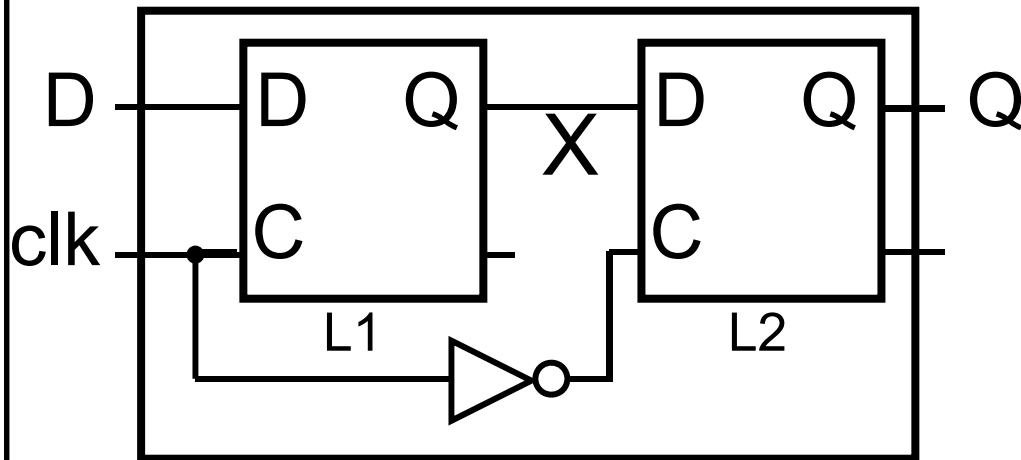


Clock = 0: L1 *opaque*  
L2 *transparent*

When **CLK falls** ( $1 \rightarrow 0$ ),  
*Q* gets *X*, *X* cannot change

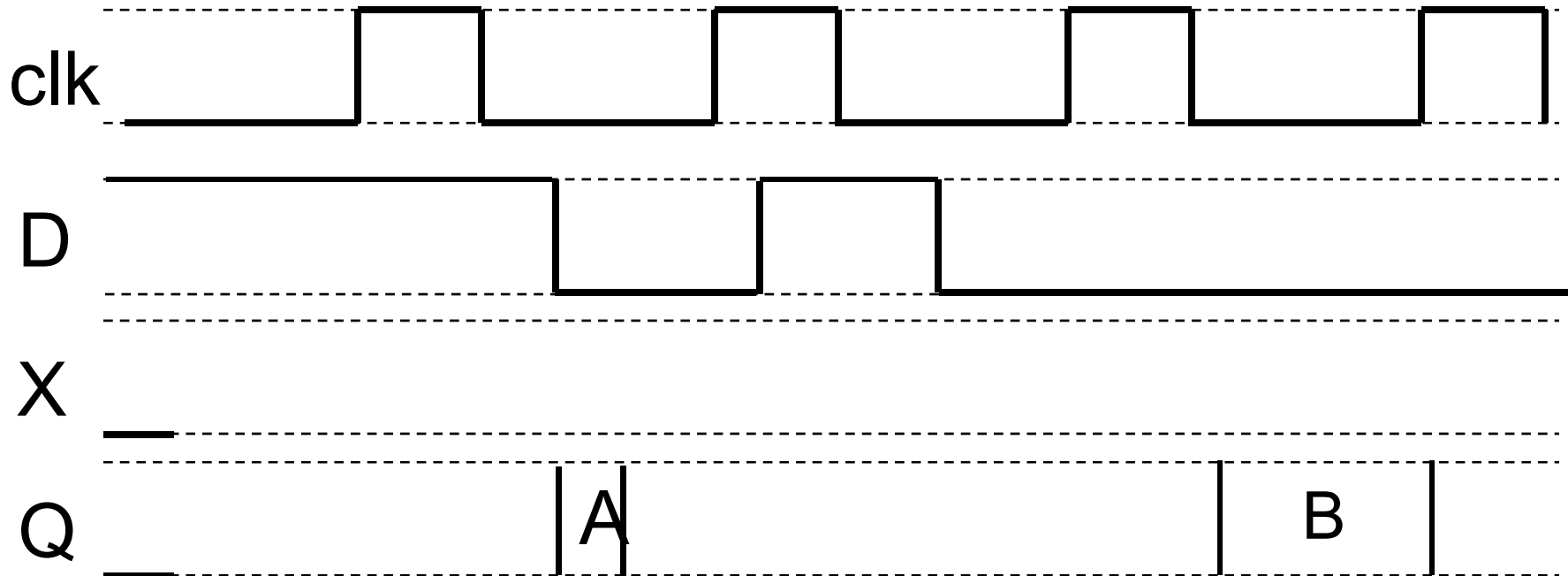


# iClicker Question – start here



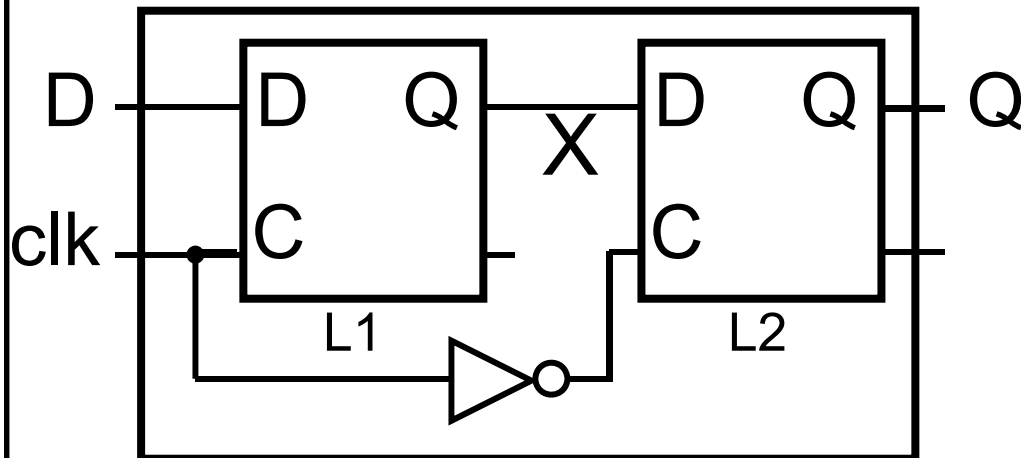
What is the value of Q at A & B?

- a)  $A = 0, B = 0$
- b)  $A = 0, B = 1$
- c)  $A = 1, B = 0$
- d)  $A = 1, B = 1$



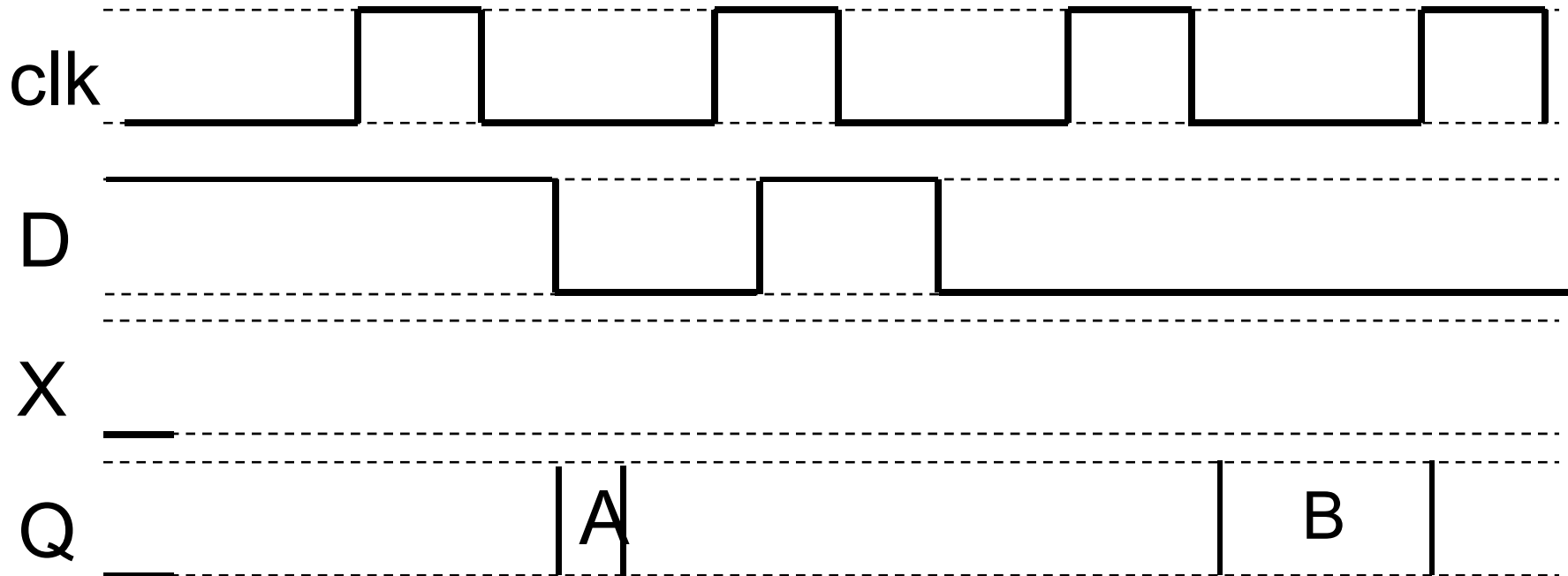


# iClicker Question – start here

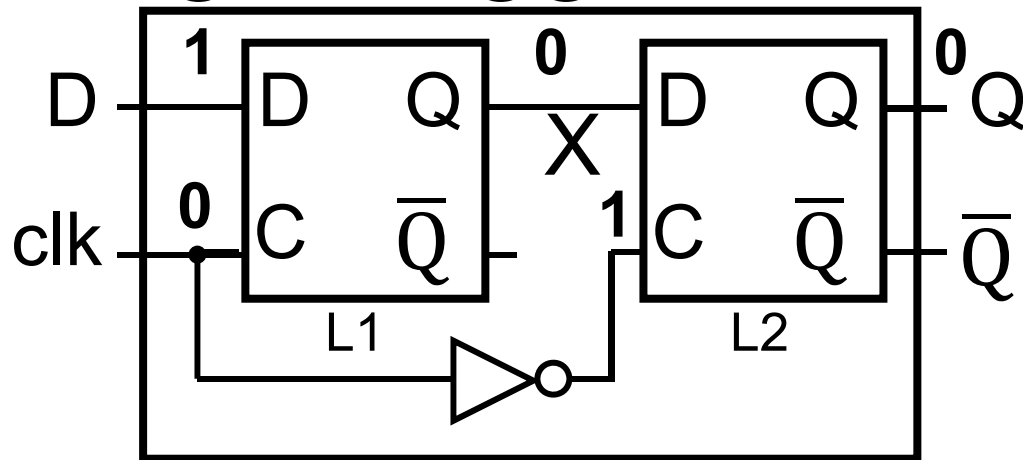


What is the value of Q at A & B?

- a)  $A = 0, B = 0$
- b)  $A = 0, B = 1$
- ☒ c)  $A = 1, B = 0$
- d)  $A = 1, B = 1$

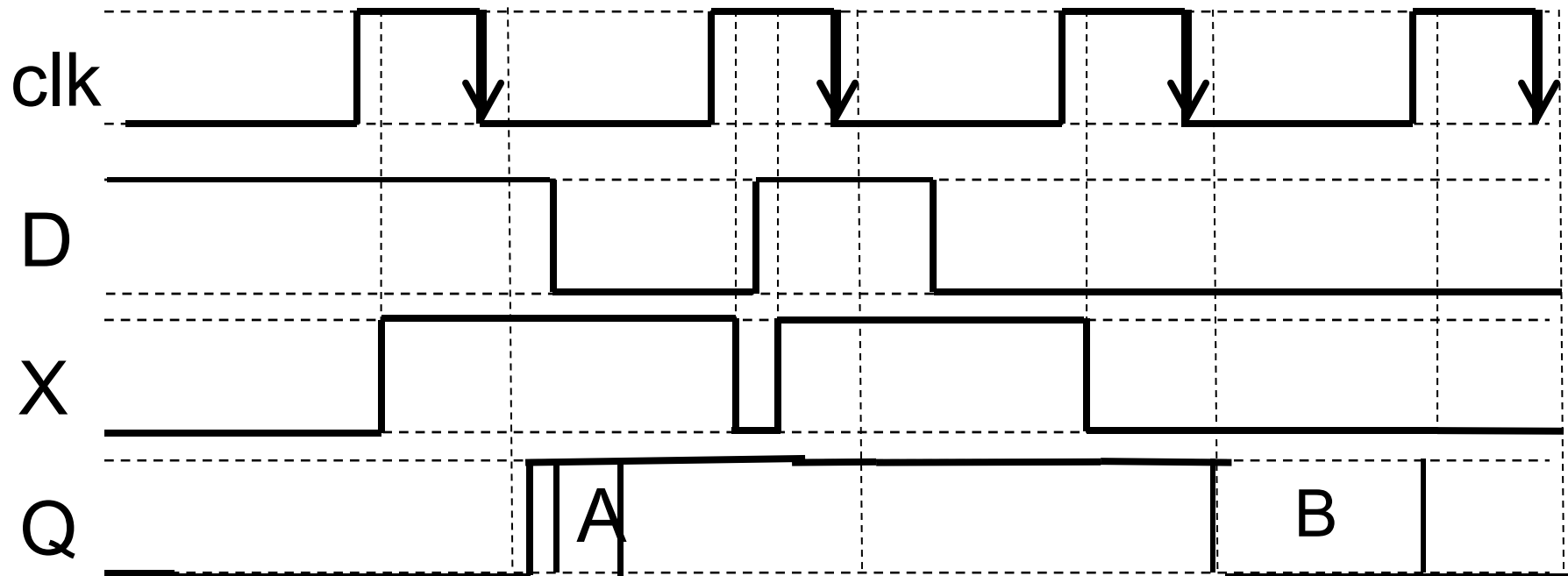


# Edge-Triggered D Flip-Flop

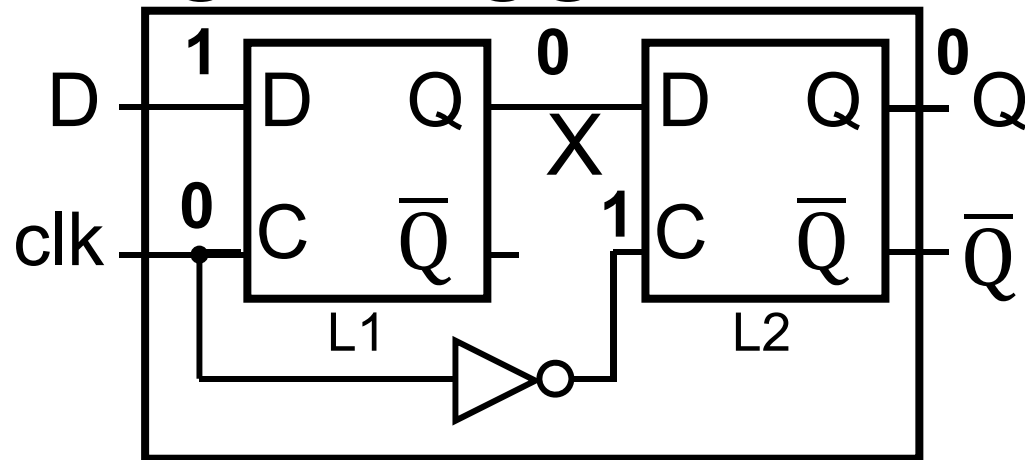


## D Flip-Flop

- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges

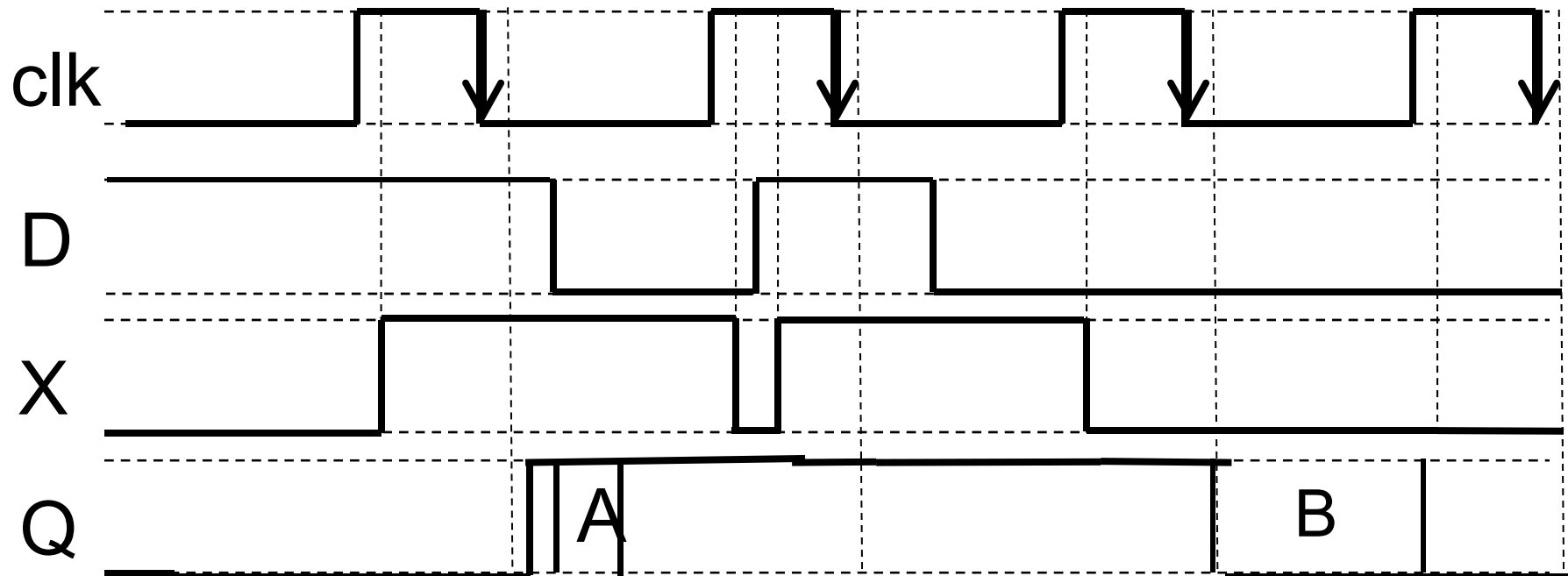


# Edge-Triggered D Flip-Flop



## D Flip-Flop

- Edge-Triggered
- Data captured when clock is high
- Output changes only on falling edges



# Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.

An Edge-Triggered D Flip-Flop (aka Master-Slave D Flip-Flop) stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.

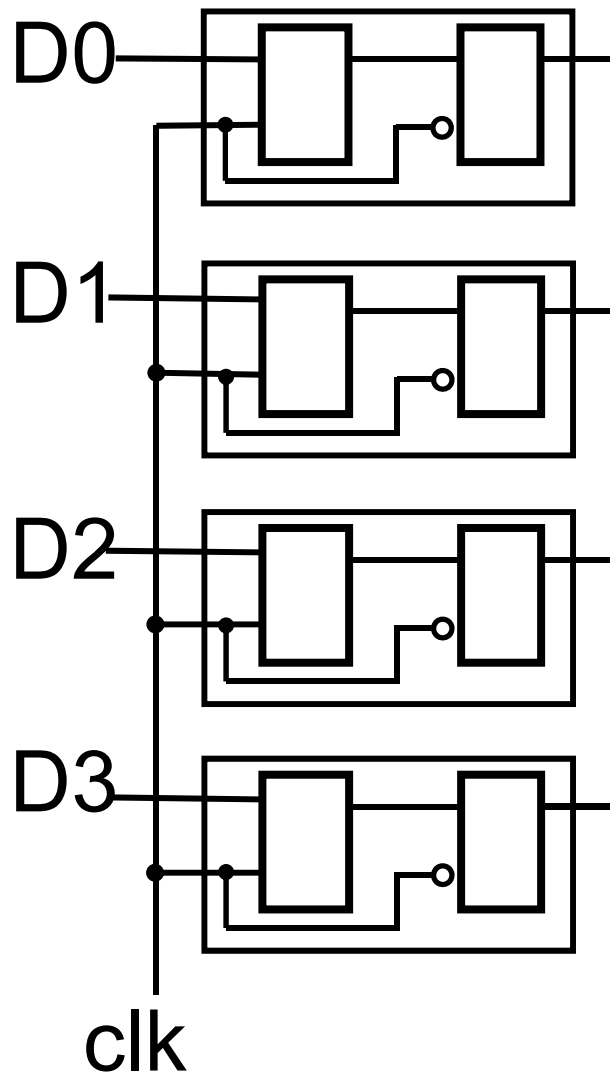


# Next Goal

How do we store more than one bit,  $N$  bits?

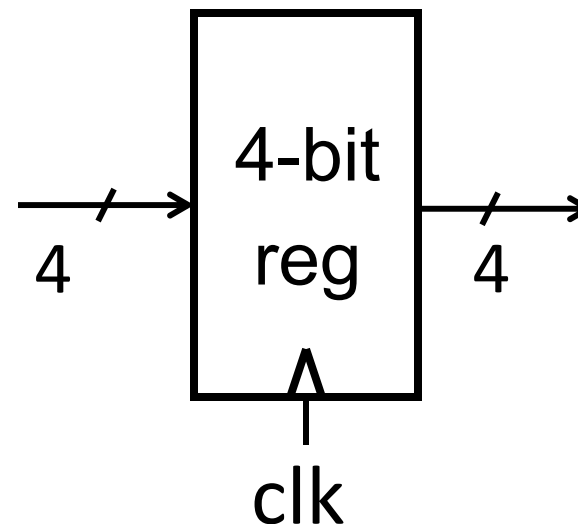


# Registers



## Register

- D flip-flops in parallel
- shared clock
- extra clocked inputs:  
write\_enable, reset, ...



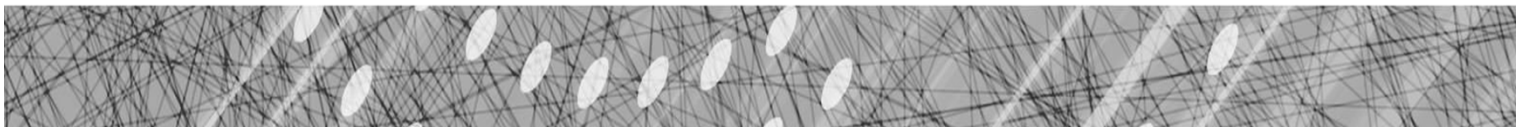
# Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

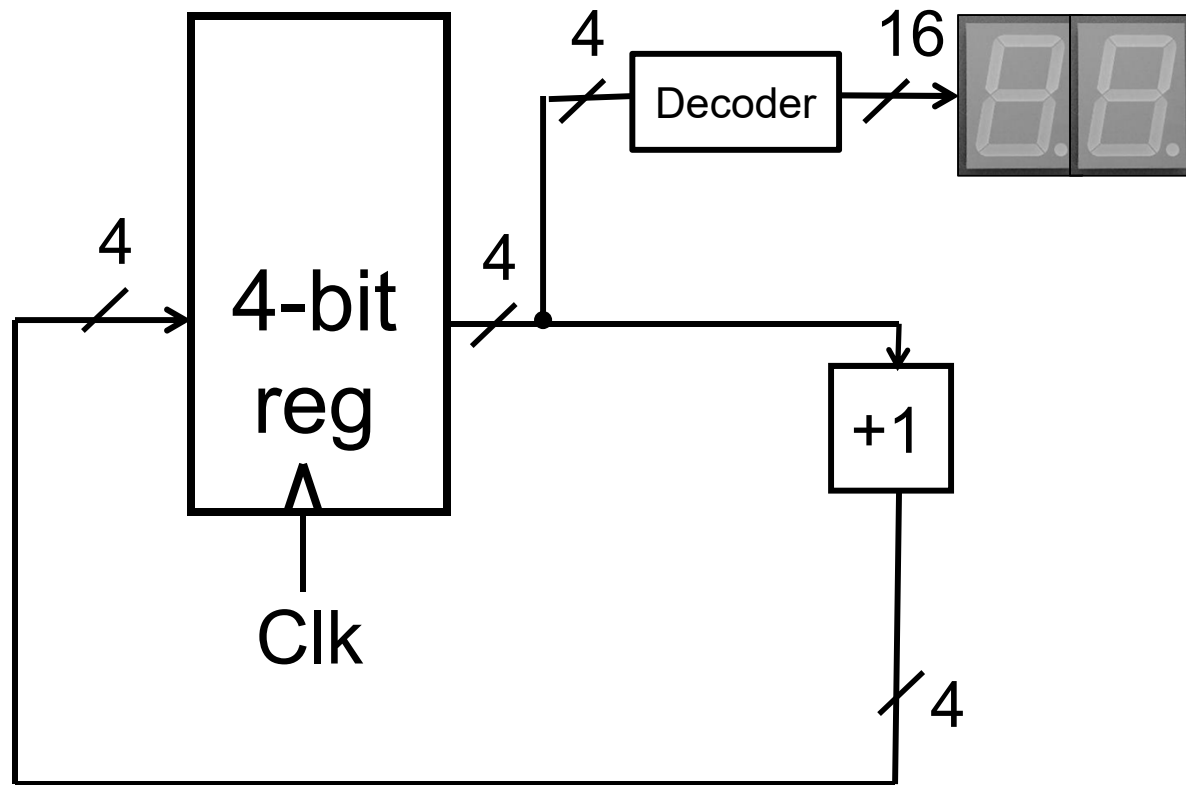
(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.

An Edge-Triggered D Flip-Flop (aka Master-Slave D Flip-Flop) stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.

An  $N$ -bit **register** stores  $N$ -bits. It is created with  $N$  D-Flip-Flops in parallel along with a shared clock.

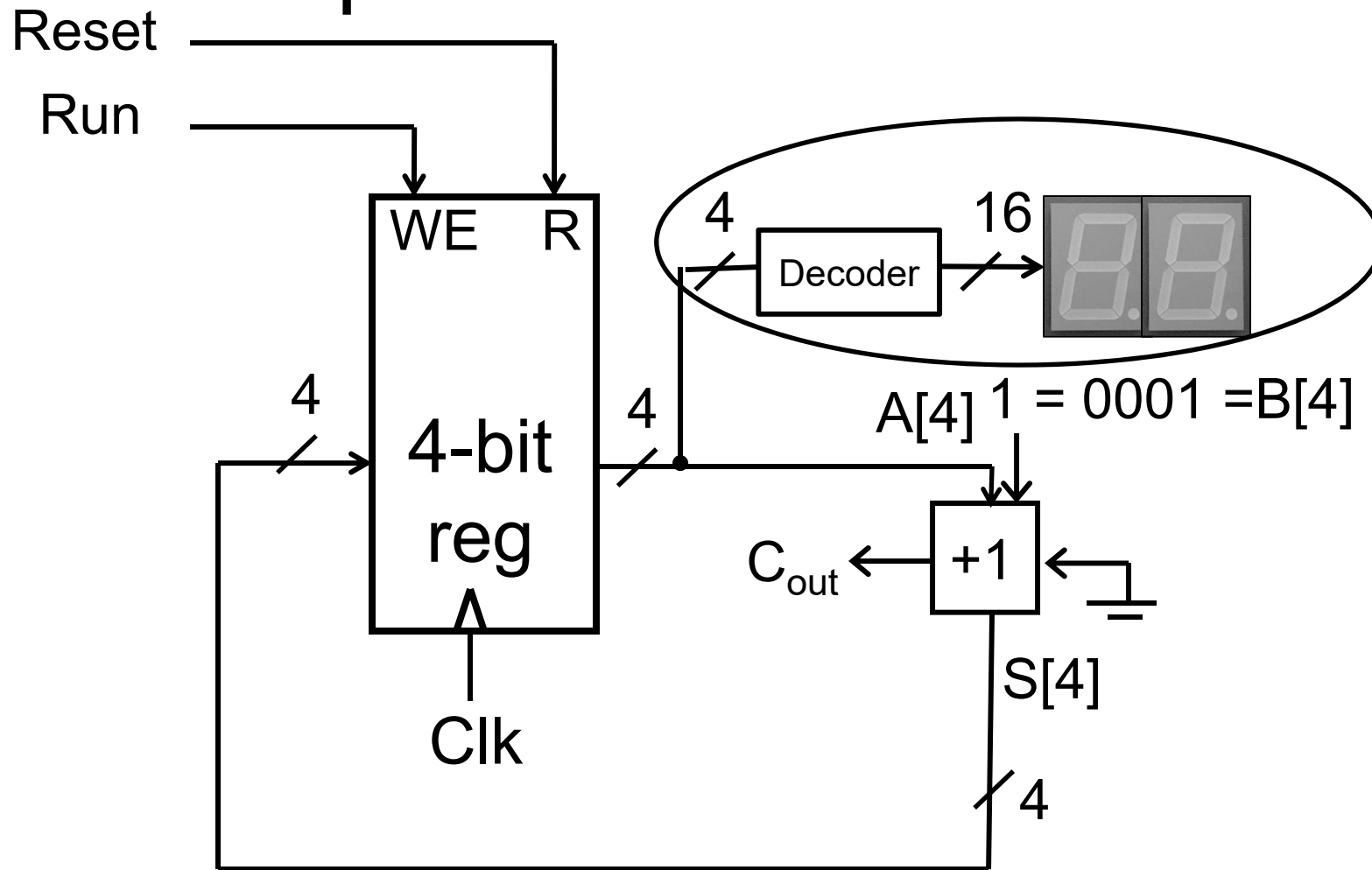


# An Example: What will this circuit do?





# An Example: What will this circuit do?



# Decoder Example: 7-Segment LED

## 7-Segment LED

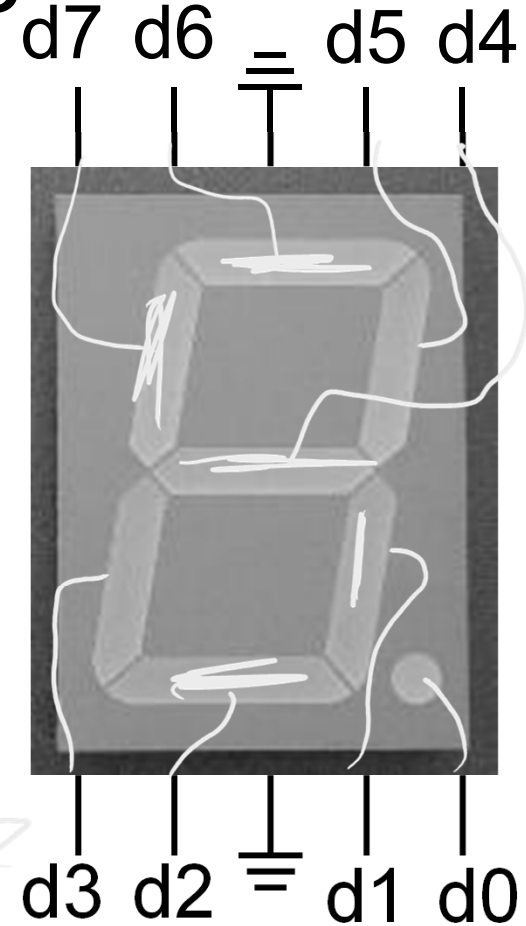
- photons emitted when electrons fall into holes



# Decoder Example: 7-Segment LED

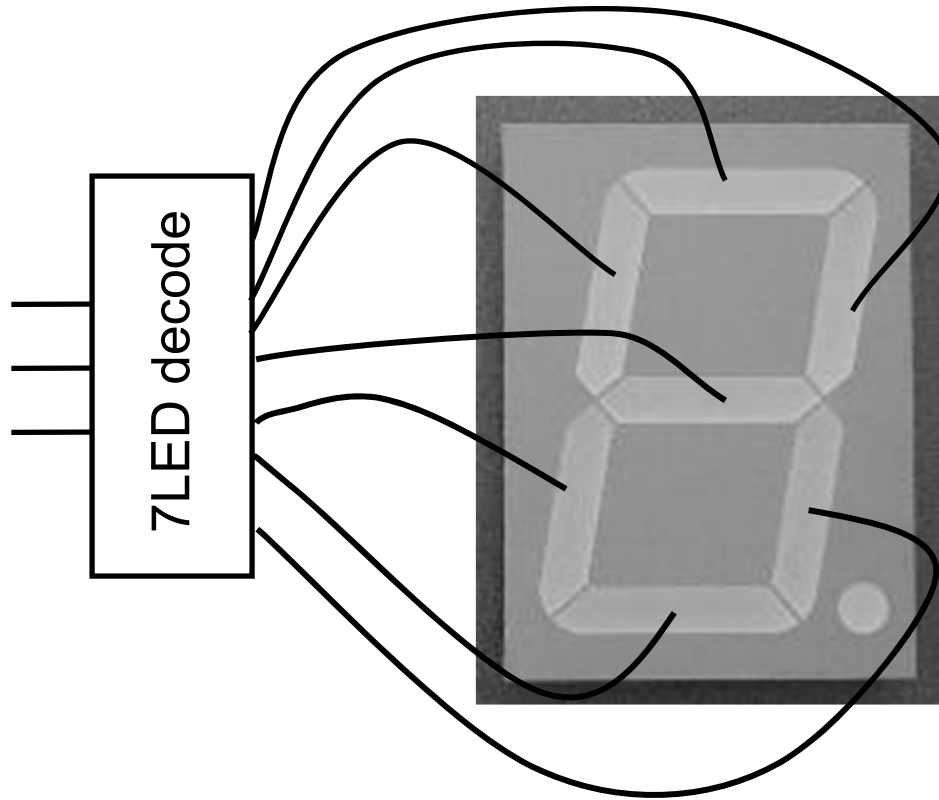
## 7-Segment LED

- photons emitted when electrons fall into holes



d0 1 2 3 4 5 6 7  
5 = 0 1 1 0 1 0 1 1

# Decoder Example: 7-Segment LED Decoder



3 inputs

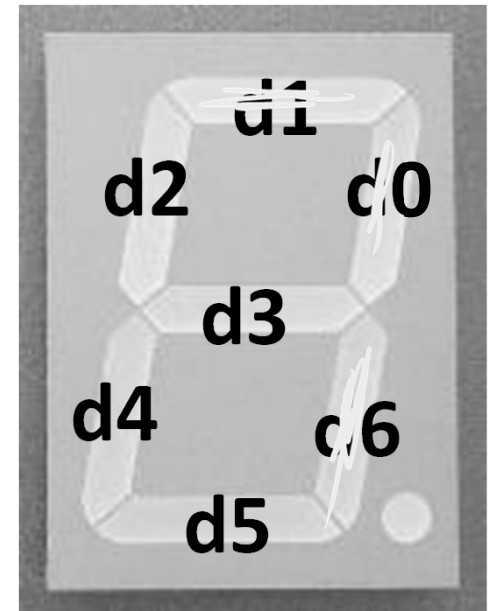
- encode 0 – 7 in binary

7 outputs

- one for each LED

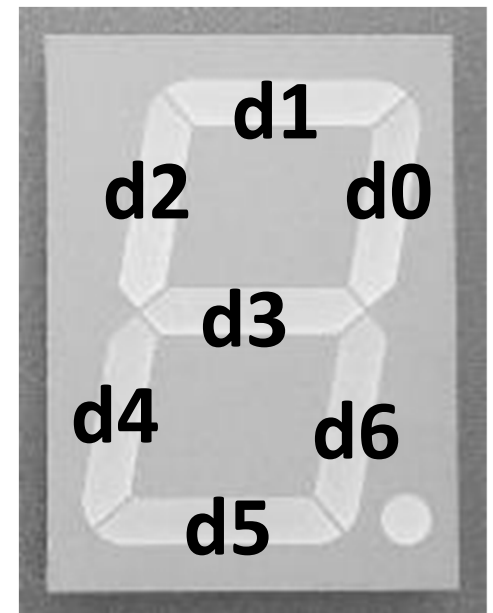
# 7 Segment LED Decoder Implementation

<b>b2</b>	<b>b1</b>	<b>b0</b>	<b>d6</b>	<b>d5</b>	<b>d4</b>	<b>d3</b>	<b>d2</b>	<b>d1</b>	<b>d0</b>
<b>0</b>	<b>0</b>	<b>0</b>							
<b>0</b>	<b>0</b>	<b>1</b>							
<b>0</b>	<b>1</b>	<b>0</b>							
<b>0</b>	<b>1</b>	<b>1</b>							
<b>1</b>	<b>0</b>	<b>0</b>							
<b>1</b>	<b>0</b>	<b>1</b>							
<b>1</b>	<b>1</b>	<b>0</b>							
<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>

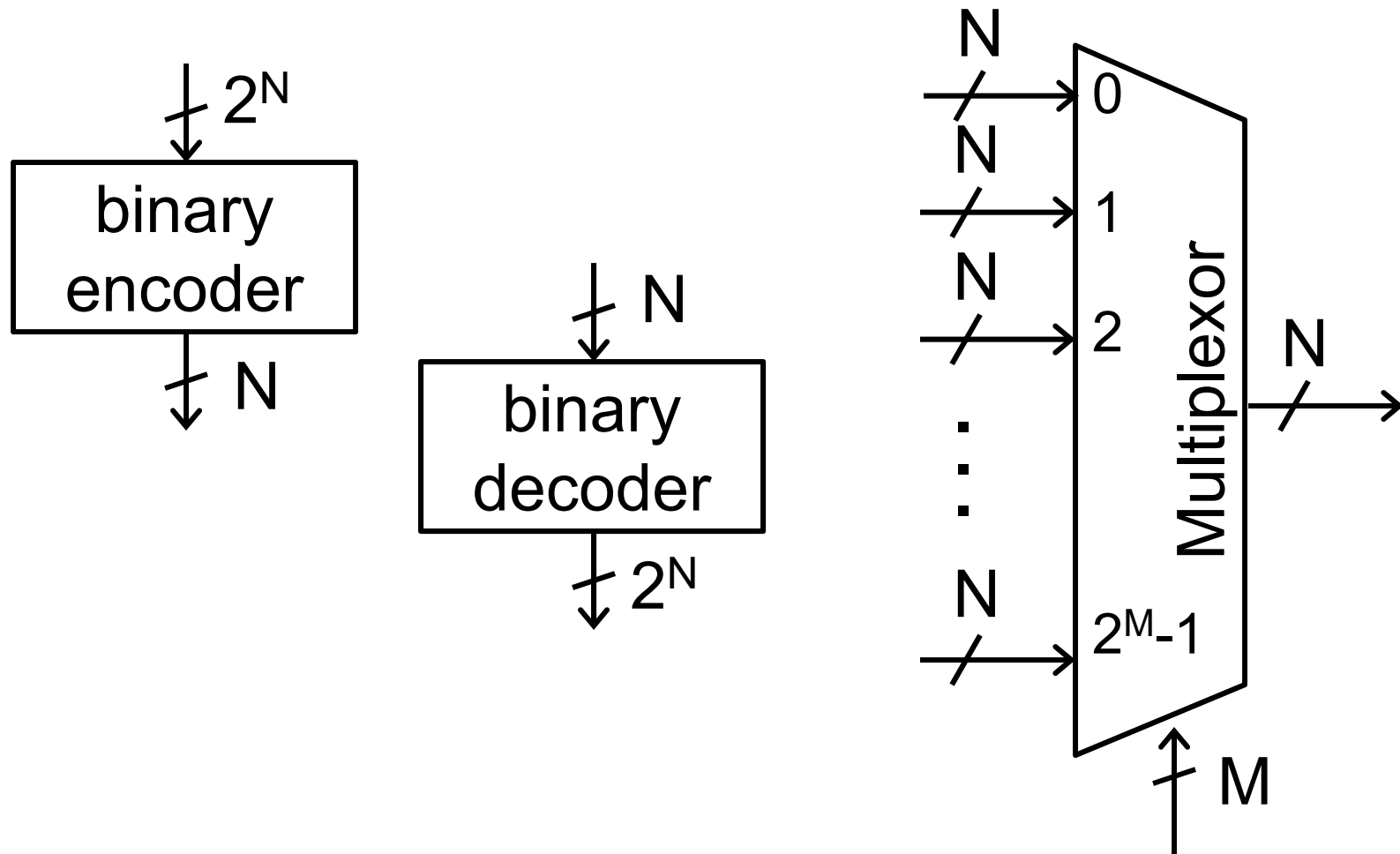


# 7 Segment LED Decoder Implementation

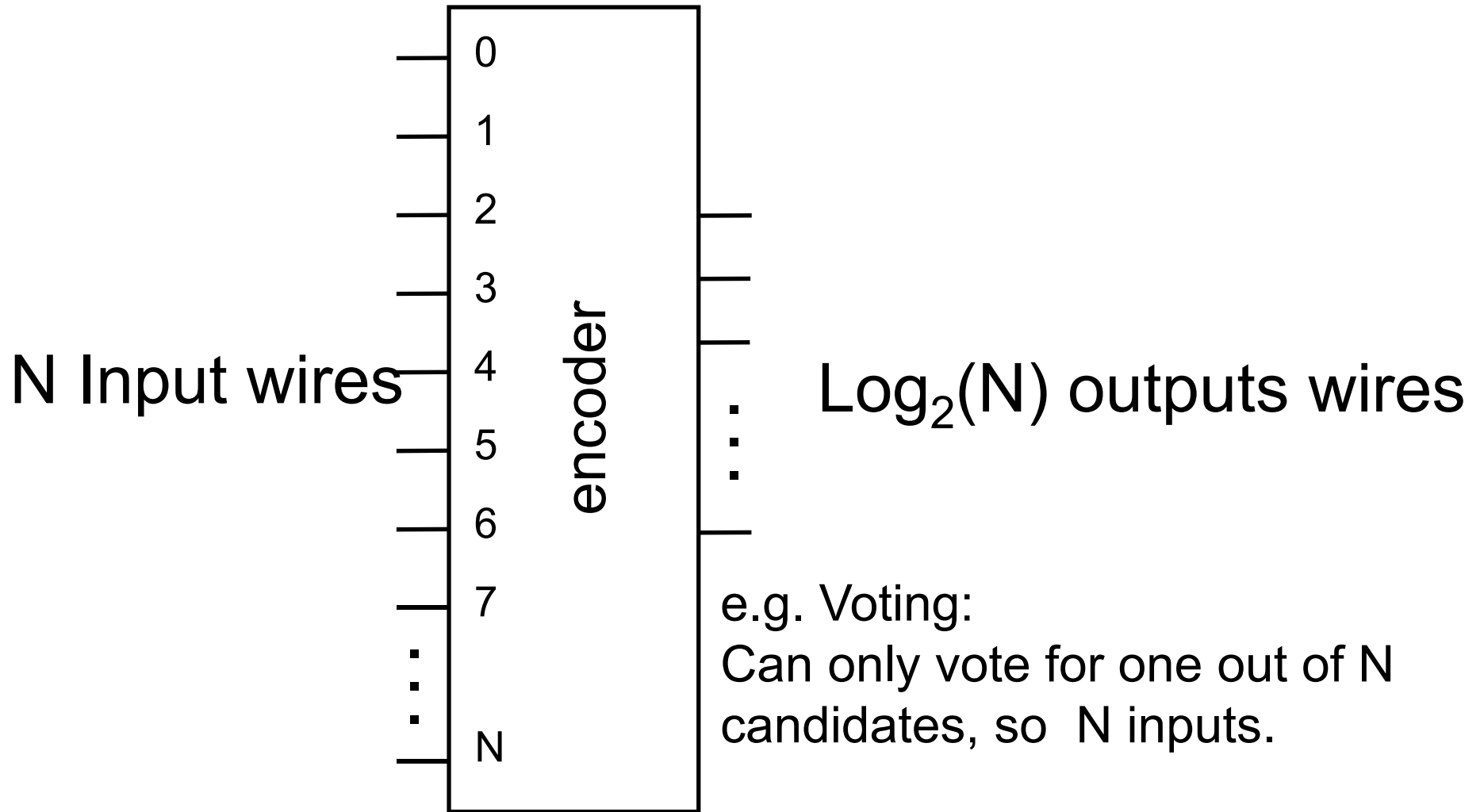
<b>b2</b>	<b>b1</b>	<b>b0</b>	<b>d6</b>	<b>d5</b>	<b>d4</b>	<b>d3</b>	<b>d2</b>	<b>d1</b>	<b>d0</b>
0	0	0	1	1	1	0	1	1	1
0	0	1	1	0	0	0	0	0	1
0	1	0	0	1	1	1	0	1	1
0	1	1	1	1	0	1	0	1	1
1	0	0	1	0	0	1	1	0	1
1	0	1	1	1	0	1	1	1	0
1	1	0	1	1	1	1	1	1	0
1	1	1	1	0	0	0	0	1	1



# Basic Building Blocks We have Seen



# Encoders



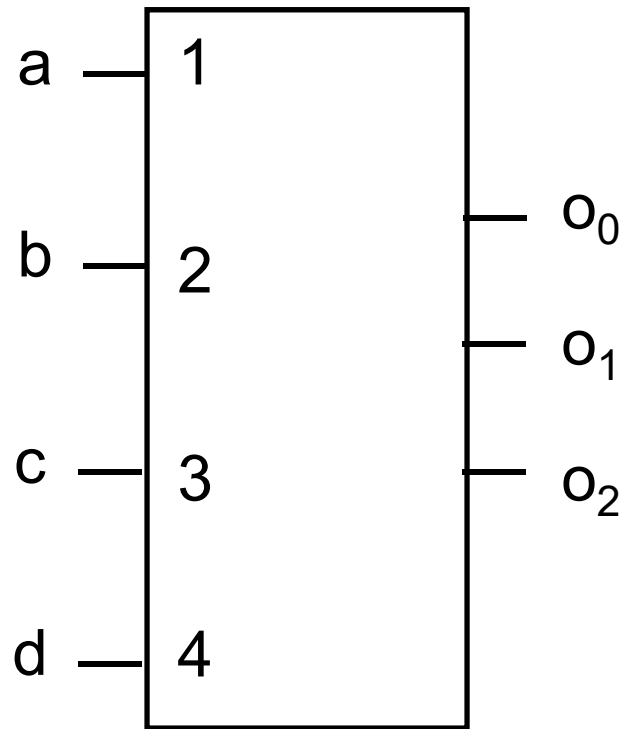
e.g. Voting:

Can only vote for one out of  $N$  candidates, so  $N$  inputs.

But can encode vote efficiently with binary encoding.



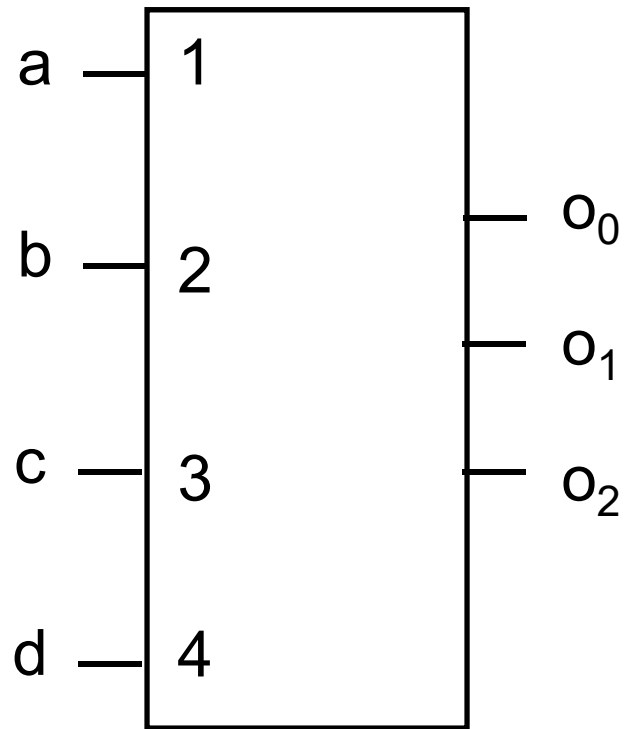
# Example Encoder Truth Table



A 3-bit  
encoder  
with 4 inputs  
for simplicity

a	b	c	d				
0	0	0	0				
1	0	0	0				
0	1	0	0				
0	0	1	0				
0	0	0	1				

# Example Encoder Truth Table

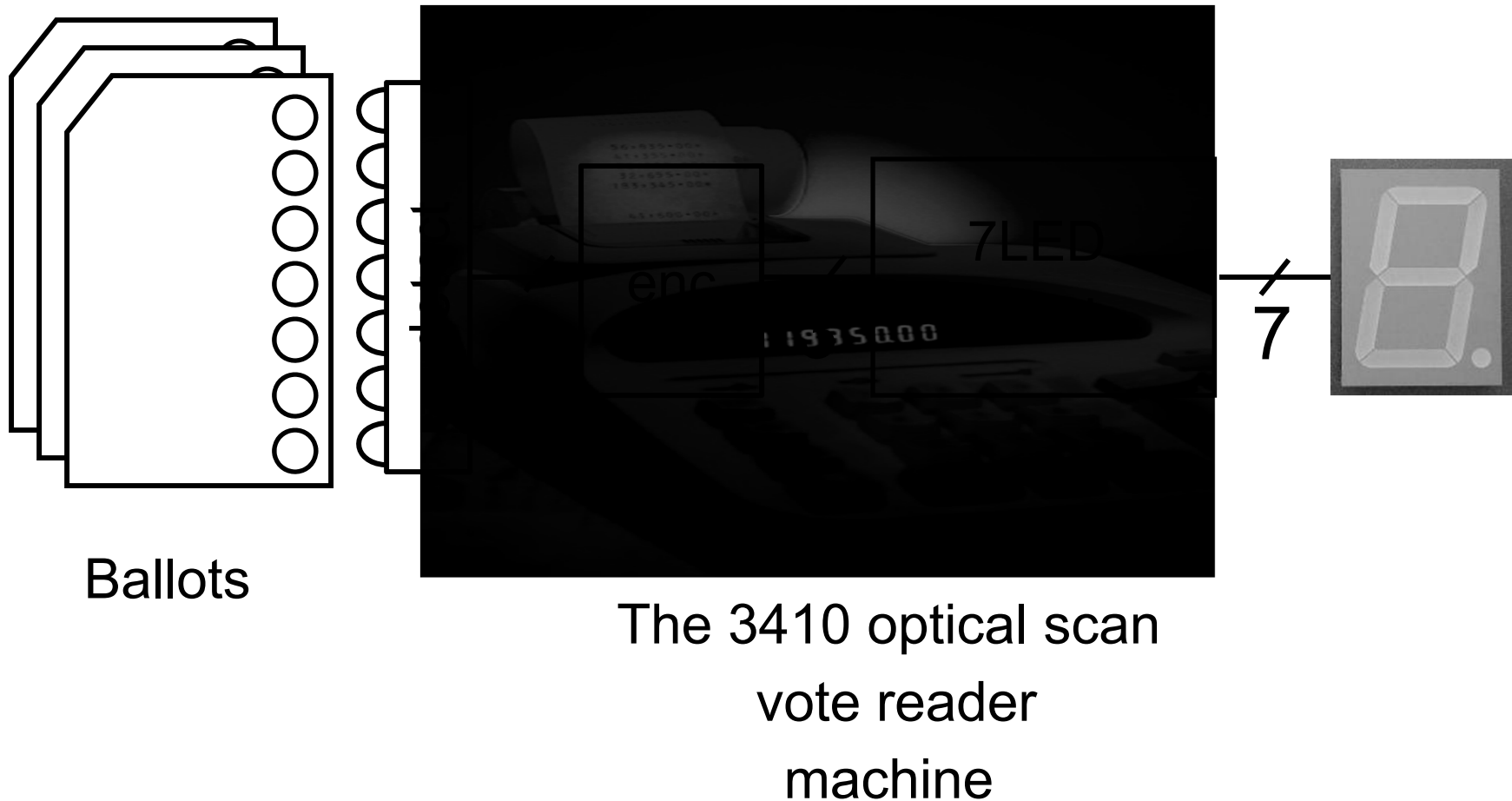


A 3-bit  
encoder  
with 4 inputs  
for simplicity

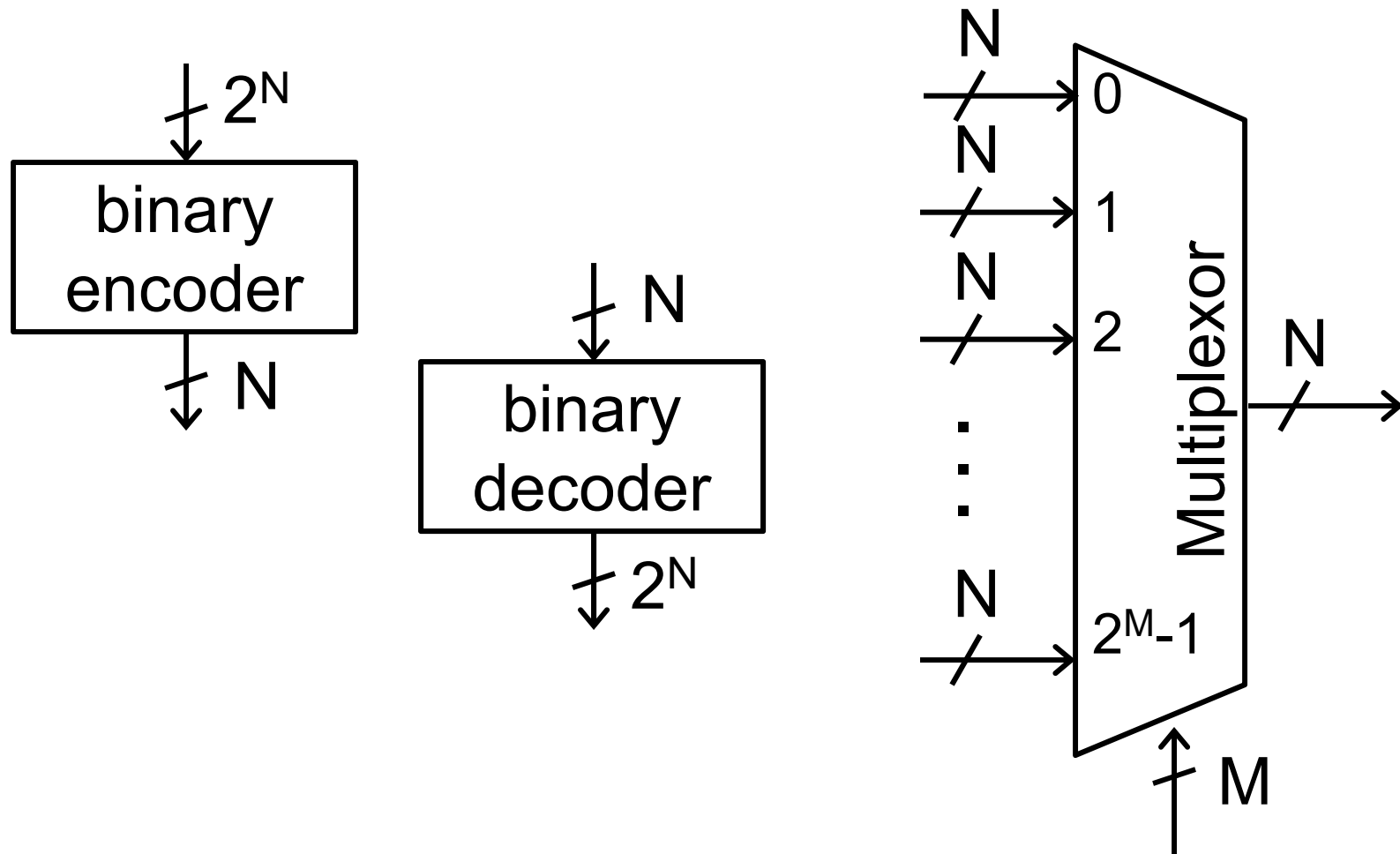
a	b	c	d		o <sub>2</sub>	o <sub>1</sub>	o <sub>0</sub>
0	0	0	0		0	0	0
1	0	0	0		0	0	1
0	1	0	0		0	1	0
0	0	1	0		0	1	1
0	0	0	1		1	0	0

- $o_2 = \overline{a}b\overline{c}d$
- $o_1 = \overline{a}b\overline{c}d + \overline{a}b\overline{c}\overline{d}$
- $o_0 = a\overline{b}\overline{c}d + a\overline{b}\overline{c}\overline{d}$

# Basic Building Blocks Example: Voting



# Basic Building Blocks We have Seen



# Recap

We can now build interesting devices with sensor

- Using combinational logic

We can also store data values (aka Sequential Logic)

- In state-holding elements
- Coupled with clocks



# Summary

We can now build interesting devices with sensor

- Using combinational logic

We can also store data values

- Stateful circuit elements (D Flip Flops, Registers, ...)
- Clock to synchronize state changes

