Gates and Logic: From Transistors to Logic Gates and Logic Circuits

Prof. Hakim Weatherspoon

CS 3410

Computer Science

Cornell University

[Weatherspoon, Bala, Bracy, and Sirer]

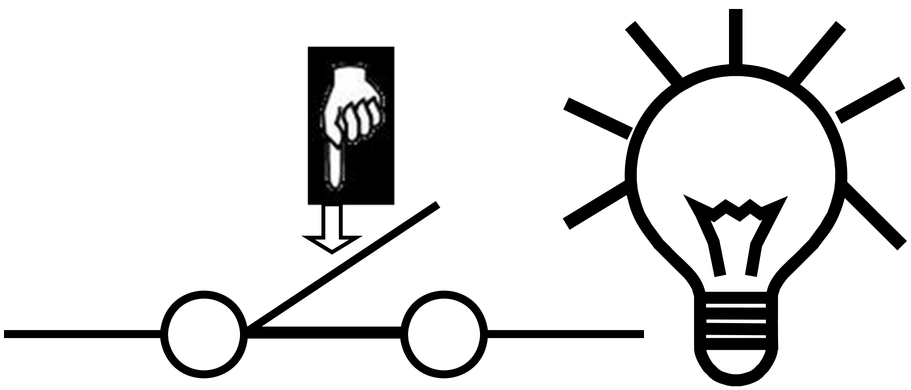
Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Logic Circuit Minimization
  - Algebraic Manipulations
  - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)

A switch

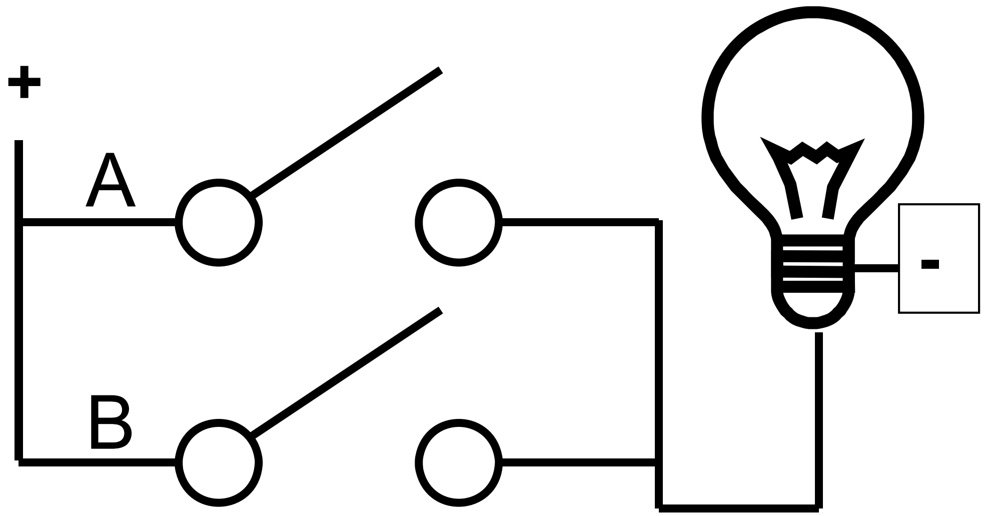
Acts as a *conductor* or *insulator*.

Can be used to build amazing things…



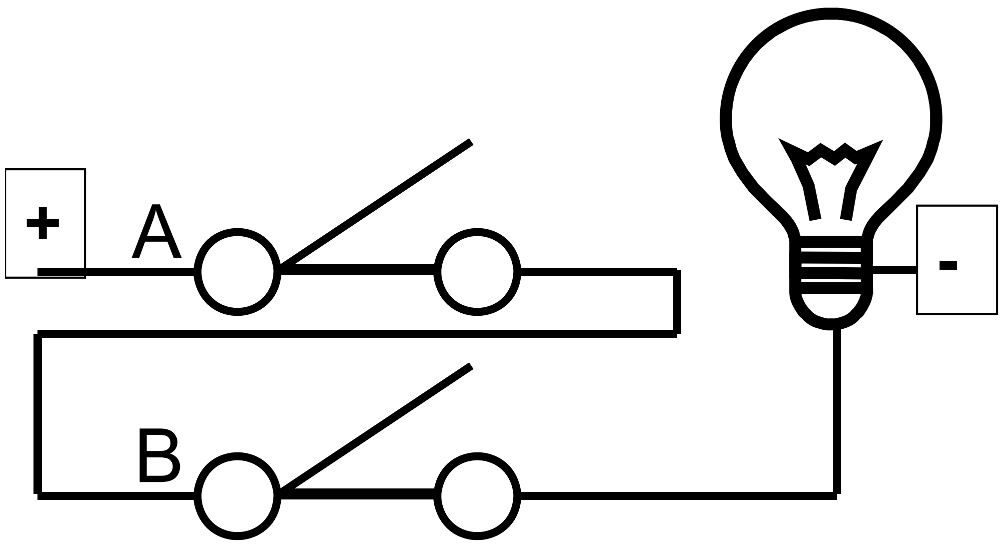
The Bombe used to break the German Enigma machine during World War II

Basic Building Blocks: Switches to Logic Gates



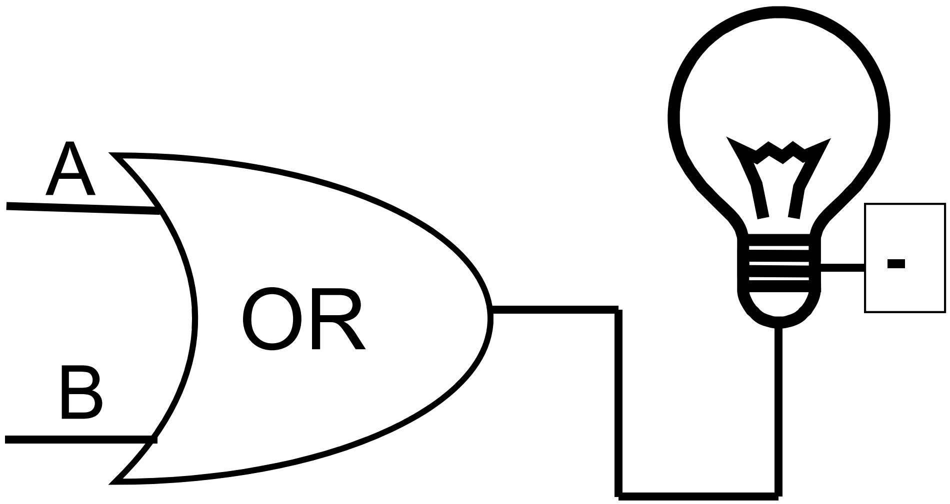
Truth Table

A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	



A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

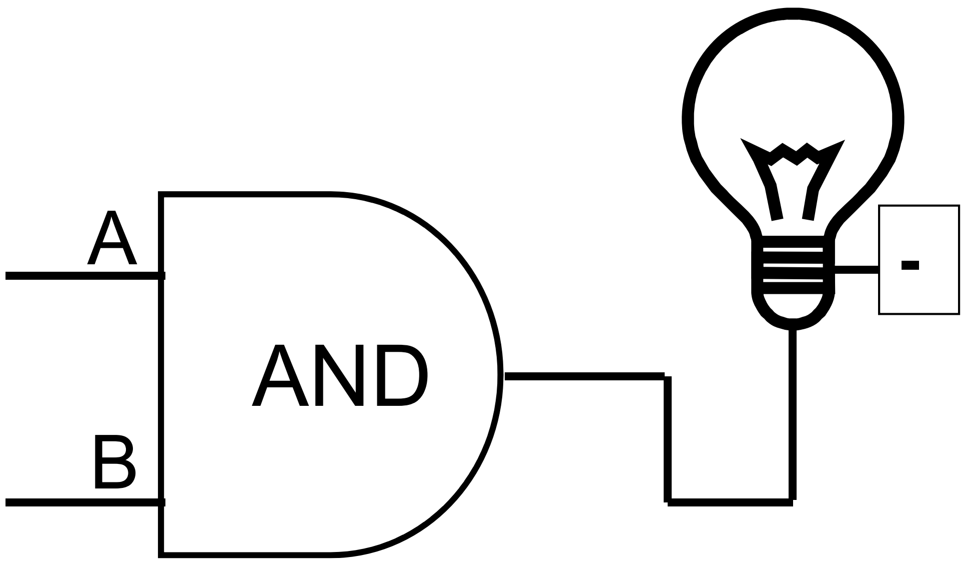
Basic Building Blocks: Switches to Logic Gates



- Either (OR)

Truth Table

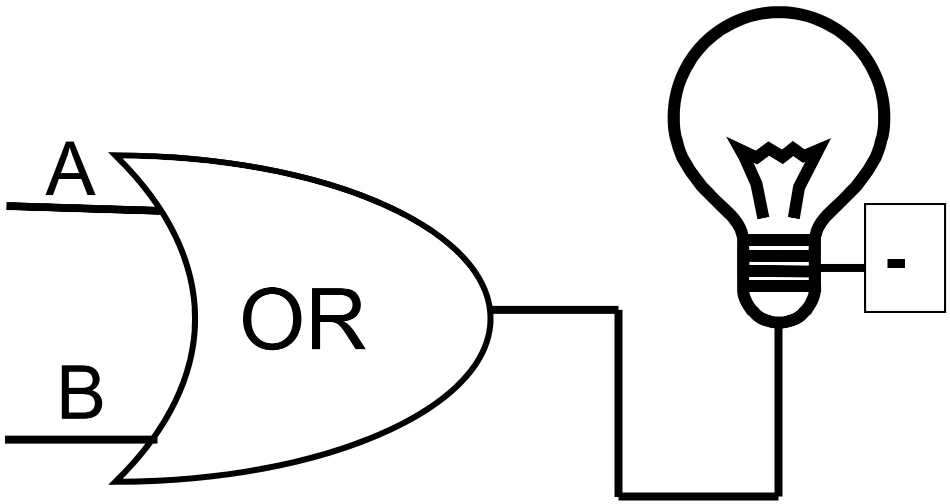
A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	



- Both (AND)

A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

Basic Building Blocks: Switches to Logic Gates

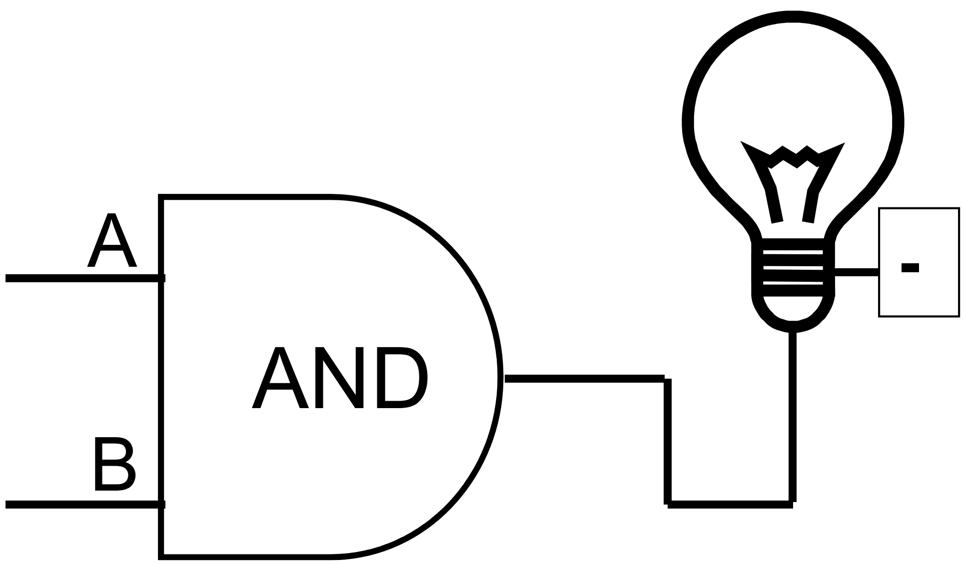


- Either (OR)

Truth Table

A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

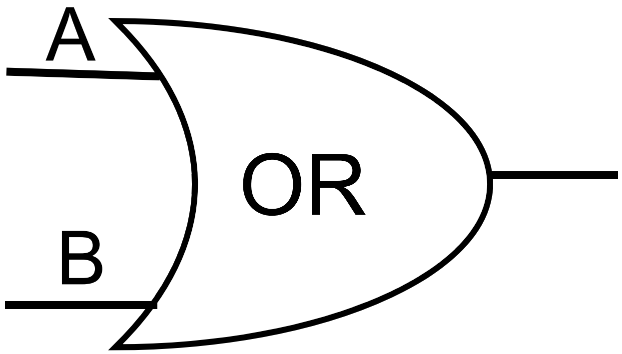
0 = OFF
1 = ON



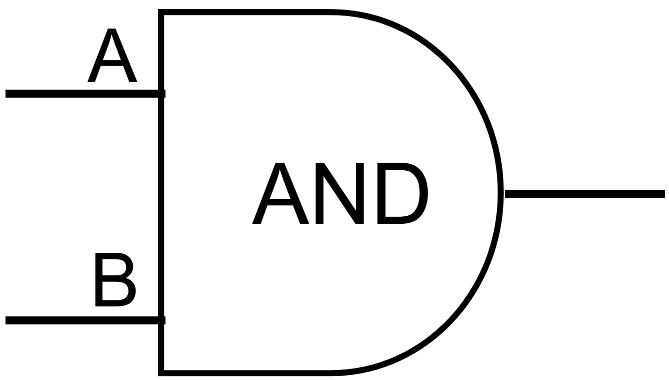
- Both (AND)

A	B	Light
0	0	
0	1	
1	0	
1	1	

Basic Building Blocks: Switches to Logic Gates



George Boole (1815-1864)



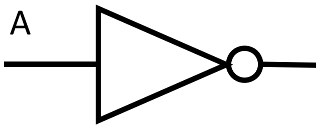
- Did you know?
- George Boole: Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.

Takeaway

- Binary (two symbols: true and false) is the basis of Logic Design

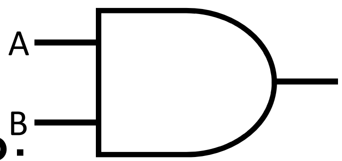
Building Functions: Logic Gates

- NOT:



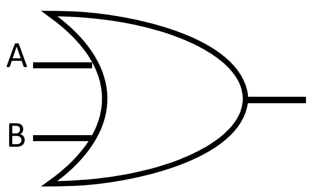
A	Out
0	1
1	0

- AND:



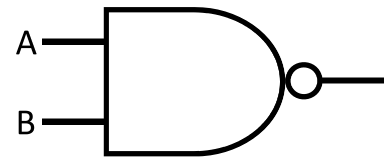
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

- OR:



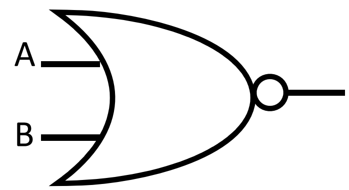
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

NAND:



A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

- Logic Gates
  - digital circuit that either allows a signal to pass through it or not.
  - Used to build logic functions
  - There are seven basic logic gates:

    AND, OR, **NOT**,

    NAND (not AND), NOR (not OR), XOR, and XNOR (not XOR) [later]

Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Logic Circuit Minimization
  - Algebraic Manipulations
  - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)

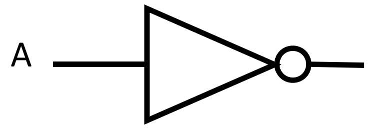
Next Goal

- Given a Logic function, create a Logic Circuit that implements the Logic Function…
- …and, *with the minimum number of logic gates*

- Fewer gates: A cheaper ($$$) circuit!

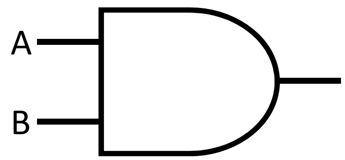
Logic Gates

NOT:



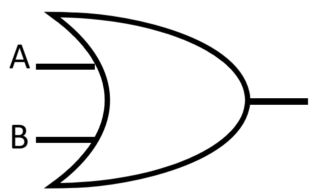
A	Out
0	1
1	0

AND:



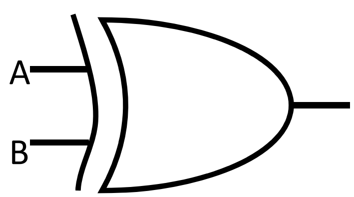
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

OR:



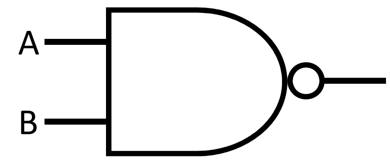
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

XOR:



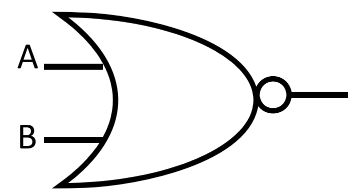
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

NAND:



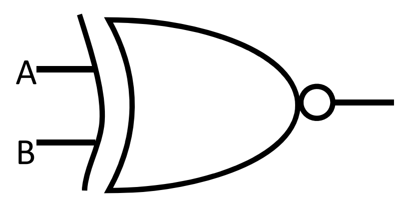
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

XNOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	1

Logic Implementation

- How to implement a desired logic function?

a	b	c	out
0	**0**	**0**	**0**
0	**0**	**1**	**1**
0	**1**	**0**	**0**
0	**1**	**1**	**1**
1	**0**	**0**	**0**
1	**0**	**1**	**1**
1	**1**	**0**	**0**
1	**1**	**1**	**0**

Logic Implementation

- How to implement a desired logic function?

a	b	c	out	minterm
0	**0**	**0**	**0**	$\bar{a}\,\bar{b}\,\bar{c}$
0	**0**	**1**	**1**	$\bar{a}\,\bar{b}\,c$
0	**1**	**0**	**0**	$\bar{a}\,b\,\bar{c}$
0	**1**	**1**	**1**	$\bar{a}\,b\,c$
1	**0**	**0**	**0**	$a\,\bar{b}\,\bar{c}$
1	**0**	**1**	**1**	$a\,\bar{b}\,c$
1	**1**	**0**	**0**	$a\,b\,\bar{c}$
1	**1**	**1**	**0**	$a\,b\,c$

1) Write minterms
2) sum of products:
   - OR of all minterms where out=1

Logic Equations

- NOT:
  - out = $\bar{a}$ = !a = ¬a
- AND:
  - out = a · b = a & b = a ∧ b
- OR:
  - out = a + b = a | b = a ∨ b
- XOR:
  - out = a ⊕ b = $a\bar{b} + \bar{a}b$

NAND:

- out = $\overline{a \cdot b}$ = !(a & b) = ¬ (a ∧ b)

NOR:

- out = $\overline{a + b}$ = !(a | b) = ¬ (a ∨ b)

XNOR:

- out = $\overline{a \oplus b}$ = ab + $\overline{ab}$

- Logic Equations
  - Constants: true = 1, false = 0
  - Variables: a, b, out, …
  - Operators (above): AND, OR, NOT, etc.

Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$a + 0 =$

$a + 1 =$

$a + \bar{a} =$

$a \cdot 0 =$

$a \cdot 1 =$

$a \cdot \bar{a} =$

Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$\overline{(a + b)}$ =

$\overline{(a \cdot b)}$ =

a + a b =

a(b+c) =

$\overline{a(b + c)}$ =

Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- **Logic Circuit Minimization – *why?***
  - Algebraic Manipulations
  - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)

Checking Equality w/Truth Tables

circuits ↔ truth tables ↔ equations

Example: $(a+b)(a+c) = a + bc$

a	b	c					
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Takeaway

- Binary (two symbols: true and false) is the basis of Logic Design
- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.

Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Logic Circuit Minimization
  - Algebraic Manipulations
  - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)

Karnaugh Maps

How does one find the most efficient equation?

- Manipulate algebraically until…?
- Use Karnaugh Maps (optimize visually)
- Use a software optimizer

For large circuits

- Decomposition & reuse of building blocks

Minimization with Karnaugh maps (1)

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

- Sum of minterms yields
  - out =

Minimization with Karnaugh maps (2)

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

c \ ab	00	01	11	10
0	0	0	0	1
1	1	1	0	1

- Sum of minterms yields
  - out = $\overline{a}\overline{b}c + \overline{a}bc + a\overline{b}\overline{c} + a\overline{b}c$
- Karnaugh map minimization
  - Cover all 1's
  - Group adjacent blocks of 2^n 1's that yield a rectangular shape
  - Encode the common features of the rectangle
    - out = $a\overline{b} + \overline{a}c$

Karnaugh Minimization Tricks (1)

c \ ab	00	01	11	10
0	0	1	1	1
1	0	0	1	0

- Minterms can overlap
  - out =

c \ ab	00	01	11	10
0	1	1	1	1
1	0	0	1	0

- Minterms can span 2, 4, 8 or more cells
  - out =

Karnaugh Minimization Tricks (2)

cd \ ab	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

- The map wraps around
  - out =

cd \ ab	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

Karnaugh Minimization Tricks (3)

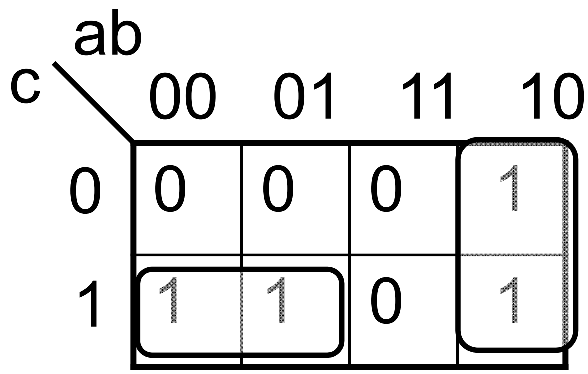
cd \ ab	00	01	11	10
00	0	0	0	0
01	1	x	x	x
11	1	x	x	1
10	0	0	0	0

- “Don’t care” values can be interpreted individually in whatever way is convenient
  - assume all x’s = 1
  - out =

cd \ ab	00	01	11	10
00	1	0	0	x
01	0	x	x	0
11	0	x	x	0
10	1	0	0	1

  - assume middle x’s = 0
  - assume 4th column x = 1
  - out =

Minimization with K-Maps



(1) Circle the 1's (see below)

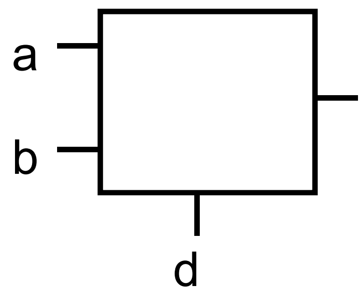
(2) Each circle is a logical component of the final equation

$$= a\bar{b} + \bar{a}c$$

Rules:

- Use fewest circles necessary to cover all 1's
- Circles must cover *only* 1's
- Circles span rectangles of size power of 2 (1, 2, 4, 8…)
- Circles should be as large as possible (all circles of 1?)
- Circles may wrap around edges of K-Map
- 1 may be circled multiple times *if* that means fewer circles

Multiplexer



- A multiplexer selects between multiple inputs
  - out = a, if d = 0
  - out = b, if d = 1

a	b	d	out
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

- Build truth table
- Minimize diagram
- Derive logic diagram

Takeaway

- Binary (two symbols: true and false) is the basis of Logic Design
- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.
- Any logic function can be implemented as “sum of products”. Karnaugh Maps minimize number of gates.

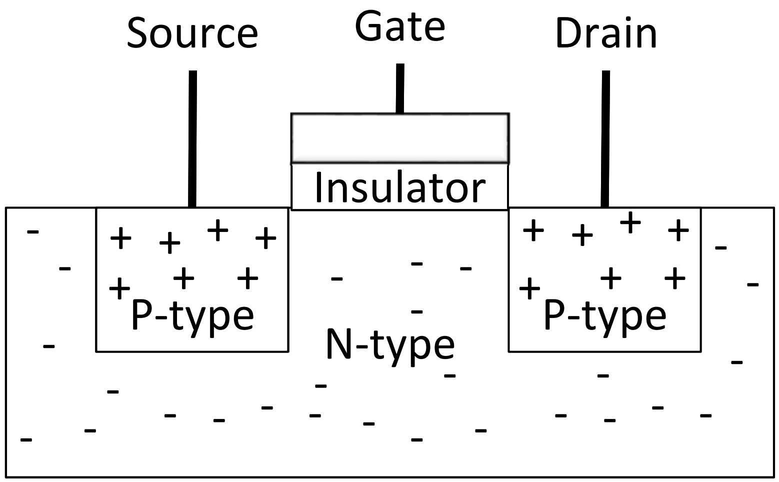
Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Logic Circuit Minimization
  - Algebraic Manipulations
  - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)

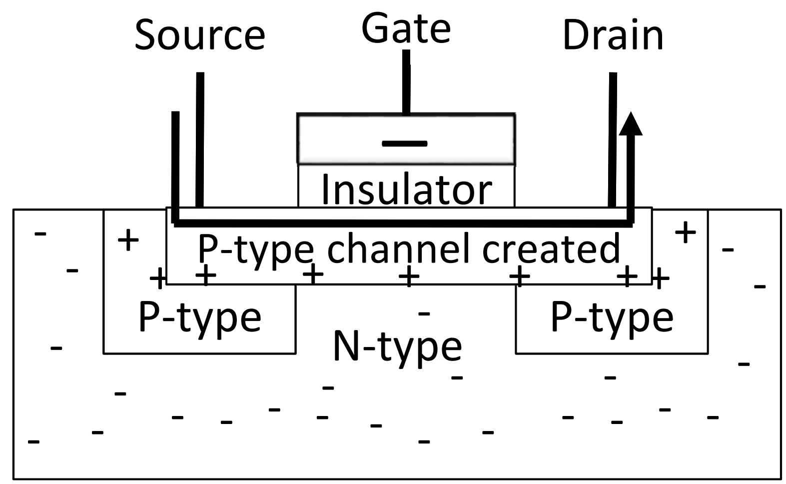
Silicon Valley & the Semiconductor Industry

- Transistors:
- Youtube video “How does a transistor work”
  https://www.youtube.com/watch?v=IcrBqCFLHIY
- Break: show some Transistor, Fab, Wafer photos

Transistors 101



P-Transistor Off

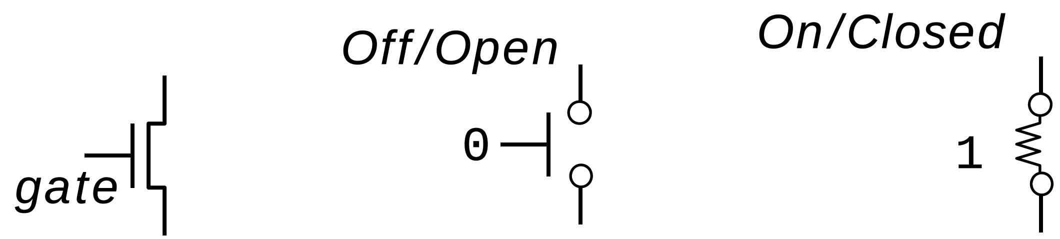


P-Transistor On

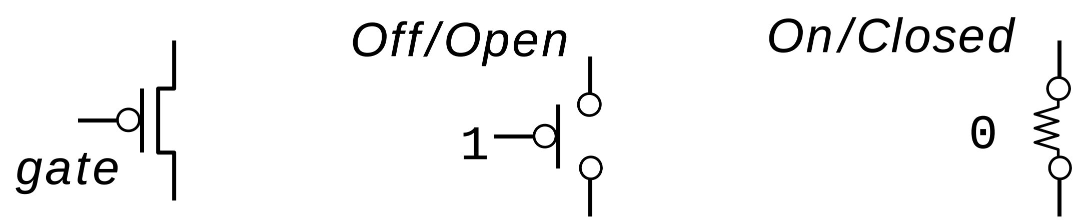
N-Type Silicon: negative free-carriers (electrons)
P-Type Silicon: positive free-carriers (holes)
P-Transistor: negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain
N-Transistor: works the opposite way
Metal-Oxide Semiconductor (Gate-Insulator-Silicon)
- Complementary MOS = **CMOS** technology uses both p- & n-type transistors

CMOS Notation

N-type



P-type



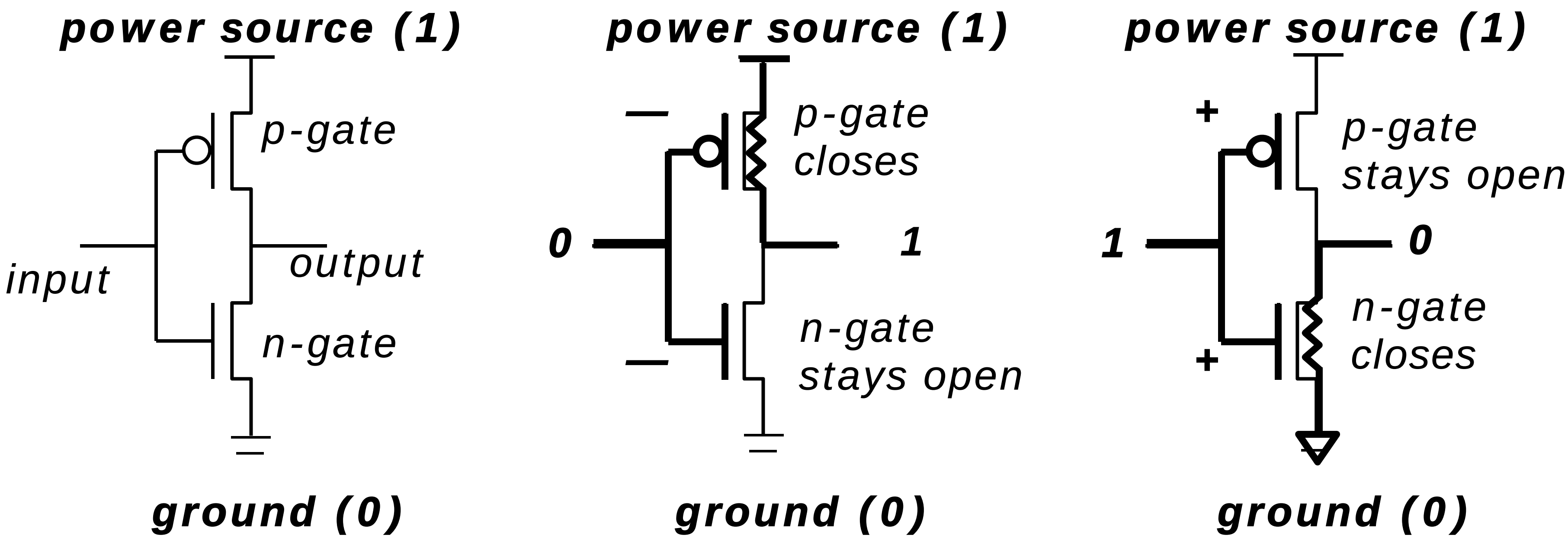
Gate input controls whether current can flow between the other two terminals or not.

Hint: the “o” bubble of the p-type tells you that this gate wants a 0 to be turned on

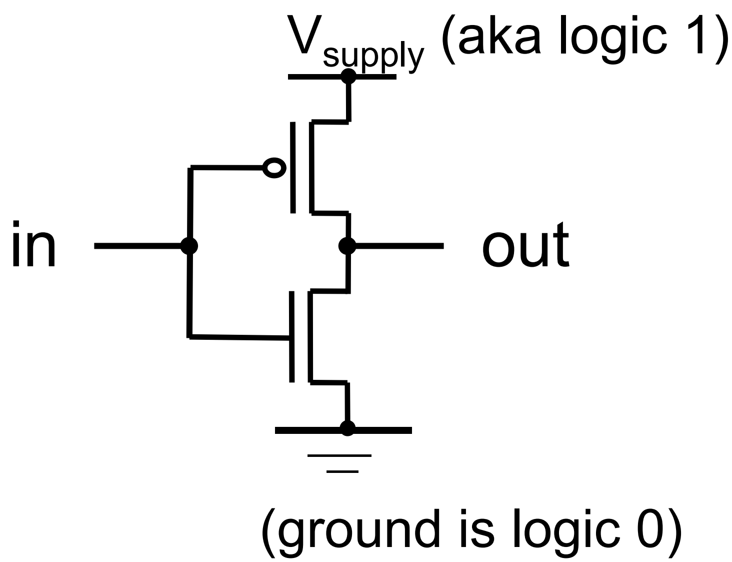
2-Transistor Combination: NOT

- Logic gates are constructed by combining transistors in complementary arrangements
- Combine p&n transistors to make a NOT gate:

CMOS Inverter :

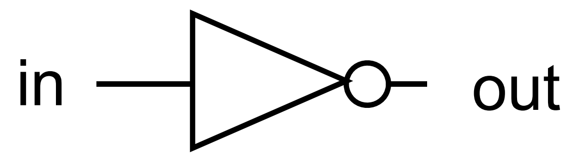


Inverter



Function: NOT

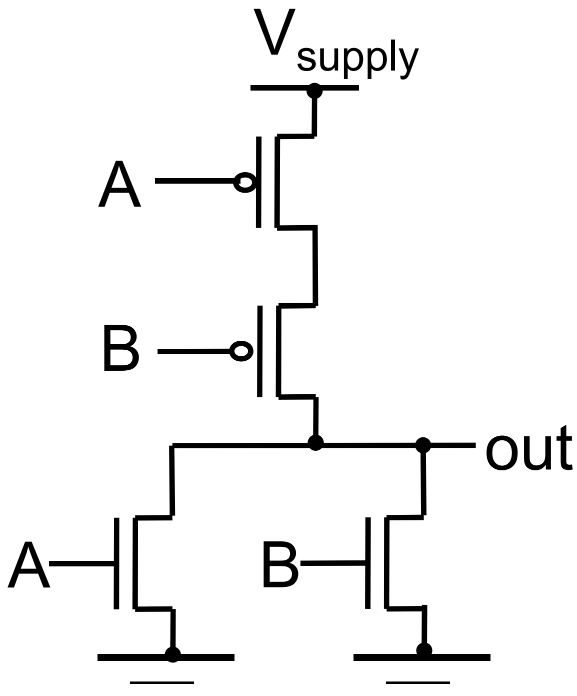
Symbol:



Truth Table:

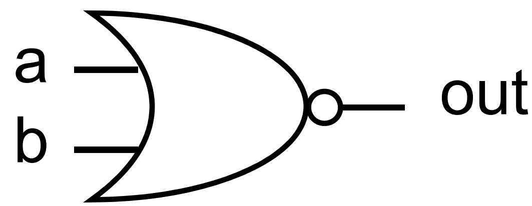
In	Out
0	1
1	0

NOR Gate



Function: NOR

Symbol:

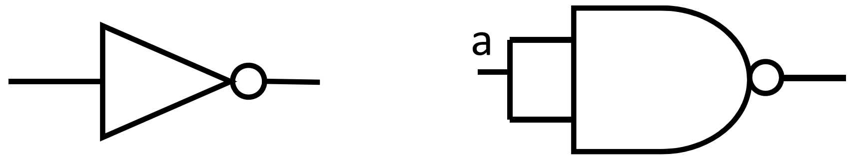


Truth Table:

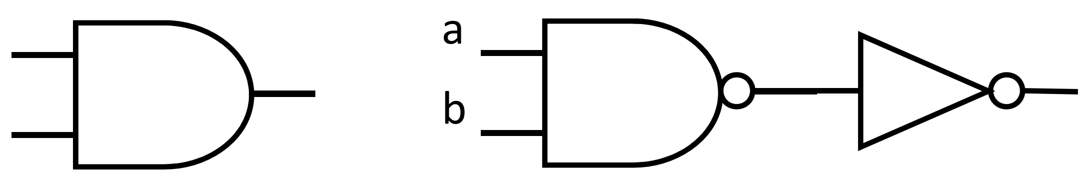
A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

Building Functions (Revisited)

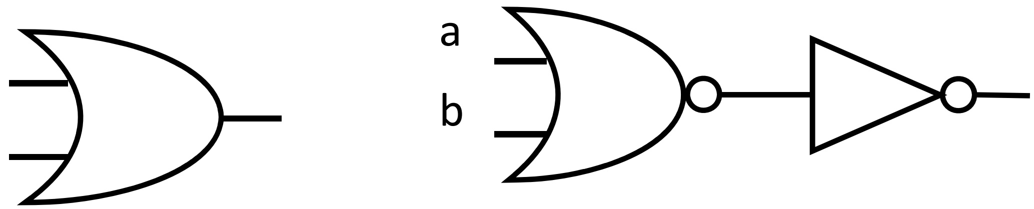
- NOT:



- AND:

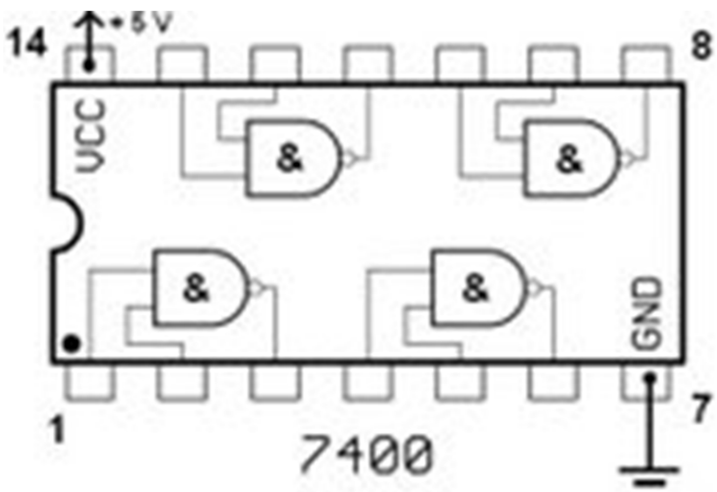


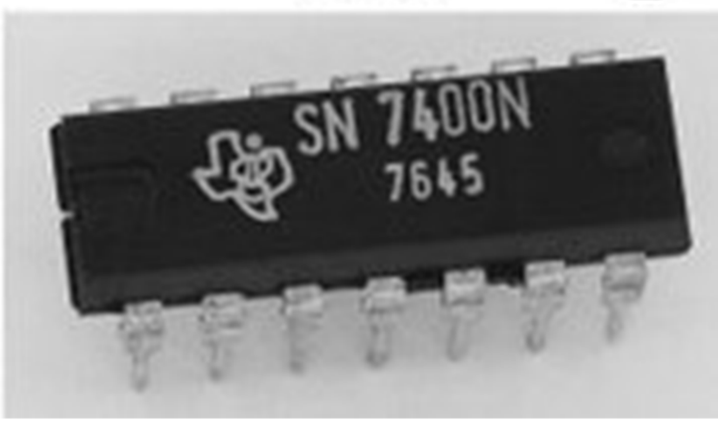
- OR:



- NAND and NOR are universal
  - Can implement ***any*** function with NAND or just NOR gates
  - useful for manufacturing

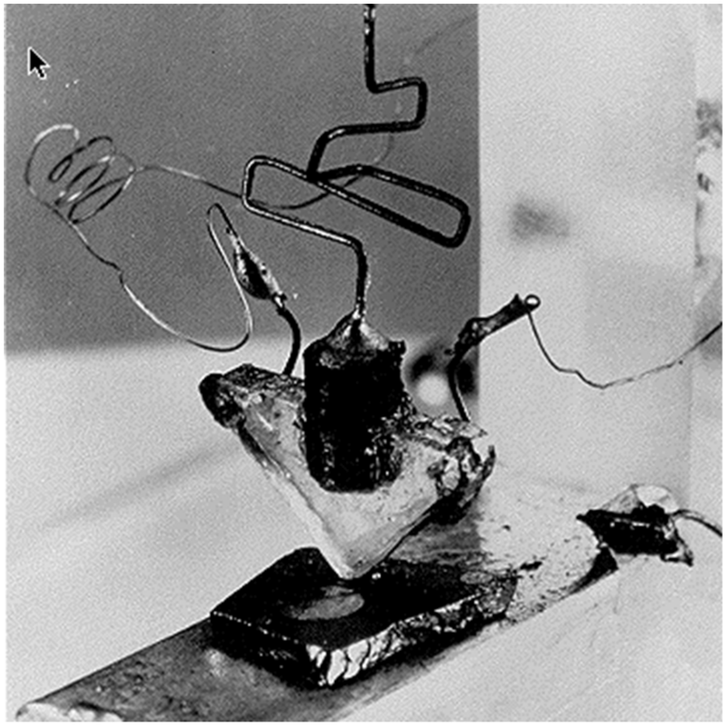
Logic Gates

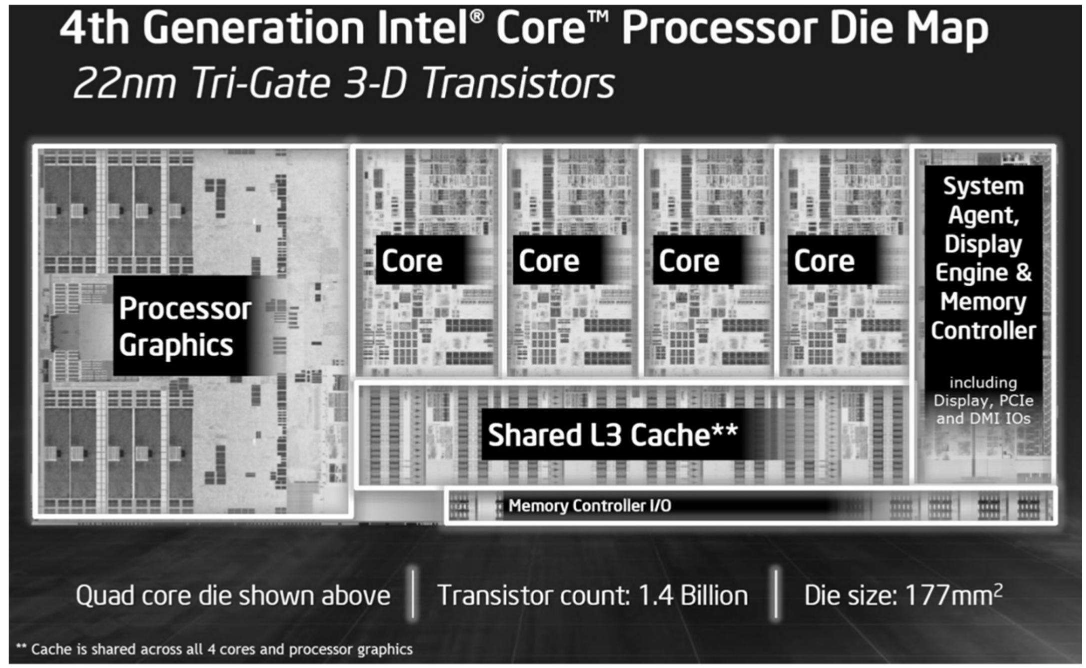




- One can buy gates separately
  - ex. 74xxx series of integrated circuits
  - cost ~$1 per chip, mostly for packaging and testing
- Cumbersome, but possible to build devices using gates put together manually

Then and Now



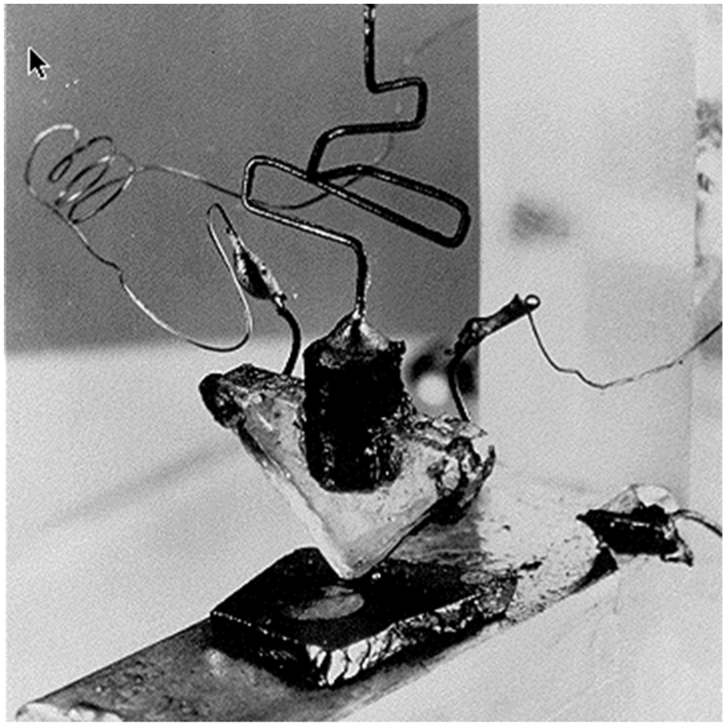


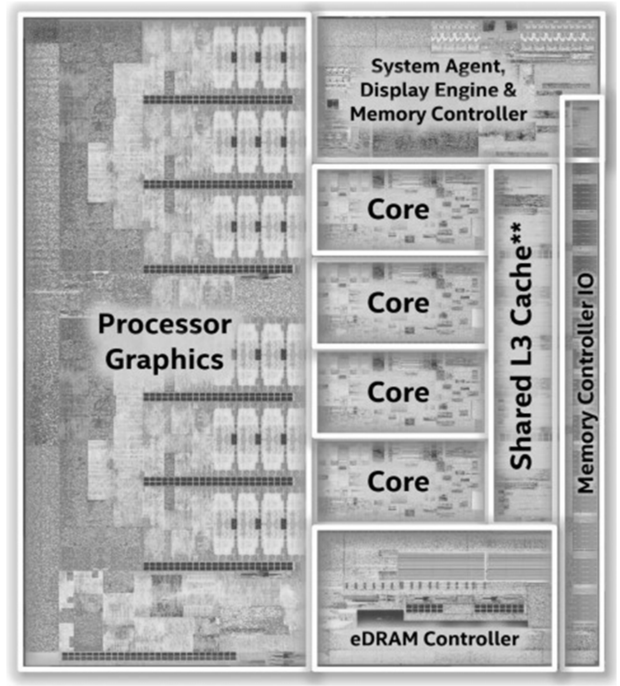
http://techguru3d.com/4th-gen-intel-haswell-processors-architecture-and-lineup/

- The first transistor
  - One workbench at AT&T Bell Labs
  - 1947
  - Bardeen, Brattain, and Shockley
- Intel Haswell
  - 1.4 billion transistors, 22nm
  - 177 square millimeters
  - Four processing cores

https://en.wikipedia.org/wiki/Transistor_count

Then and Now





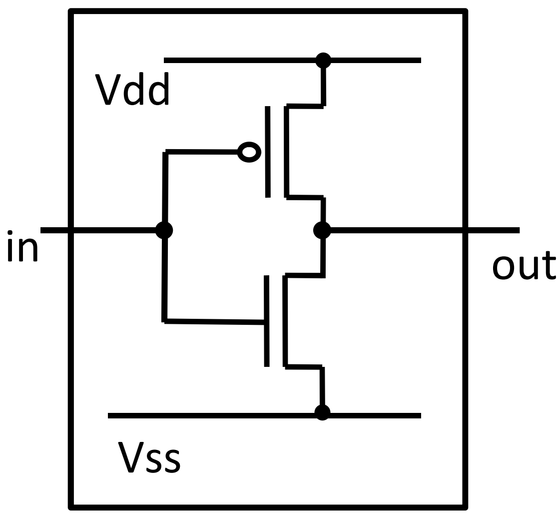
https://www.computershopper.com/computex-2015/performance-preview-desktop-broadwell-at-computex-2(

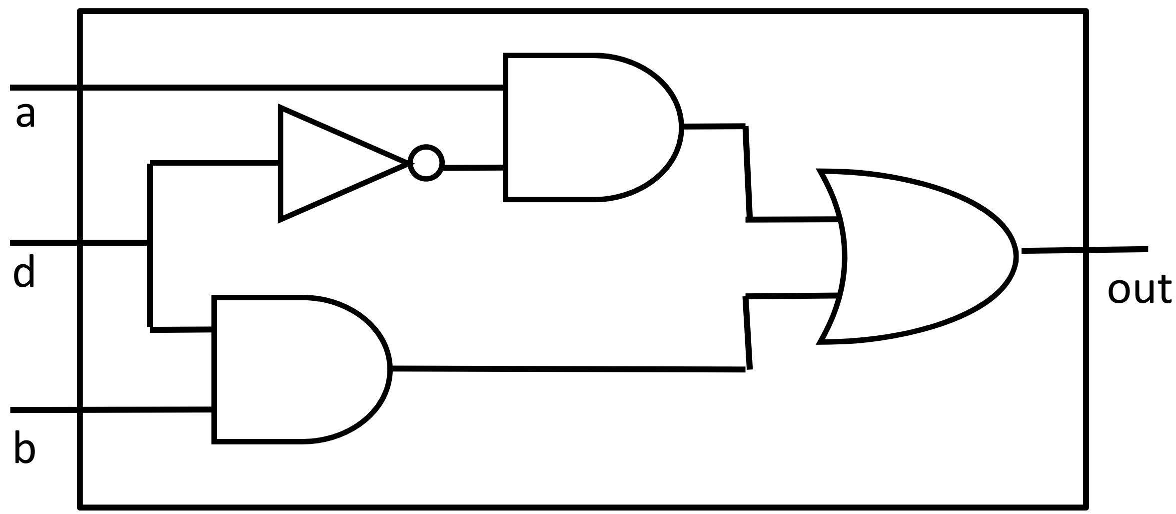
- The first transistor
  - One workbench at AT&T Bell Labs
  - 1947
  - Bardeen, Brattain, and Shockley
- Intel Broadwell
  - 7.2 billion transistors, 14nm
  - 456 square millimeters
  - Up to 22 processing cores

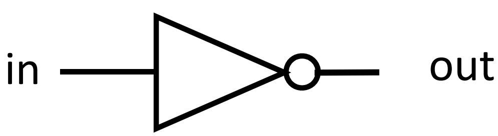
https://en.wikipedia.org/wiki/Transistor_count

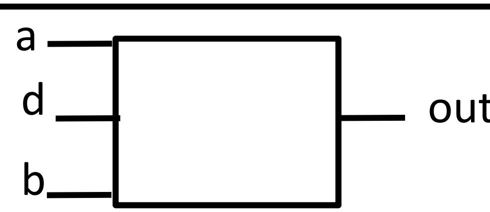
Big Picture: Abstraction

- Hide complexity through simple abstractions
  - Simplicity
    - Box diagram represents inputs and outputs
  - Complexity
    - Hides underlying NMOS- and PMOS-transistors and atomic interactions









Summary

- Most modern devices made of billions of transistors
  - You will build a processor in this course!
  - Modern transistors made from semiconductor materials
  - Transistors used to make logic gates and logic circuits

- We can now implement any logic circuit
  - Use P- & N-transistors to implement NAND/NOR gates
  - Use NAND or NOR gates to implement the logic circuit
  - *Efficiently*: use K-maps to find required minimal terms