

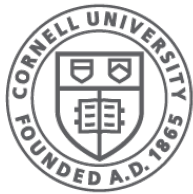
Gates and Logic: From Transistors to Logic Gates and Logic Circuits

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CS 3410

Computer Science

Cornell University



Cornell CIS
COMPUTING AND INFORMATION SCIENCE

[Weatherspoon, Bala, Bracy, and Sirer]

Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Understanding the foundations of
 - Computer Systems Organization and Programming



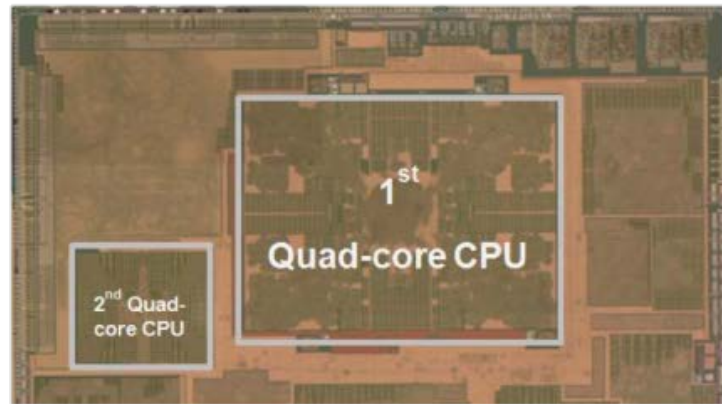
Goals for Today

- From Switches to Logic Gates to Logic Circuits
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 - Computer Systems Organization and Programming
 - e.g. Galaxy Note 9



Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Understanding the foundations of
 - Computer Systems Organization and Programming
 - e.g. Galaxy Note 9
 - with the big.LITTLE DynamicIQ 8-core ARM processor



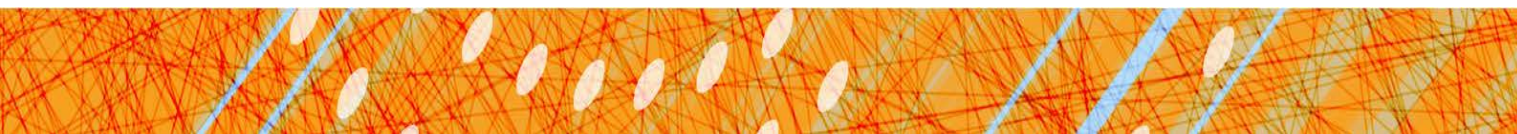
Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Understanding the foundations of
 - Computer Systems Organization and Programming
 - e.g. Galaxy Note 9
 - with the big.LITTLE DynamicIQ 8-core ARM processor

	big Quad Core	LITTLE Quad Core
Architecture	ARM v8	ARM v8
Process	Samsung 10nm	Samsung 10nm
Frequency	2.9GHz+	1.9GHz
Area	3.5mm ²	
Power-ratio	1	0.17
L1 Cache Size	64 KB I/D Cache	64 KB I/D Cache
L2 Cache Size	2 MB Data Cache	512 KB Data Cache

Goals for Today

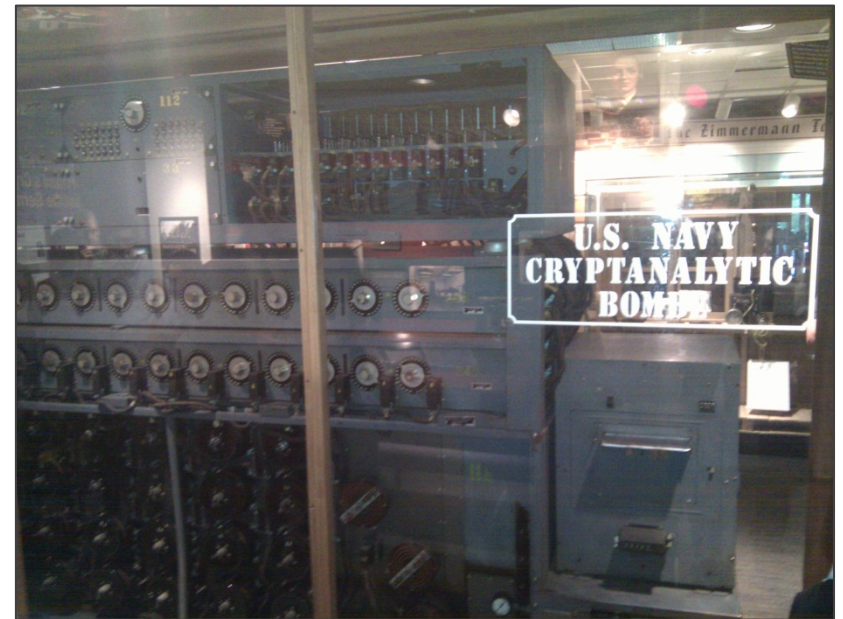
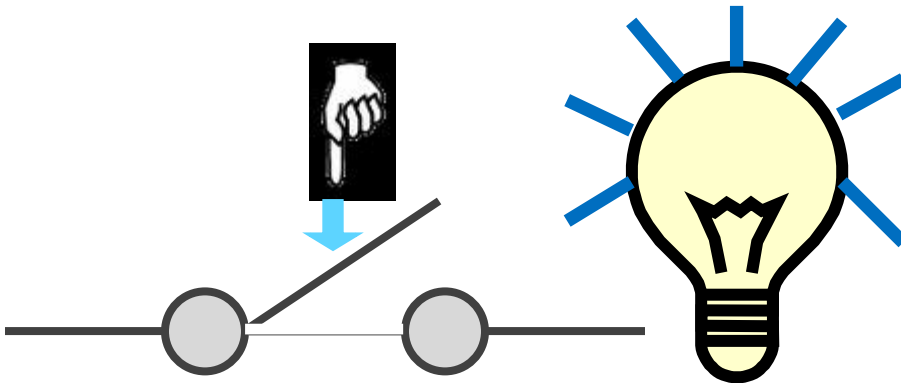
- From Switches to Logic Gates to Logic Circuits
- Logic Gates
 - From switches
 - Truth Tables
- Logic Circuits
 - From Truth Tables to Circuits (Sum of Products)
 - Identity Laws
- Logic Circuit Minimization
 - Algebraic Manipulations
 - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)



A switch

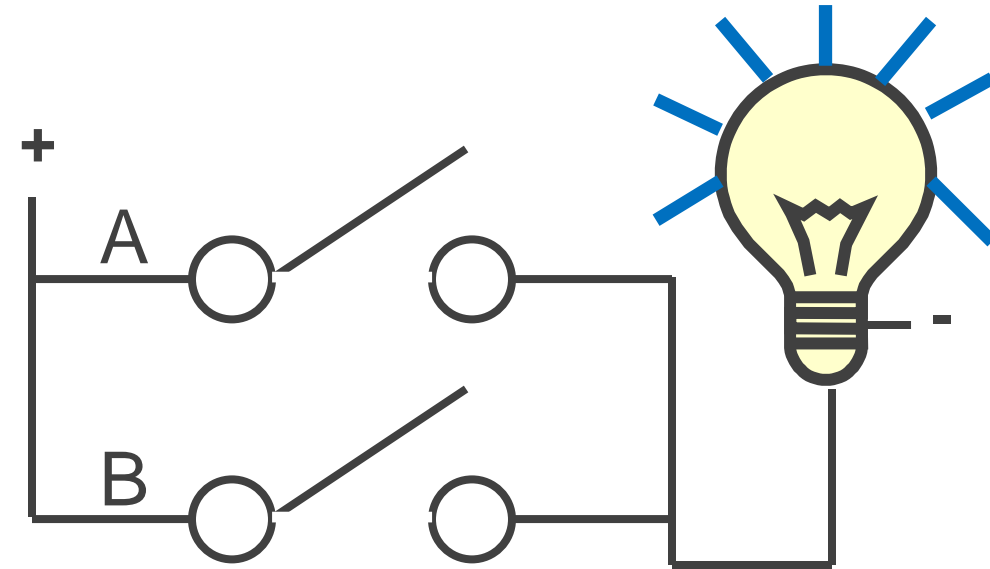
Acts as a *conductor* or *insulator*.

Can be used to build amazing things...



The Bombe used to break the German Enigma machine during World War II

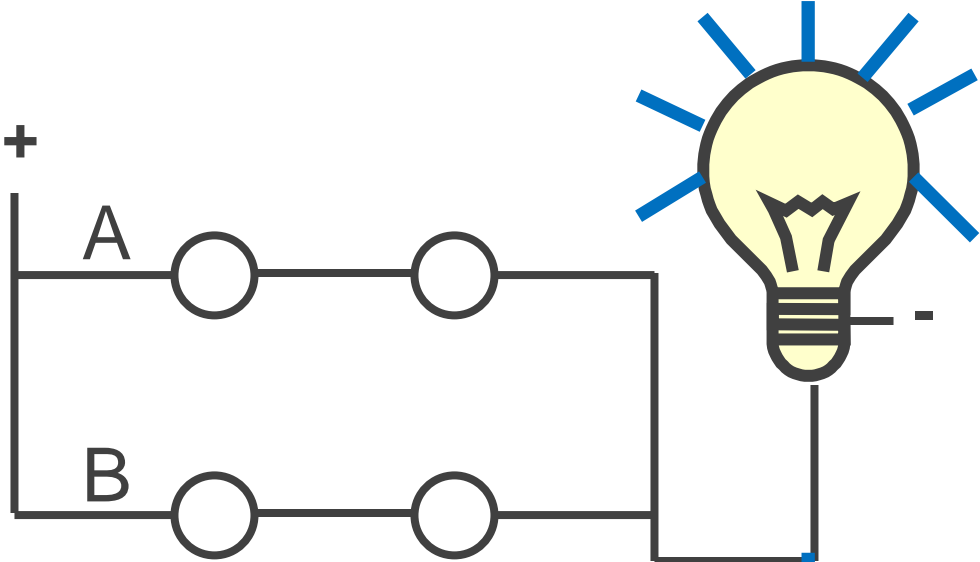
Basic Building Blocks: Switches to Logic Gates



Truth Table

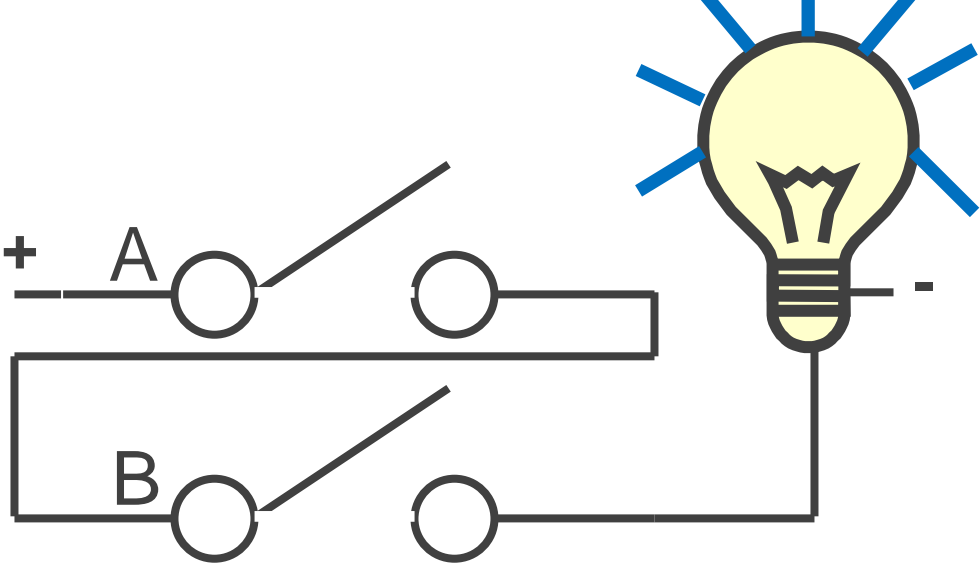
A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

Basic Building Blocks: Switches to Logic Gates



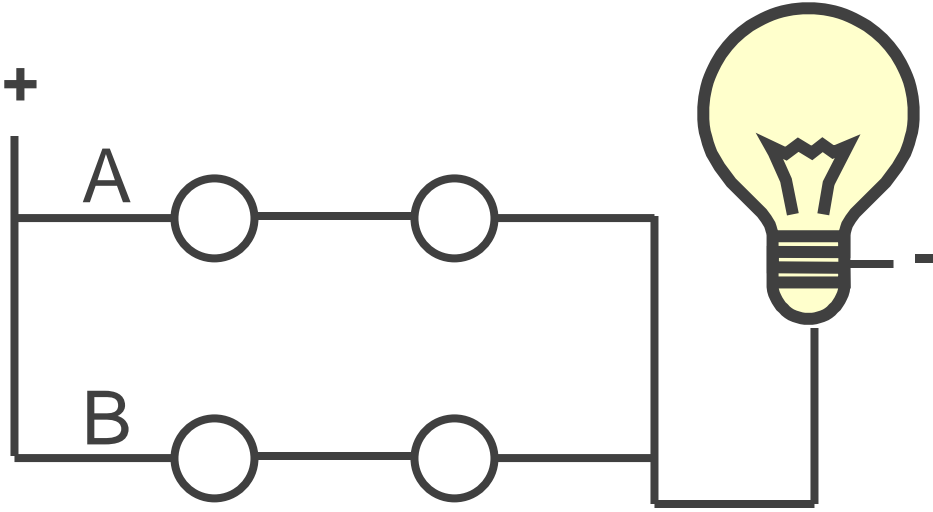
Truth Table

A	B	Light
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON



A	B	Light
OFF	OFF	
OFF	ON	
ON	OFF	
ON	ON	

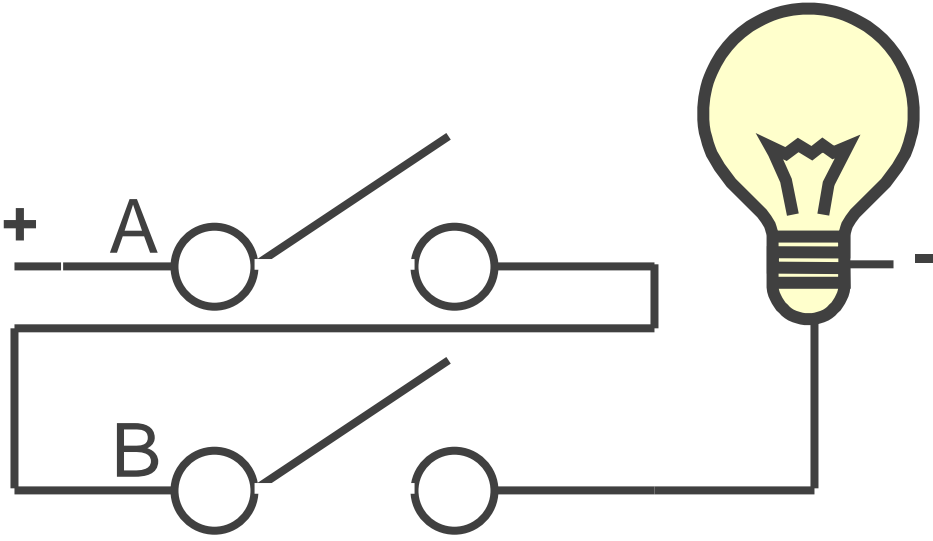
Basic Building Blocks: Switches to Logic Gates



- **Either (OR)**

Truth Table

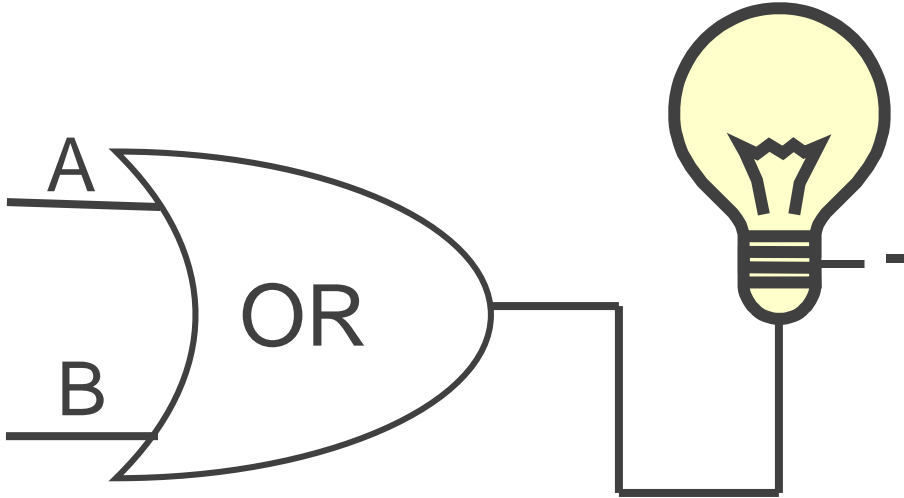
A	B	Light
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON



- **Both (AND)**

A	B	Light
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

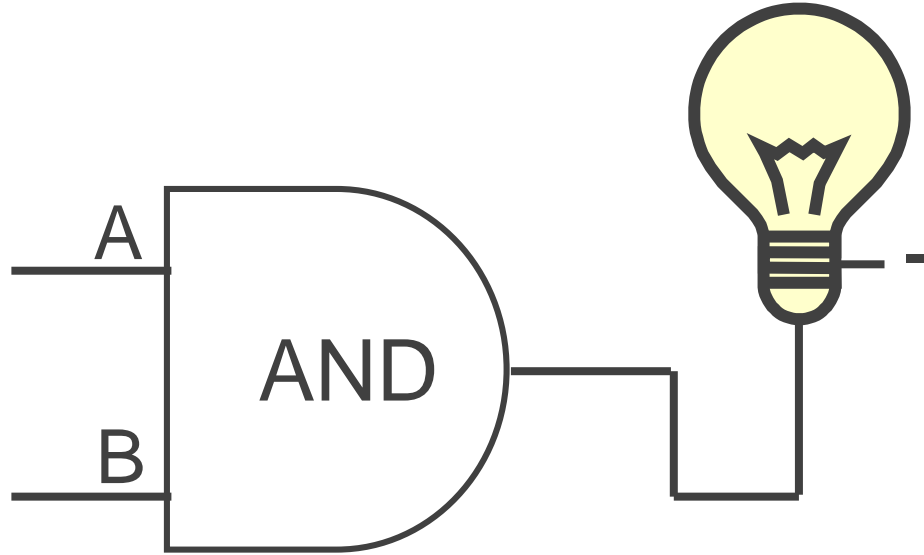
Basic Building Blocks: Switches to Logic Gates



- **Either (OR)**

Truth Table

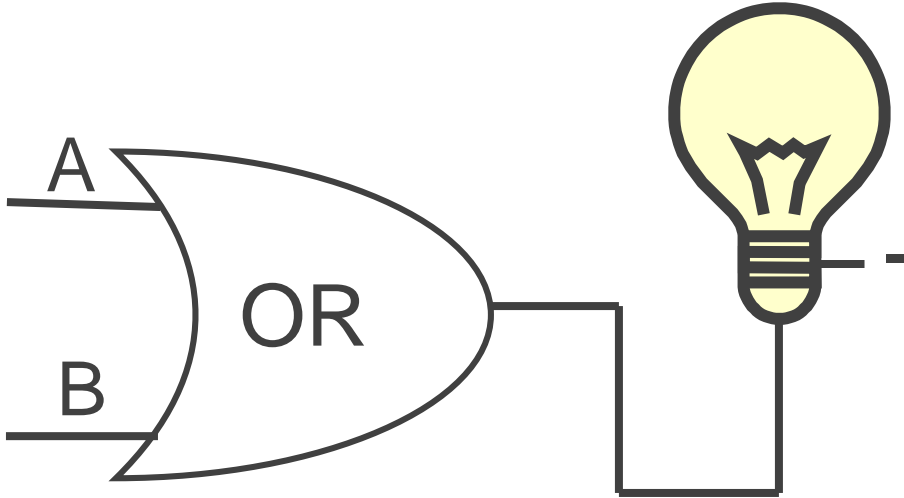
A	B	Light
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON



- **Both (AND)**

A	B	Light
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

Basic Building Blocks: Switches to Logic Gates

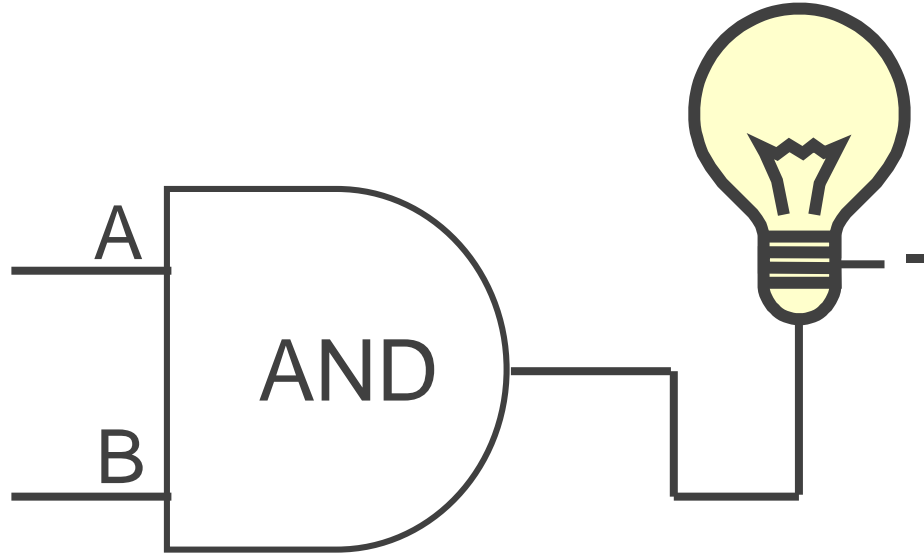


- **Either (OR)**

Truth Table

A	B	Light
0	0	0
0	1	1
1	0	1
1	1	1

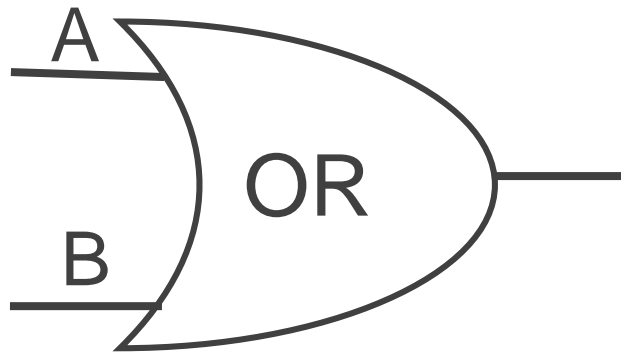
0 = OFF
1 = ON



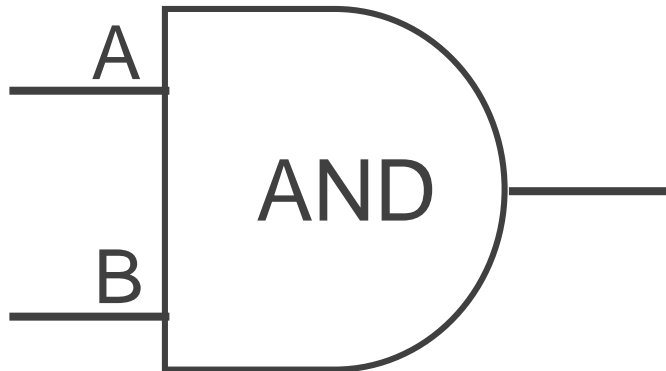
- **Both (AND)**

A	B	Light
0	0	0
0	1	0
1	0	0
1	1	1

Basic Building Blocks: Switches to Logic Gates



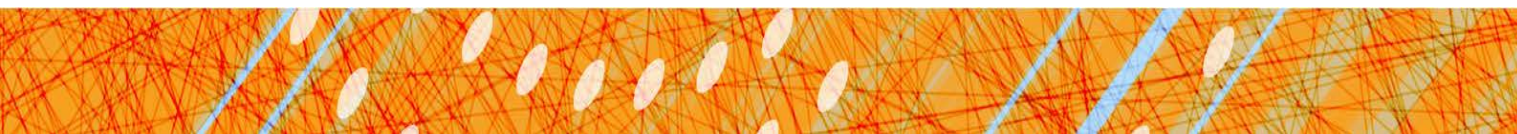
George Boole (1815-1864)



- **Did you know?**
- **George Boole:** Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.

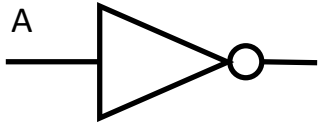
Takeaway

- Binary (two symbols: **true** and **false**) is the basis of Logic Design



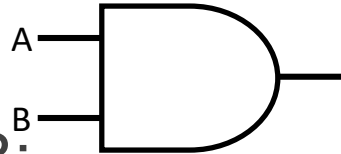
Building Functions: Logic Gates

- NOT:



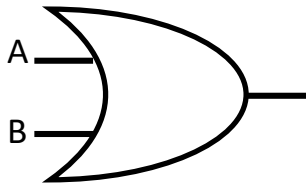
A	Out

- AND:



A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

- OR:



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

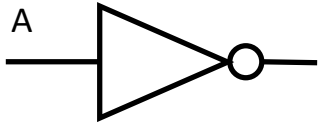
- Logic Gates

- digital circuit that either allows a signal to pass through it or not.
- Used to build logic functions
- There are seven basic logic gates:

AND, OR, NOT,

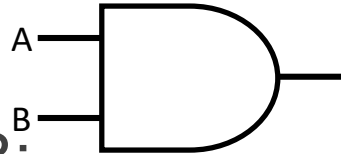
Building Functions: Logic Gates

- NOT:



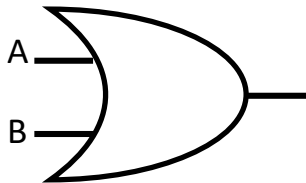
A	Out
0	1
1	0

- AND:



A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

- OR:



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

- Logic Gates

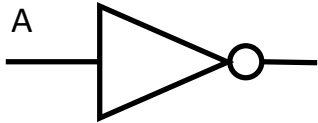
- digital circuit that either allows a signal to pass through it or not.
- Used to build logic functions
- There are seven basic logic gates:

AND, OR, NOT,

NAND (not AND), **NOR** (not OR), **XOR**, and **XNOR** (not XOR) [later]

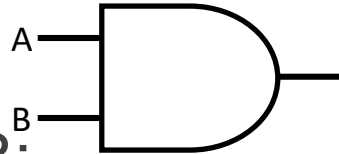
Building Functions: Logic Gates

- NOT:



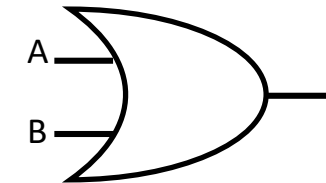
A	Out
0	1
1	0

- AND:



A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

- OR:



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

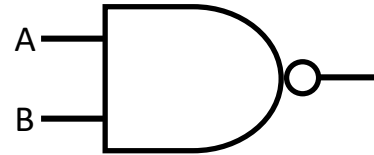
- Logic Gates

- digital circuit that either allows a signal to pass through it or not.
- Used to build logic functions
- There are seven basic logic gates:

AND, OR, NOT,

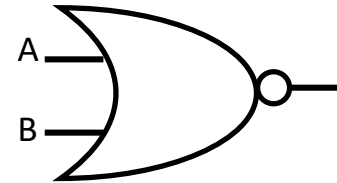
NAND (not AND), **NOR** (not OR), **XOR**, and **XNOR** (not XOR) [later]

NAND:



A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0






Which Gate is this? iClicker Question

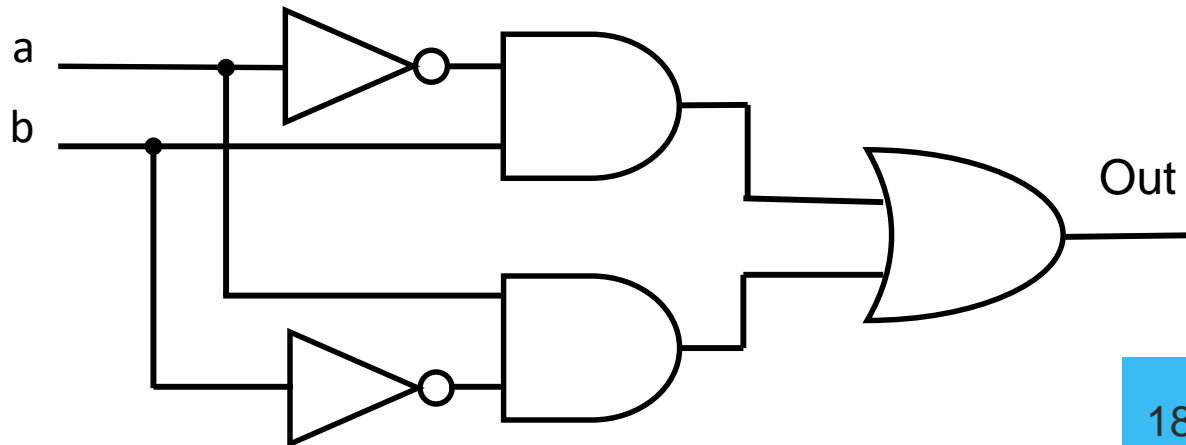
Function:

Symbol:

a	b	Out

Truth Table:






- (A) NOT 
- (B) OR 
- (C) XOR 
- (D) AND 
- (E) NAND 

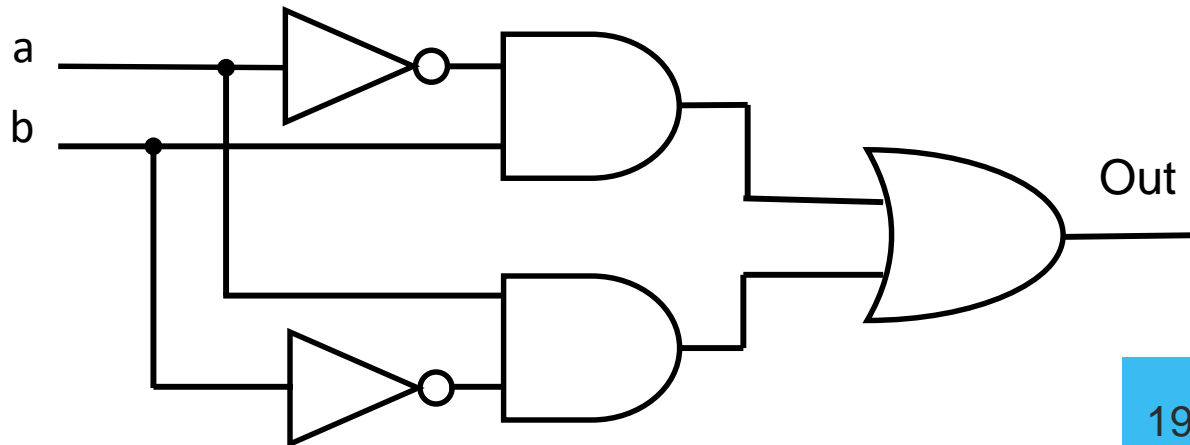


Which Gate is this? iClicker Question

- XOR: out = 1 if a or b is 1, but not both;
- out = 0 otherwise.
- out = 1, only if a = 1 AND b = 0
- OR a = 0 AND b = 1

a	b	Out
0	0	0
0	1	1
1	0	1
1	1	0

- (A) NOT 
- (B) OR 
- (C) XOR 
- (D) AND 
- (E) NAND 



Which Gate is this? iClicker Question

- XOR: out = 1 if a or b is 1, but not both;
- out = 0 otherwise.
- out = 1, only if a = 1 AND b = 0
- OR a = 0 AND b = 1

a	b	Out
0	0	0
0	1	1
1	0	1
1	1	0

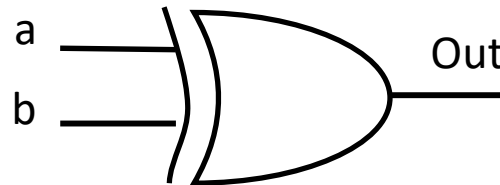
(A) NOT 

(B) OR 

(C) XOR 

(D) AND 

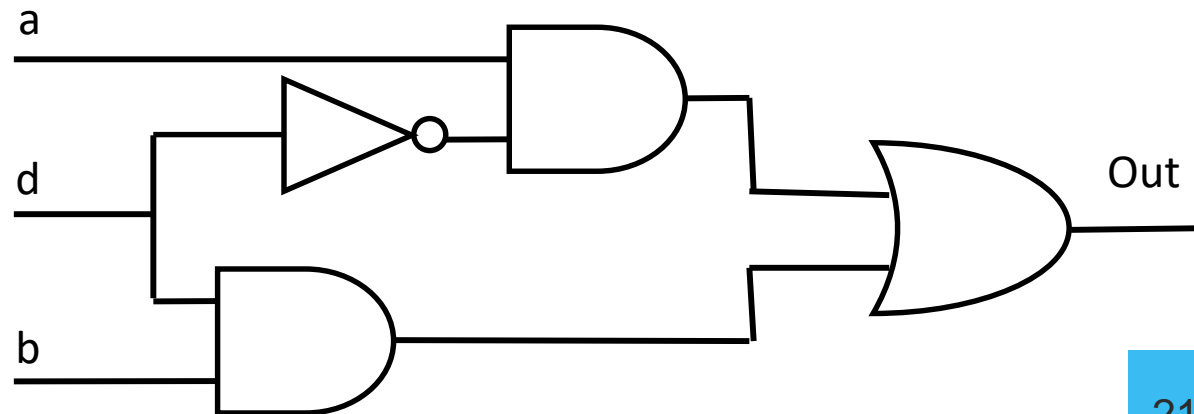
(E) NAND 



Activity#2: Logic Gates

- Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.
- What does the logic circuit do?

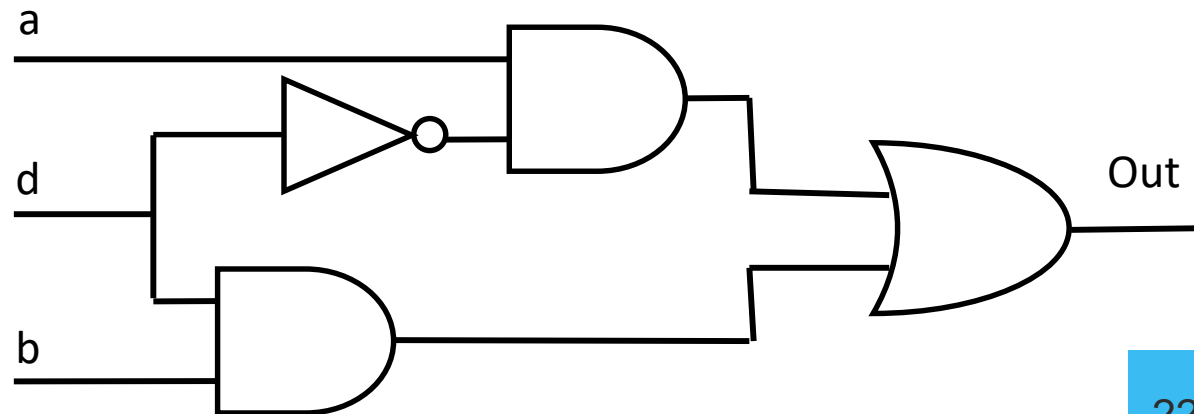
a	b	d	Out
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



Activity#2: Logic Gates

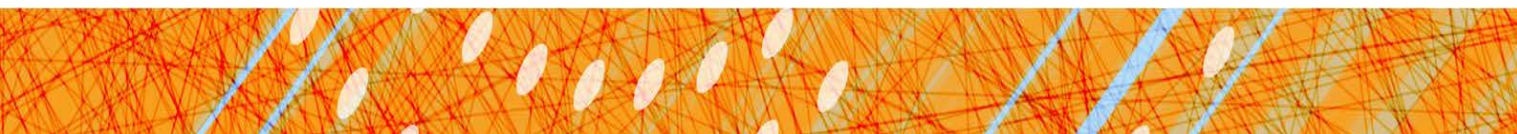
- **Multiplexor**: select (**d**) between two inputs (**a** and **b**) and set one as the output (**out**)?
- $\text{out} = a$, if $d = 0$
- $\text{out} = b$, if $d = 1$

a	b	d	Out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



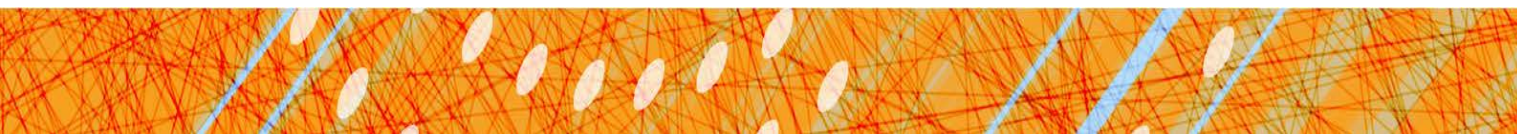
Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
 - From switches
 - Truth Tables
- Logic Circuits
 - From Truth Tables to Circuits (Sum of Products)
 - Identity Laws
- Logic Circuit Minimization
 - Algebraic Manipulations
 - Truth Tables (Karnaugh Maps)
- Transistors (electronic switch)



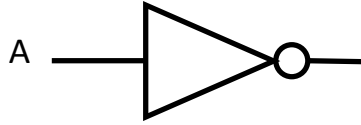
Next Goal

- Given a Logic function, create a Logic Circuit that implements the Logic Function...
- ...and, *with the minimum number of logic gates*
- Fewer gates: A cheaper (\$\$\$) circuit!



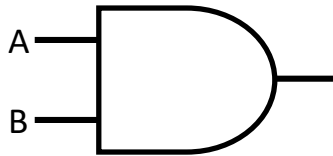
Logic Gates

NOT:



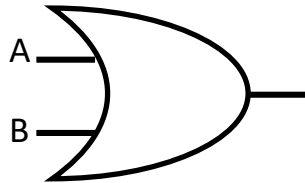
A	Out
0	1
1	0

AND:



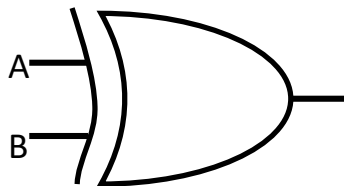
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

OR:



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

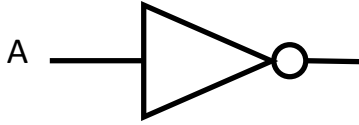
XOR:



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

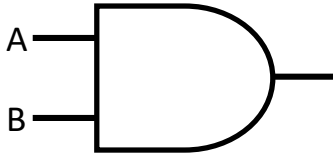
Logic Gates

NOT:



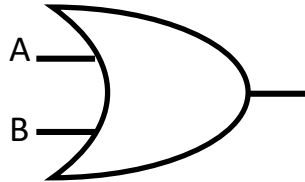
A	Out
0	1
1	0

AND:



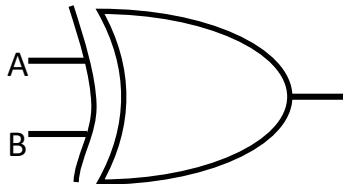
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

OR:



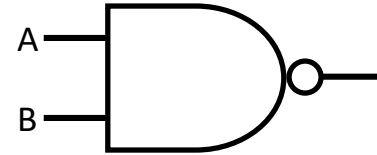
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

XOR:



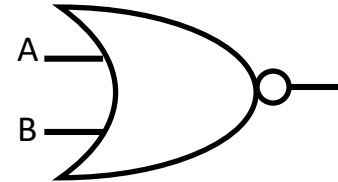
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

NAND:



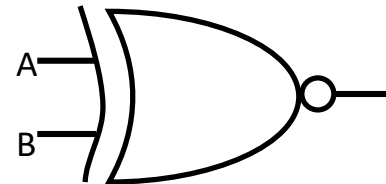
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

NOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

XNOR:



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	1

Logic Implementation

- How to implement a desired logic function?

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Logic Implementation

- How to implement a desired logic function?

a	b	c	out	minterm
0	0	0	0	$\bar{a} \bar{b} \bar{c}$
0	0	1	1	$\bar{a} \bar{b} c$
0	1	0	0	$\bar{a} b \bar{c}$
0	1	1	1	$\bar{a} b c$
1	0	0	0	$a \bar{b} \bar{c}$
1	0	1	1	$a \bar{b} c$
1	1	0	0	$a b \bar{c}$
1	1	1	0	$a b c$

1) Write **minterms**

2) **sum of products:**

- OR of all minterms where out=1

Logic Implementation

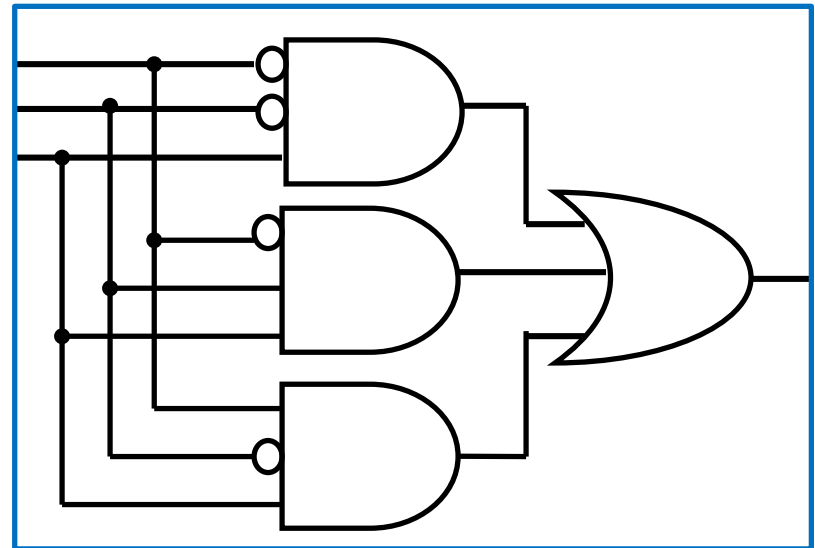
- How to implement a desired logic function?

a	b	c	out	minterm
0	0	0	0	$\bar{a} \bar{b} \bar{c}$
0	0	1	1	$\bar{a} b c$
0	1	0	0	$\bar{a} b \bar{c}$
0	1	1	1	$\bar{a} b c$
1	0	0	0	$a \bar{b} \bar{c}$
1	0	1	1	$a \bar{b} c$
1	1	0	0	$a b \bar{c}$
1	1	1	0	$a b c$

1) Write **minterms**

2) **sum of products:**

- OR of all minterms where out=1
 - E.g. $out = \bar{a} b c + \bar{a} b c + a \bar{b} c$



corollary: *any* combinational circuit *can be* implemented in two levels of logic (ignoring inverters)

Logic Equations

- NOT:
 - $\text{out} = \bar{a} = !a = \neg a$
- AND:
 - $\text{out} = a \cdot b = a \& b = a \wedge b$
- OR:
 - $\text{out} = a + b = a | b = a \vee b$
- XOR:
 - $\text{out} = a \oplus b = a\bar{b} + \bar{a}b$
- Logic Equations
 - Constants: true = 1, false = 0
 - Variables: a, b, out, ...
 - Operators (above): AND, OR, NOT, etc.

Logic Equations

- NOT:

- $\text{out} = \bar{a} = !a = \neg a$

- AND:

- $\text{out} = a \cdot b = a \& b = a \wedge b$

- NAND:

- $\text{out} = \overline{a \cdot b} = !(a \& b) = \neg (a \wedge b)$

- OR:

- $\text{out} = a + b = a | b = a \vee b$

- NOR:

- $\text{out} = \overline{a + b} = !(a | b) = \neg (a \vee b)$

- XOR:

- $\text{out} = a \oplus b = a\bar{b} + \bar{a}b$

- XNOR:

- $\text{out} = \overline{a \oplus b} = ab + \bar{a}\bar{b}$

- Logic Equations

- Constants: true = 1, false = 0
- Variables: a, b, out, ...
- Operators (above): AND, OR, NOT, etc.

Identities

Identities useful for manipulating logic equations

– For optimization & ease of implementation

$$a + 0 =$$

$$a + 1 =$$

$$a + \bar{a} =$$

$$a \cdot 0 =$$

$$a \cdot 1 =$$

$$a \cdot \bar{a} =$$

Identities

Identities useful for manipulating logic equations

– For optimization & ease of implementation

$$a + 0 = a$$

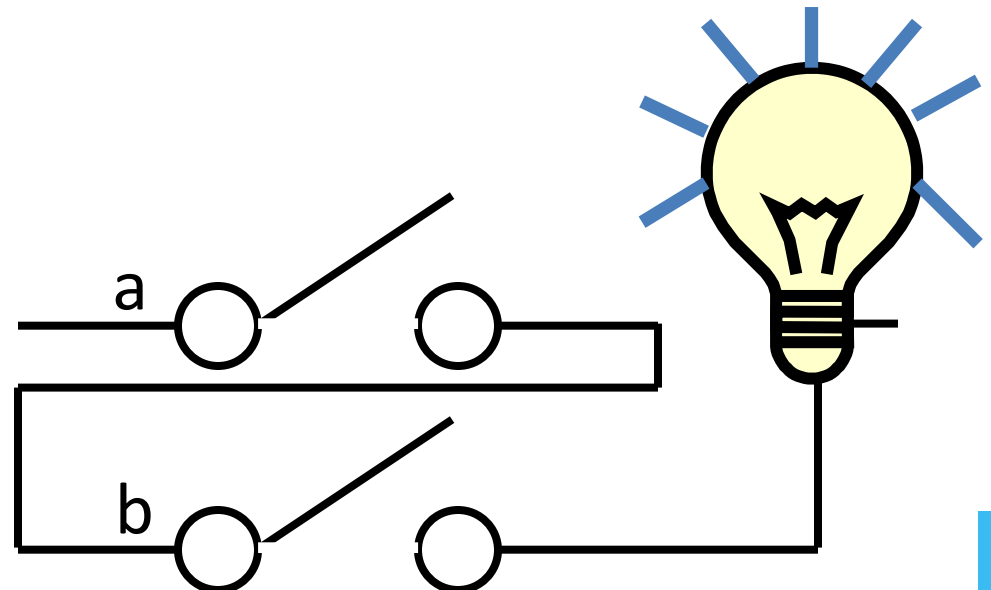
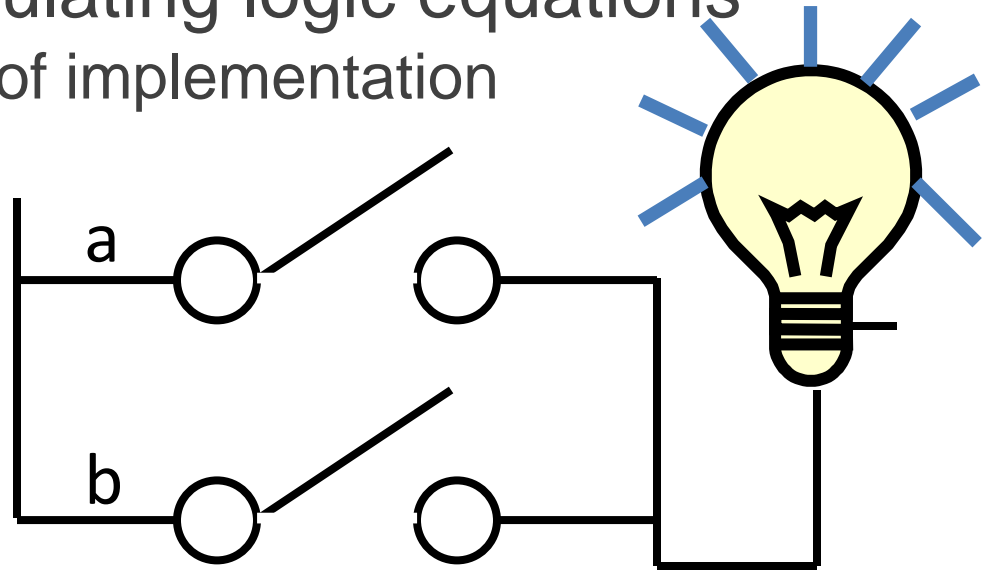
$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$



Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$$\overline{(a + b)} =$$

$$\overline{(a \cdot b)} =$$

$$a + a b =$$

$$a(b+c) =$$

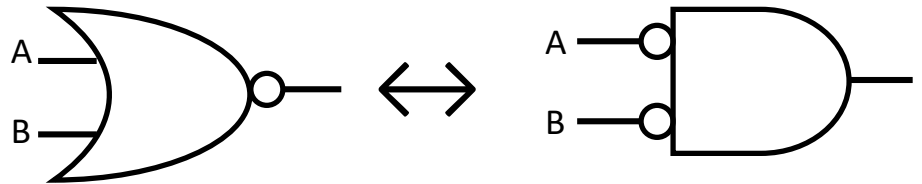
$$\overline{a(b + c)} =$$

Identities

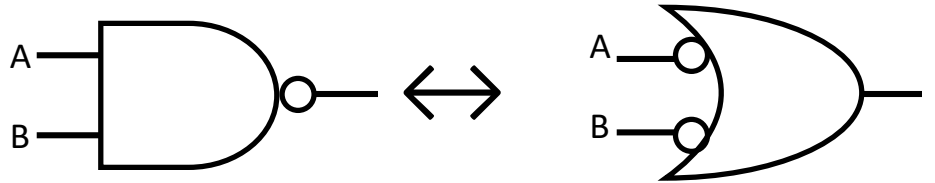
Identities useful for manipulating logic equations

- For optimization & ease of implementation

$$\overline{(a + b)} = \bar{a} \cdot \bar{b}$$



$$\overline{(a \cdot b)} = \bar{a} + \bar{b}$$



$$a + a b = a$$

$$a(b+c) = ab + ac$$

$$\overline{a(b + c)} = \bar{a} + \bar{b} \cdot \bar{c}$$

Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
 - From switches
 - Truth Tables
- Logic Circuits
 - From Truth Tables to Circuits (Sum of Products)
 - Identity Laws
- **Logic Circuit Minimization – *why?***
 - Algebraic Manipulations
 - Truth Tables (Karnaugh Maps)
 - Transistors (electronic switch)

Minimization Example

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$

$$a + ab = a$$

$$a(b+c) = ab + ac$$

$$\overline{(a + b)} = \bar{a} \bullet \bar{b}$$

$$\overline{(ab)} = \bar{a} + \bar{b}$$

$$\overline{a(b + c)} = \bar{a} + \bar{b} \bullet \bar{c}$$

Minimize this logic equation:

$$\begin{aligned}(a+b)(a+c) &= (a+b)a + (a+b)c \\ &= aa + ba + ac + bc \\ &= a + a(b+c) + bc \\ &= a + bc\end{aligned}$$

iClicker Question

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$

$$a + a b = a$$

$$a(b+c) = ab + ac$$

$$\overline{(a + b)} = \bar{a} \cdot \bar{b}$$

$$\overline{(ab)} = \bar{a} + \bar{b}$$

$$\overline{a(b + c)} = \bar{a} + \bar{b} \cdot \bar{c}$$

$$(a+b)(a+c) \rightarrow a + bc$$

How many gates were required before and after?

BEFORE

AFTER

(A) 2 OR

1 OR

(B) 2 OR, 1 AND

2 OR

(C) 2 OR, 1 AND

1 OR, 1 AND

(D) 2 OR, 2 AND

2 OR

(E) 2 OR, 2 AND

2 OR, 1 AND

iClicker Question

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$

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$$\overline{(a + b)} = \bar{a} \cdot \bar{b}$$

$$\overline{(ab)} = \bar{a} + \bar{b}$$

$$\overline{a(b + c)} = \bar{a} + \bar{b} \cdot \bar{c}$$

$$(a+b)(a+c) \rightarrow a + bc$$

How many gates were required before and after?

BEFORE

AFTER

(A) 2 OR

1 OR

(B) 2 OR, 1 AND

2 OR

(C) 2 OR, 1 AND

1 OR, 1 AND

(D) 2 OR, 2 AND

2 OR

(E) 2 OR, 2 AND

2 OR, 1 AND

Checking Equality w/Truth Tables

circuits \leftrightarrow truth tables \leftrightarrow equations

Example: $(a+b)(a+c) = a + bc$

a	b	c					
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Checking Equality w/Truth Tables

circuits \leftrightarrow truth tables \leftrightarrow equations

Example: $(a+b)(a+c) = a + bc$

a	b	c					
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Checking Equality w/Truth Tables

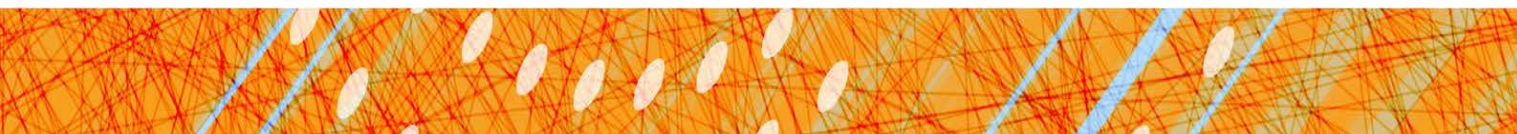
circuits \leftrightarrow truth tables \leftrightarrow equations

Example: $(a+b)(a+c) = a + bc$

a	b	c	a+b	a+c	LHS	bc	RHS
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Takeaway

- Binary (two symbols: **true** and **false**) is the basis of Logic Design
- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.



Goals for Today

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
 - From switches
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 - Identity Laws
- **Logic Circuit Minimization**
 - Algebraic Manipulations
 - Truth Tables (Karnaugh Maps)
- **Transistors (electronic switch)**

Karnaugh Maps

How does one find the most efficient equation?

- Manipulate algebraically until...?
- Use **Karnaugh Maps** (optimize visually)
- Use a software optimizer

For large circuits

- Decomposition & reuse of building blocks

Minimization with Karnaugh maps (1)

◆ Sum of minterms yields

■ out =

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Minimization with Karnaugh maps (2)

◆ Sum of minterms yields

- $out = \overline{a}bc + a\overline{b}c + a\overline{b}\overline{c} + a\overline{b}c$

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

◆ Karnaugh maps identify which inputs are (ir)relevant to the output

		ab			
		00	01	11	10
c	0				
	1				

Minimization with Karnaugh maps (2)

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

		ab			
		00	01	11	10
c	0	0	0	0	1
	1	1	1	0	1

◆ Sum of minterms yields

- $out = \overline{a}bc + a\overline{b}c + \overline{a}\overline{b}c + a\overline{b}\overline{c}$

◆ Karnaugh map minimization

- Cover all 1's
- Group adjacent blocks of 2^n 1's that yield a rectangular shape
- Encode the common features of the rectangle

- ◆ $out =$

Karnaugh Minimization Tricks (1)

c \ ab	00	01	11	10
0	0	1	1	1
1	0	0	1	0

◆ Minterms can overlap

■ out =

c \ ab	00	01	11	10
0	1	1	1	1
1	0	0	1	0

◆ Minterms can span 2, 4, 8 or more cells

■ out =

Karnaugh Minimization Tricks (2)

		ab			
		00	01	11	10
cd	00	0	0	0	0
	01	1	0	0	1
	11	1	0	0	1
	10	0	0	0	0

- The map wraps around
 - out =

		ab			
		00	01	11	10
cd	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

Karnaugh Minimization Tricks (3)

	ab			
cd	00	01	11	10
00	0	0	0	0
01	1	x	x	x
11	1	x	x	1
10	0	0	0	0

- “Don’t care” values can be interpreted individually in whatever way is convenient

- assume all x’s = 1
- out = $\bar{c}d$

	ab			
cd	00	01	11	10
00	1	0	0	x
01	0	x	x	0
11	0	x	x	0
10	1	0	0	1

- assume middle x’s = 0
- assume 4th column x = 1
- out = $\bar{c}d + cd$

Minimization with K-Maps

	ab			
c	00	01	11	10
0	0	0	0	1
1	1	1	0	1

(1) Circle the 1's (see below)

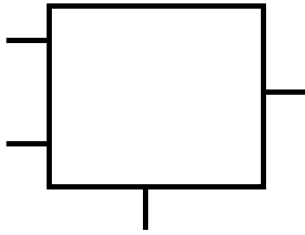
(2) Each circle is a logical component of the final equation

$$= a\bar{b} + \bar{a}c$$

Rules:

- Use fewest circles necessary to cover all 1's
- Circles must cover *only* 1's
- Circles span rectangles of size power of 2 (1, 2, 4, 8...)
- Circles should be as large as possible (all circles of 1?)
- Circles may wrap around edges of K-Map
- 1 may be circled multiple times *if* that means fewer circles

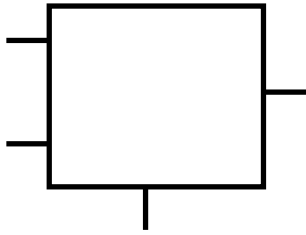
Multiplexer



a	b	d	out
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

- A multiplexer selects between multiple inputs
 - $\text{out} = a$, if $d = 0$
 - $\text{out} = b$, if $d = 1$
- Build truth table
- Minimize diagram
- Derive logic diagram

Multiplexer Implementation

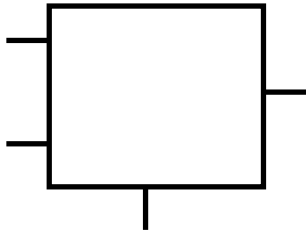


- Build a truth table

$$\text{out} = \bar{a}bd + a\bar{b}\bar{d} + ab\bar{d} + abd$$

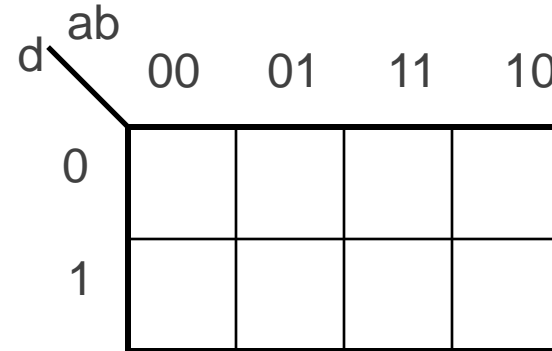
a	b	d	out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Multiplexer Implementation

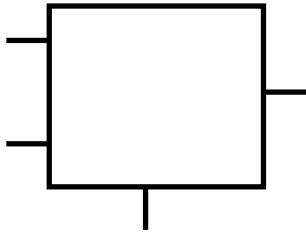


- Build the Karnaugh map

a	b	d	out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Multiplexer Implementation

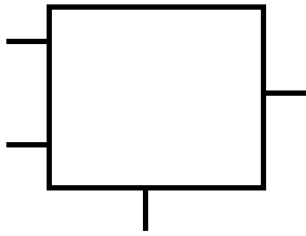


- Build the Karnaugh map

a	b	d	out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		ab			
		00	01	11	10
d	0	0	0	1	1
	1	0	1	1	0

Multiplexer Implementation

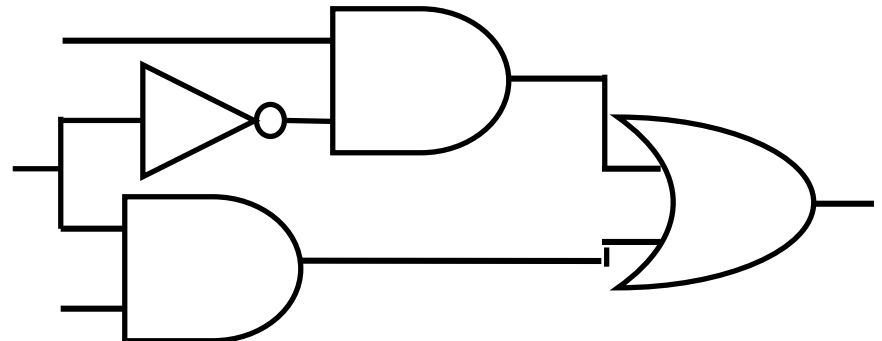


a	b	d	out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

- Derive Minimal Logic Equation

d \ ab	00	01	11	10
0	0	0	1	1
1	0	1	1	0

- $out = a\bar{d} + bd$



Takeaway

- Binary (two symbols: **true** and **false**) is the basis of Logic Design
- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.
- Any logic function can be implemented as “sum of products”. Karnaugh Maps minimize number of gates.

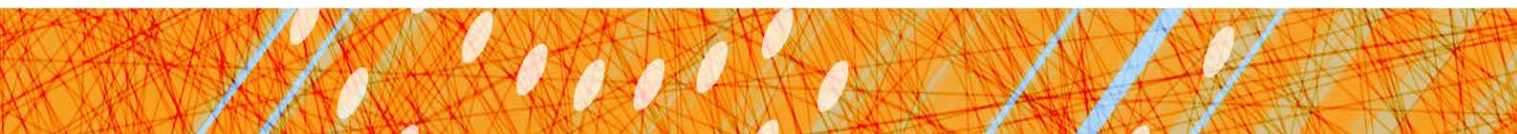
Administrivia

- **Dates to keep in Mind**

- Prelims: Tue Mar 5th and Thur May 2nd
- Proj 1: Due Fri Feb 15th
- Proj 2: Due Fri Mar 11th
- Proj 3: Due Thur Mar 28th before Spring break
- Final Project: Due Tue May 16th

Goals for Today

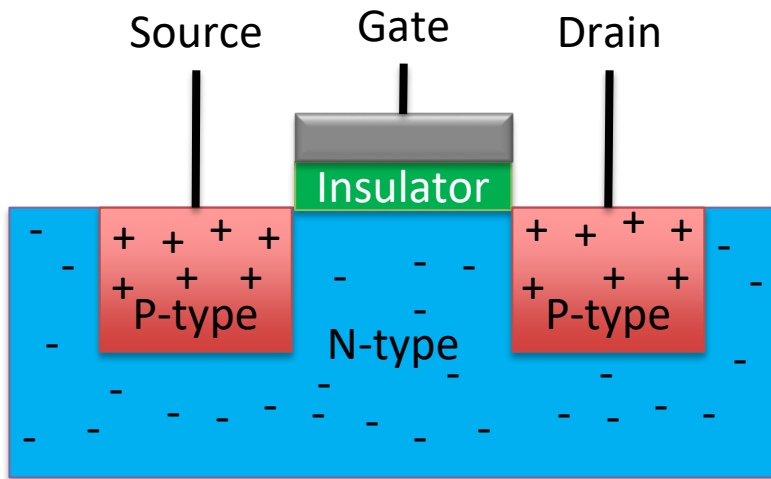
- From Switches to Logic Gates to Logic Circuits
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 - Identity Laws
- Logic Circuit Minimization
 - Algebraic Manipulations
 - Truth Tables (Karnaugh Maps)
- **Transistors (electronic switch)**



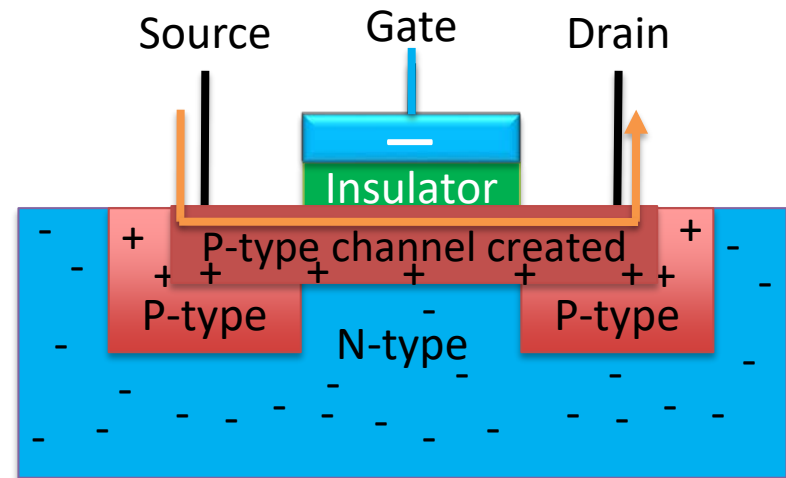
Silicon Valley & the Semiconductor Industry

- Transistors:
- Youtube video “How does a transistor work”
<https://www.youtube.com/watch?v=lcrBqCFLHIY>
- Break: show some Transistor, Fab, Wafer photos

Transistors 101



P-Transistor Off



P-Transistor On

N-Type Silicon: negative free-carriers (electrons)

P-Type Silicon: positive free-carriers (holes)

P-Transistor: negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain

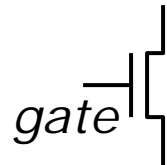
N-Transistor: works the opposite way

Metal-Oxide Semiconductor (Gate-Insulator-Silicon)

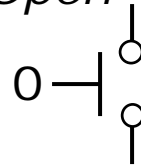
- Complementary MOS = **CMOS** technology uses both p- & n-type transistors

CMOS Notation

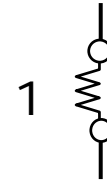
N-type



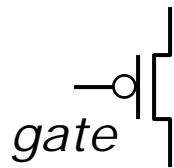
Off/Open



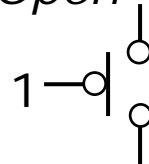
On/Closed



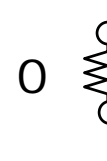
P-type



Off/Open



On/Closed



Gate input controls whether current can flow between the other two terminals or not.

Hint: the “o” bubble of the p-type tells you that this gate wants a 0 to be turned on

iClicker Question

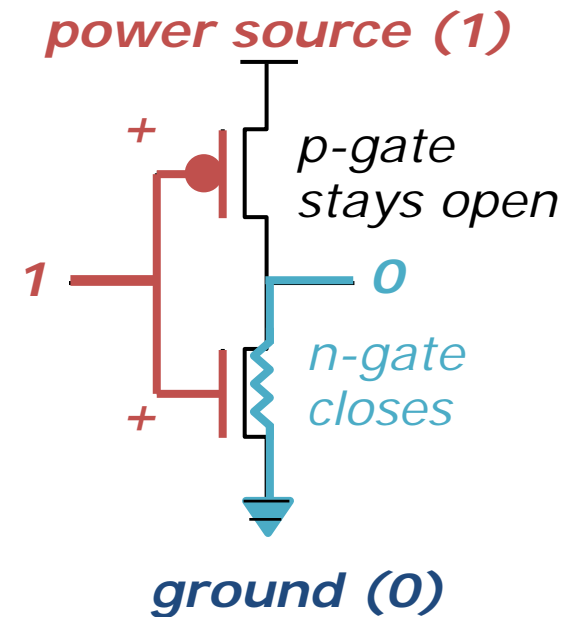
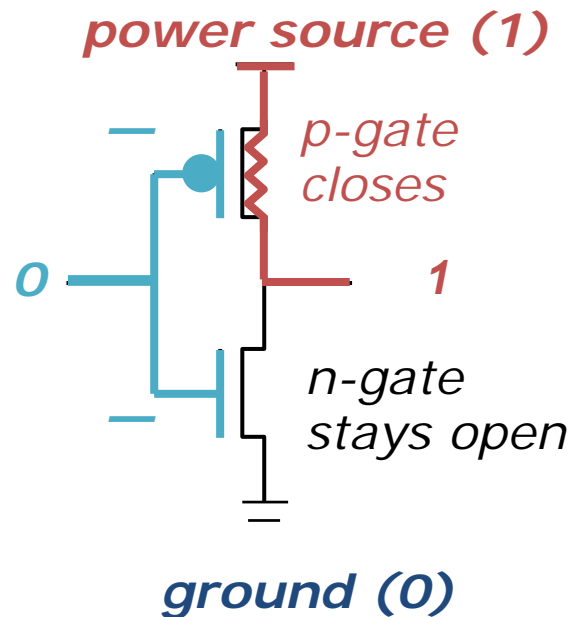
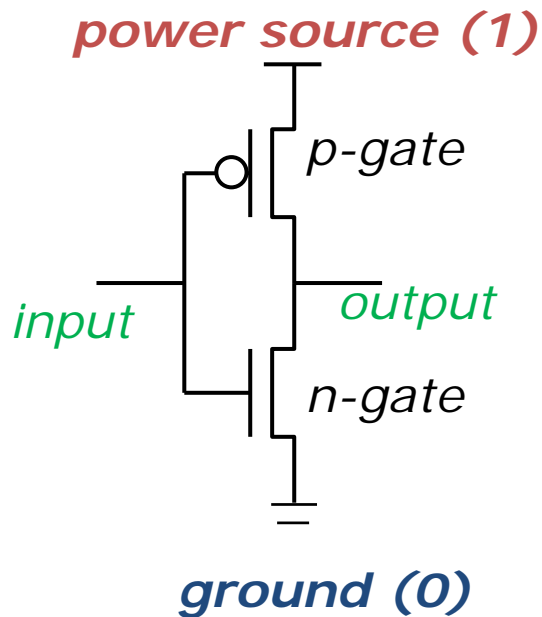
Which of the following statements is **false**?

- (A) P- and N-type transistors are both used in CMOS designs.
- (B) As transistors get smaller, the frequency of your processor will keep getting faster.
- (C) As transistors get smaller, you can fit more and more of them on a single chip.
- (D) Pure silicon is a semi conductor.
- (E) Experts believe that Moore's Law will soon end.

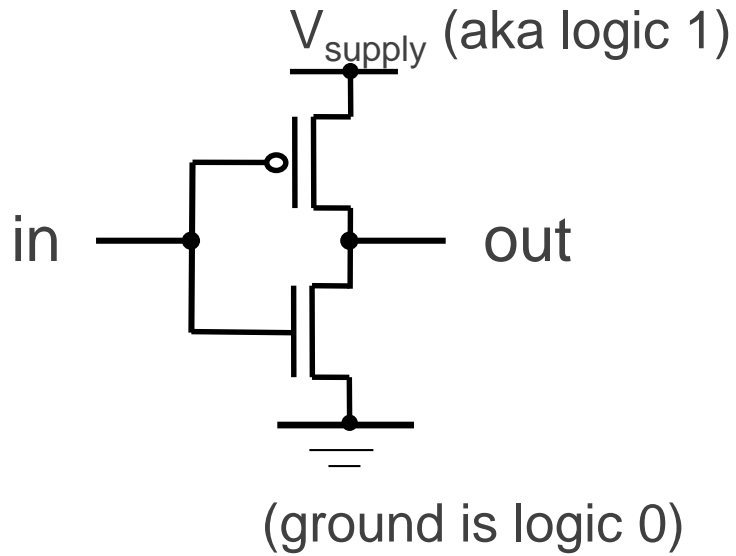
2-Transistor Combination: NOT

- Logic gates are constructed by combining transistors in complementary arrangements
- Combine p&n transistors to make a NOT gate:

CMOS Inverter :

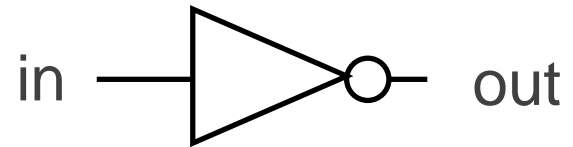


Inverter



Function: NOT

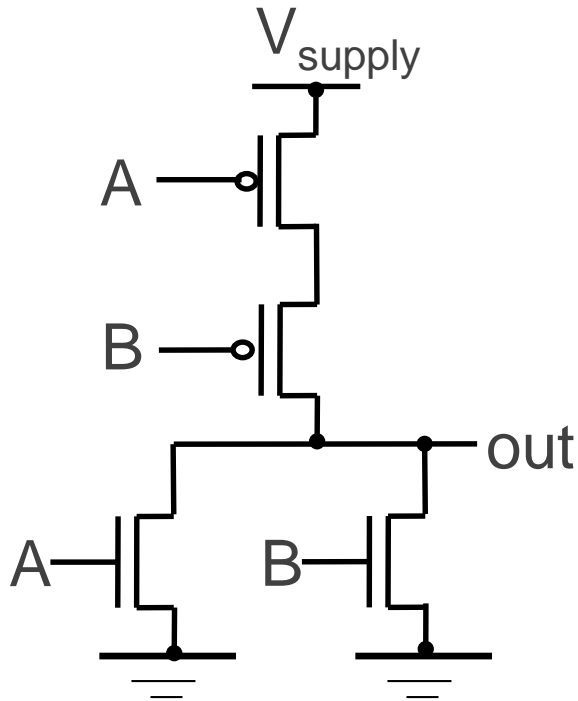
Symbol:



Truth Table:

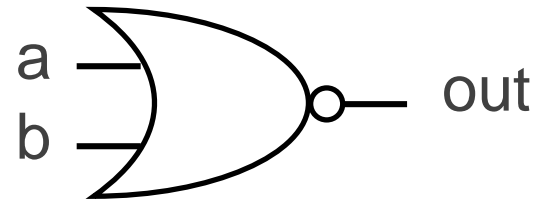
In	Out
0	1
1	0

NOR Gate



Function: NOR

Symbol:

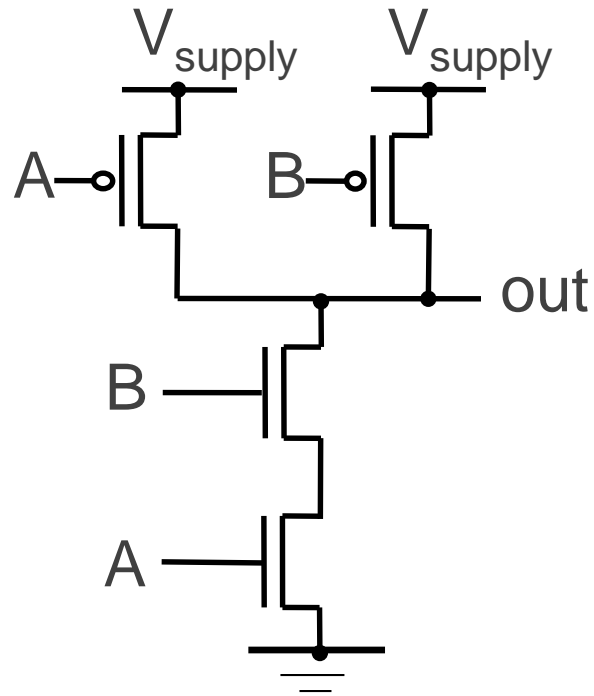


Truth Table:

A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

iClicker Question

Which Gate is this?








Function:

Symbol:

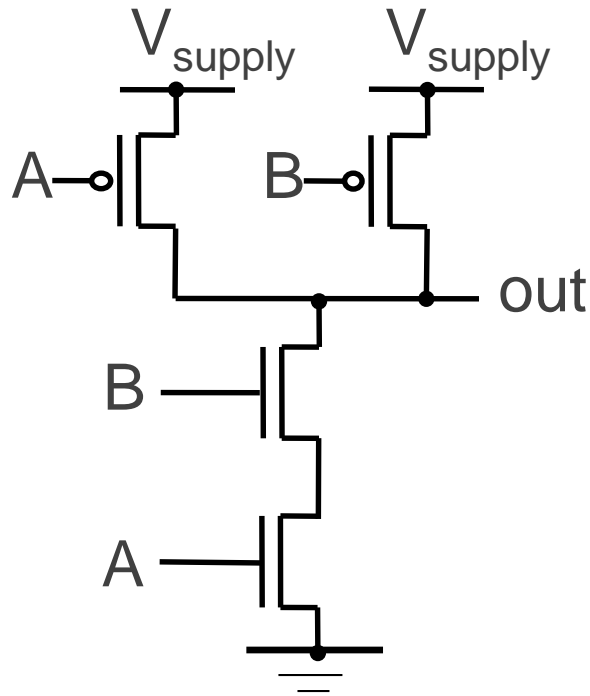
Truth Table:

A	B	out
0	0	
0	1	
1	0	
1	1	

- (A) NOT 
- (B) OR 
- (C) XOR 
- (D) AND 
- (E) NAND 

iClicker Question

Which Gate is this?



Function:

Symbol:

Truth Table:

A	B	out
0	0	1
0	1	1
1	0	1
1	1	0

(A) NOT 

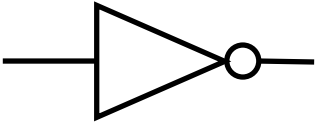
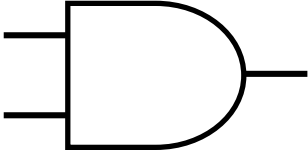
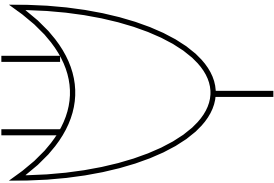
(B) OR 

(C) XOR 

(D) AND 

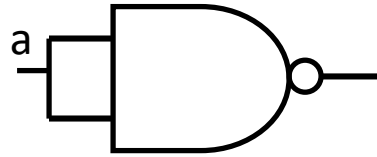
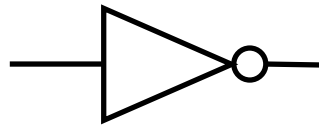
(E) NAND 

Building Functions (Revisited)

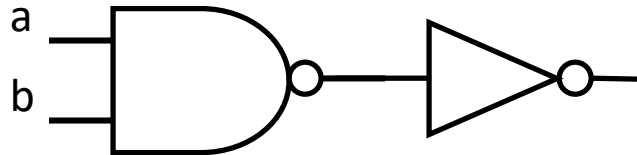
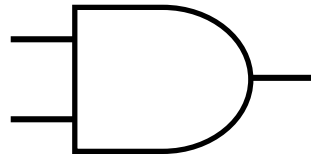
- NOT: A NOT gate symbol, consisting of a triangle pointing to the right with a small circle at its tip, and a single input line on the left and a single output line on the right.
- AND: An AND gate symbol, consisting of a D-shaped symbol with two input lines on the left and one output line on the right.
- OR: An OR gate symbol, consisting of a symbol with a curved left side and a pointed right side, with two input lines on the left and one output line on the right.
- NAND and NOR are universal
 - Can implement *any* function with NAND or just NOR gates
 - useful for manufacturing

Building Functions (Revisited)

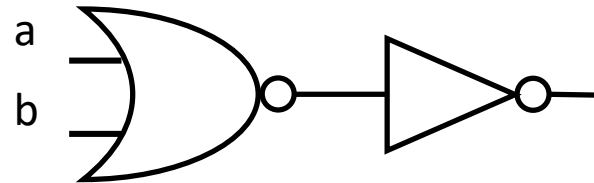
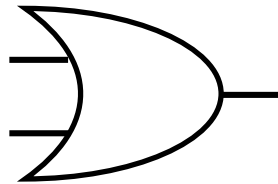
- NOT:



- AND:

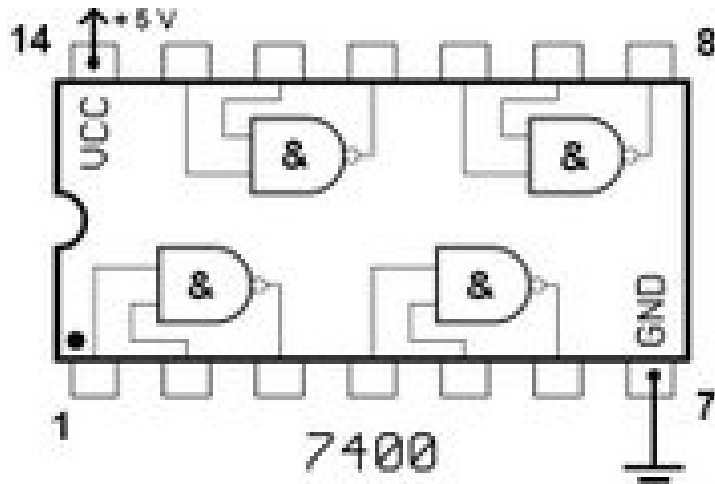


- OR:



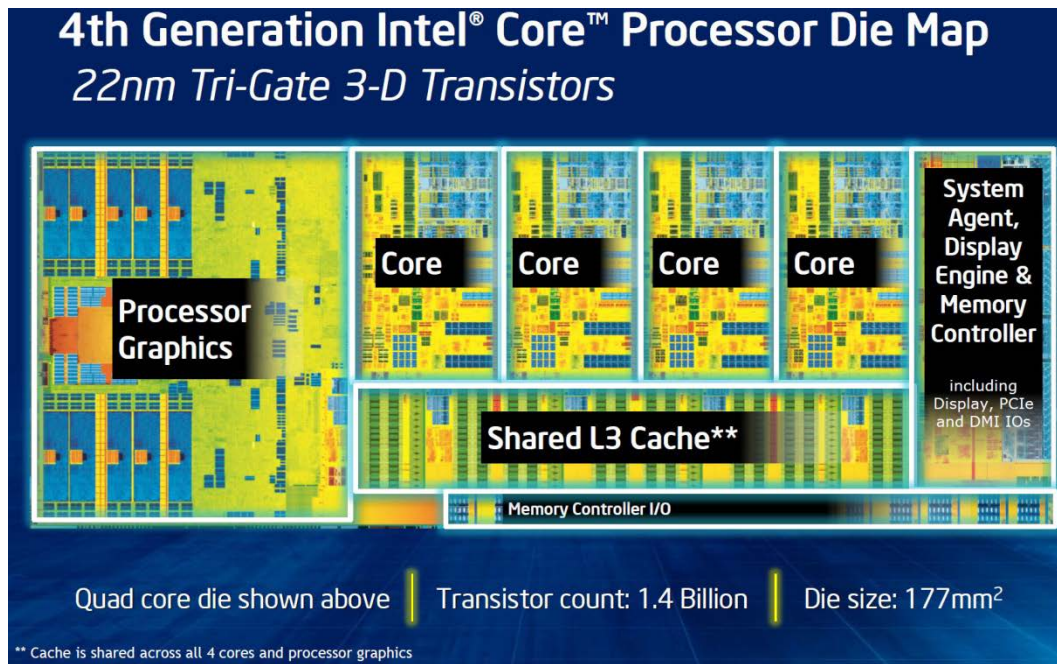
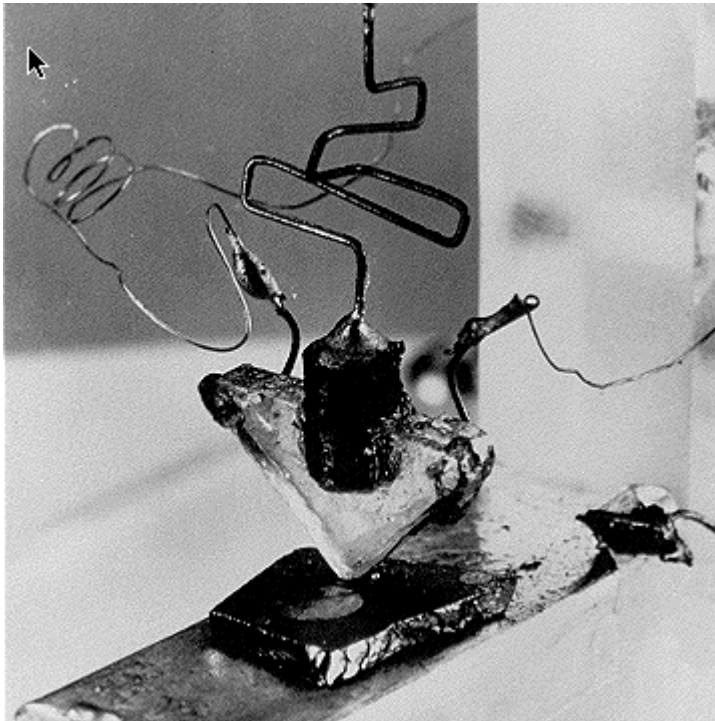
- NAND and NOR are universal
 - Can implement **any** function with NAND or just NOR gates
 - useful for manufacturing

Logic Gates



- One can buy gates separately
 - ex. 74xxx series of integrated circuits
 - cost ~\$1 per chip, mostly for packaging and testing
- Cumbersome, but possible to build devices using gates put together manually

Then and Now



<http://techguru3d.com/4th-gen-intel-haswell-processors-architecture-and-lineup/>

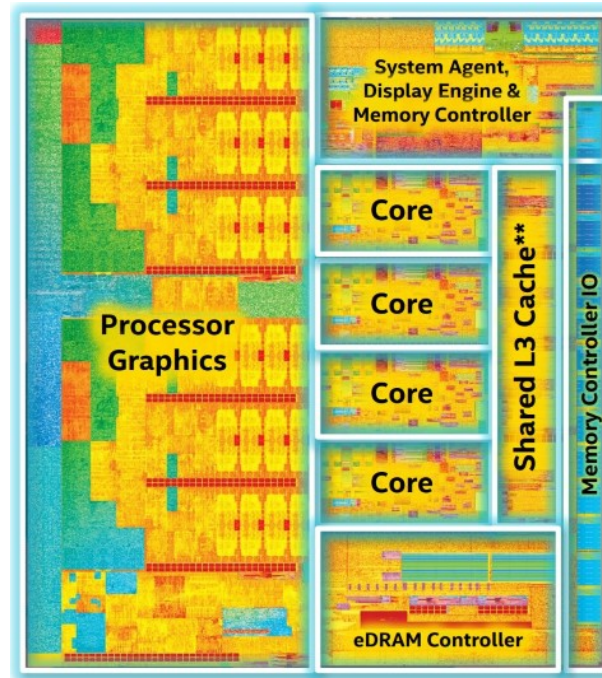
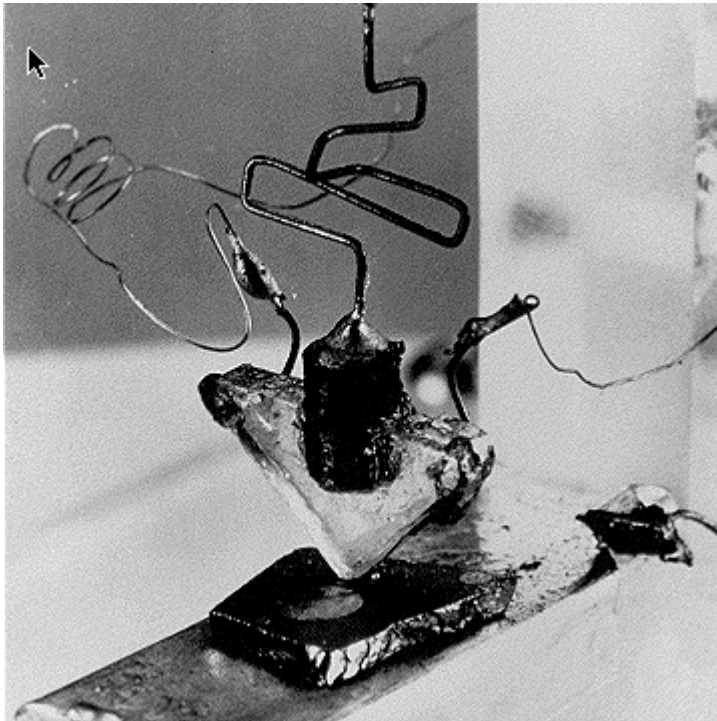
- The first transistor

- One workbench at AT&T Bell Labs
- 1947
- Bardeen, Brattain, and Shockley

- Intel Haswell

- 1.4 billion transistors, 22nm
- 177 square millimeters
- Four processing cores

Then and Now



<https://www.computershopper.com/computex-2015/performance-preview-desktop-broadwell-at-computex-2015>

- The first transistor

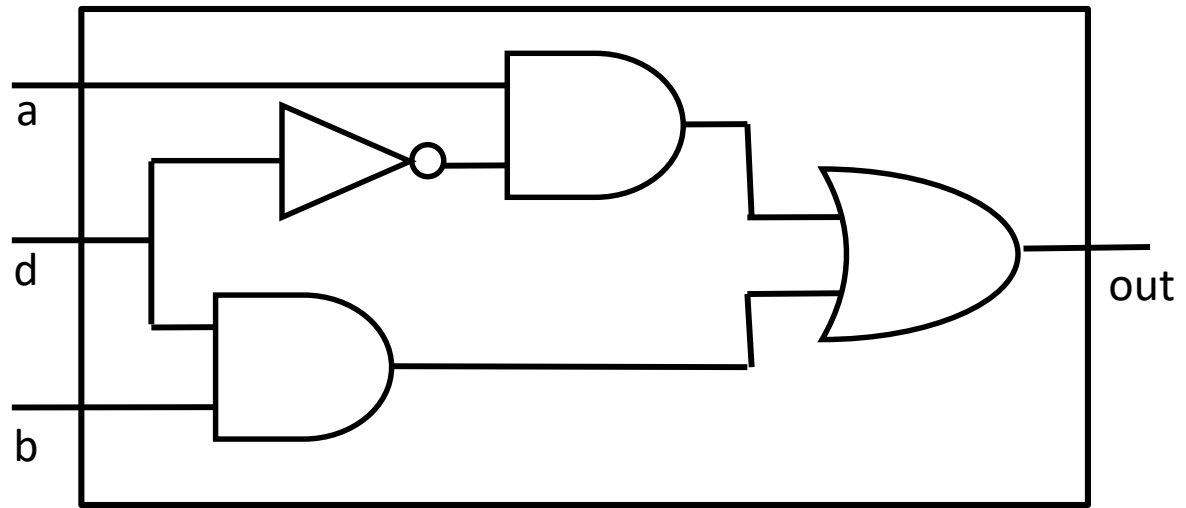
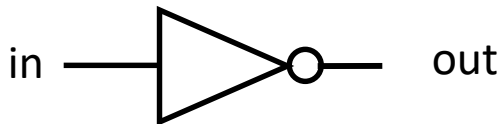
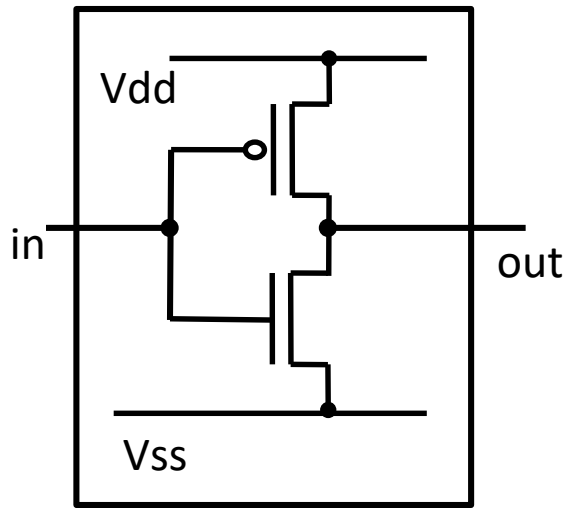
- One workbench at AT&T Bell Labs
- 1947
- Bardeen, Brattain, and Shockley

- Intel Broadwell

- 7.2 billion transistors, 14nm
- 456 square millimeters
- Up to 22 processing cores

Big Picture: Abstraction

- Hide complexity through simple abstractions
 - **Simplicity**
 - Box diagram represents inputs and outputs
 - **Complexity**
 - Hides underlying NMOS- and PMOS-transistors and atomic interactions



Summary

- Most modern devices made of billions of transistors
 - You will build a processor in this course!
 - Modern transistors made from semiconductor materials
 - Transistors used to make logic gates and logic circuits
- We can now implement any logic circuit
 - Use P- & N-transistors to implement NAND/NOR gates
 - Use NAND or NOR gates to implement the logic circuit
 - *Efficiently*: use K-maps to find required minimal terms

