Gates and Logic: From Transistors to Logic Gates and Logic Circuits

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The slides are the product of many rounds of teaching CS 3410 by Professors Weatherspoon, Bala, Bracy, and Sirer.

See: P&H Appendix B.2 and B.3 (Also, see B.1)

Goals for Today

From Switches to Logic Gates to Logic Circuits

Transistors (electronic switch)

Logic Gates

- Truth Tables

Logic Circuits

- Identity Laws
- From Truth Tables to Circuits (Sum of Products)

Logic Circuit Minimization

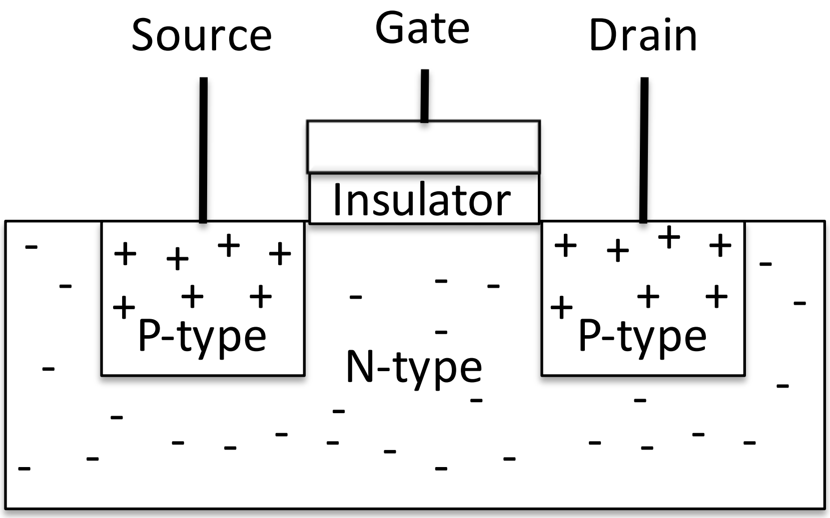
- Algebraic Manipulations
- Karnaugh Maps

Silicon Valley & the Semiconductor Industry

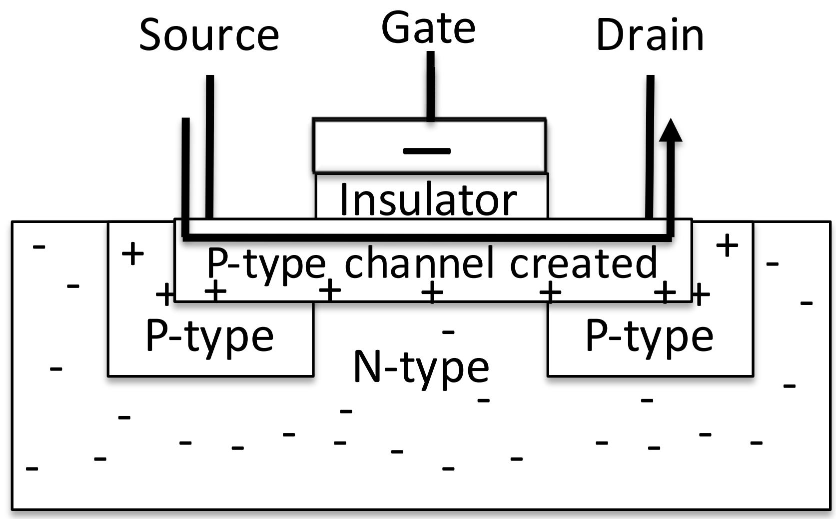
Transistors:

- Youtube video from last week
  https://www.youtube.com/watch?v=IcrBqCFLHIY

Transistors 101



Off



On

N-Type Silicon: negative free-carriers (electrons)

P-Type Silicon: positive free-carriers (holes)

P-Transistor: negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain

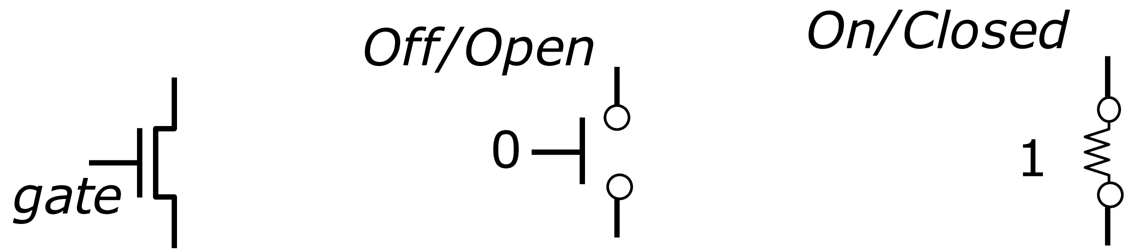
N-Transistor: works the opposite way

Metal-Oxide Semiconductor (Gate-Insulator-Silicon)

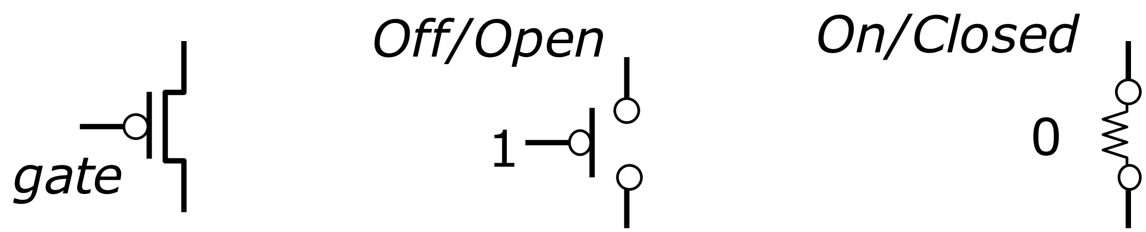
Complementary MOS = **CMOS** technology uses both p- & n-type transistors

CMOS Notation

N-type



P-type



Gate input controls whether current can flow between the other two terminals or not.

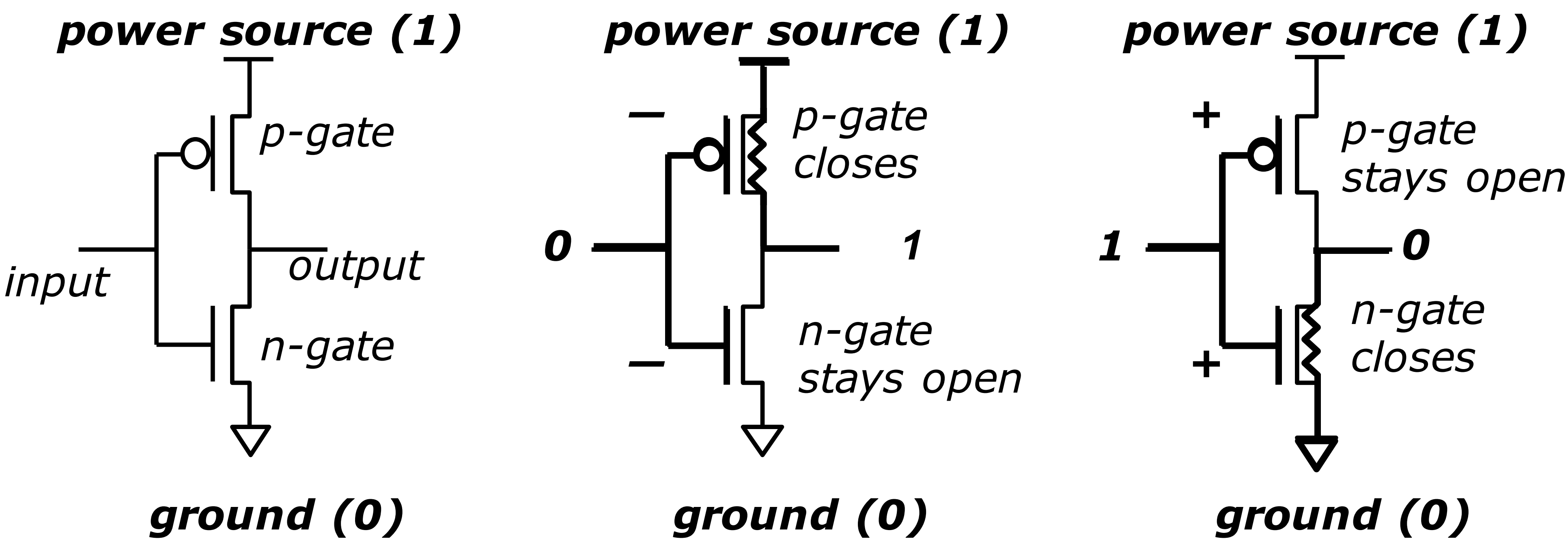
Hint: the "o" bubble of the p-type tells you that this gate wants a 0 to be turned on

2-Transistor Combination: NOT

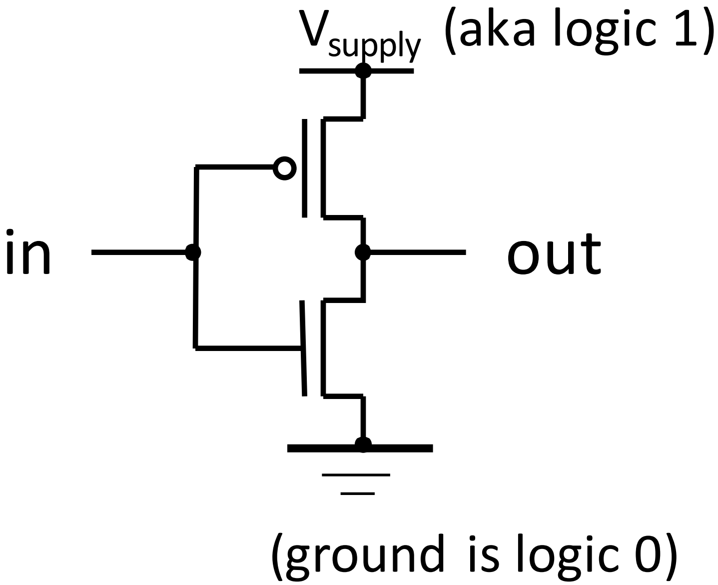
Logic gates are constructed by combining transistors in complementary arrangements

Combine p&n transistors to make a NOT gate:

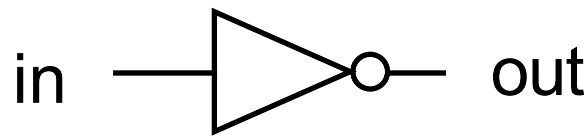
CMOS Inverter :



Inverter



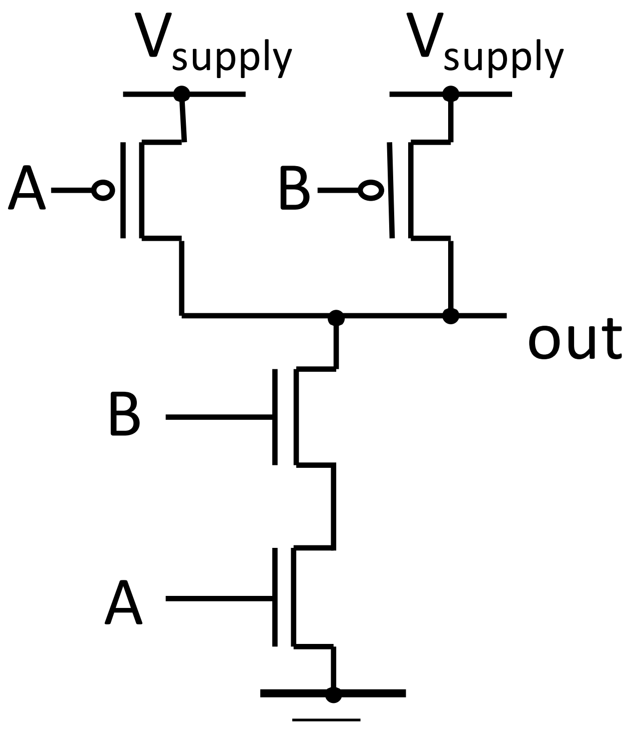
- Function: NOT
- Symbol:



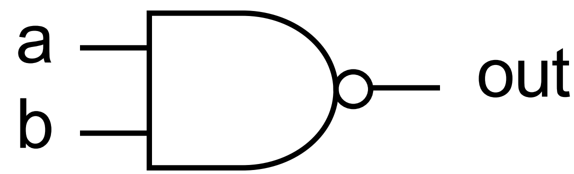
- Truth Table:

In	Out
0	1
1	0

Activity #1: Which Gate is this?



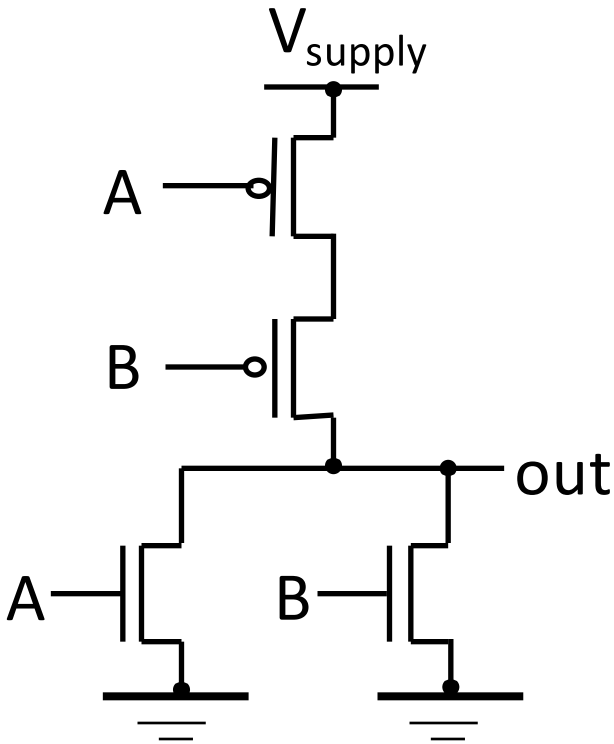
- Function:
- Symbol:



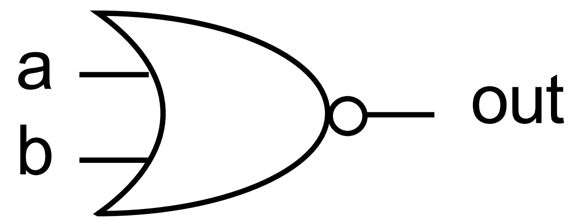
- Truth Table:

A	B	out
0	0	
1	0	
0	1	
1	1	

NOR Gate



- Function: NOR
- Symbol:



- Truth Table:

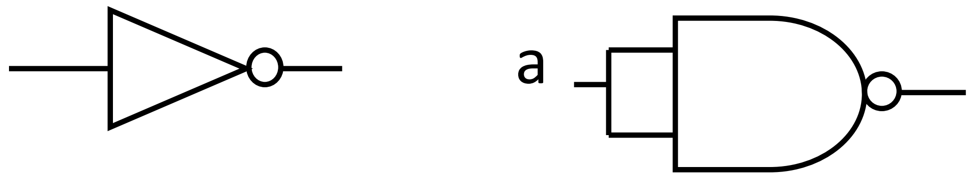
A	B	out
0	0	1
1	0	0
0	1	0
1	1	0

Building Functions

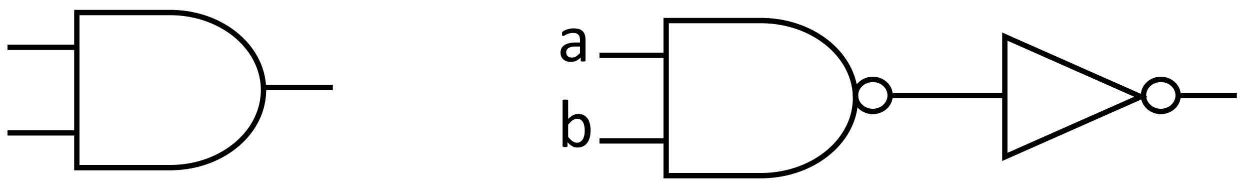
NAND and NOR are universal

- Can implement ***any*** function with NAND or just NOR gates
- useful for manufacturing

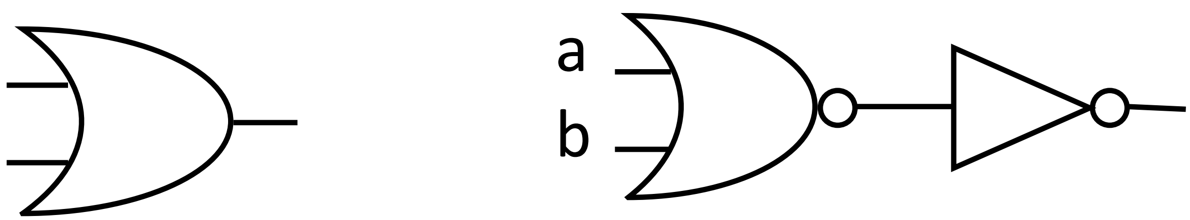
NOT:



AND:



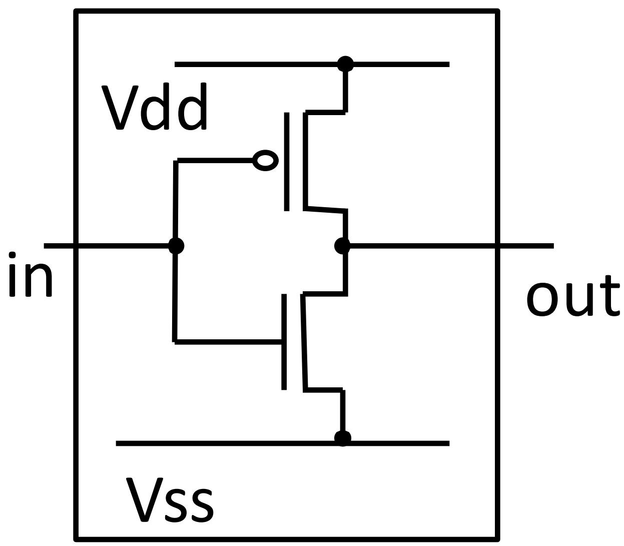
OR:

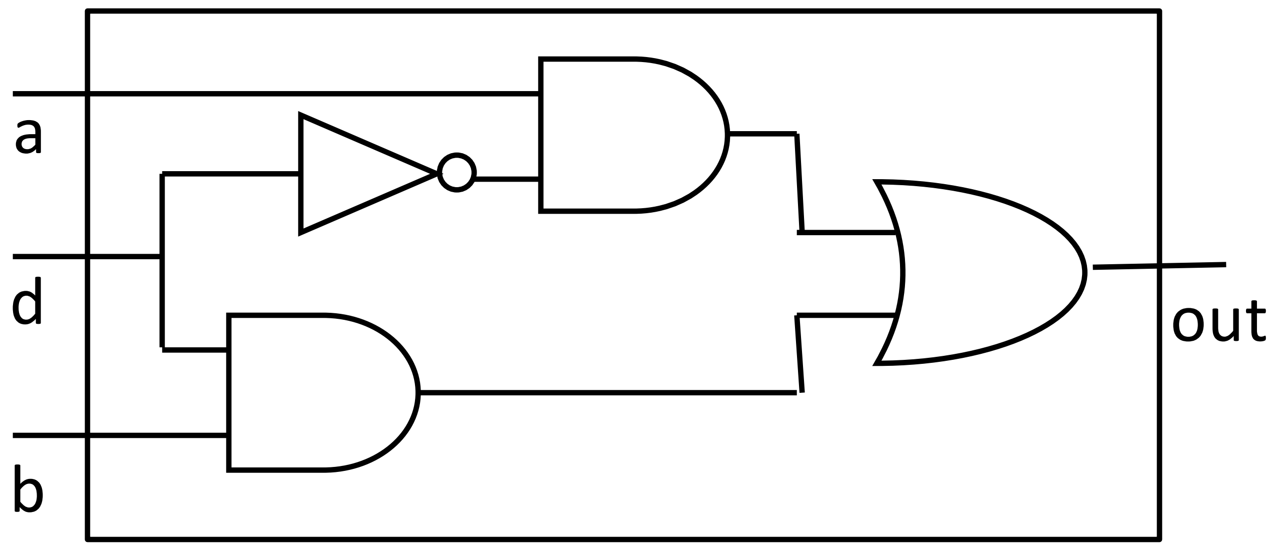


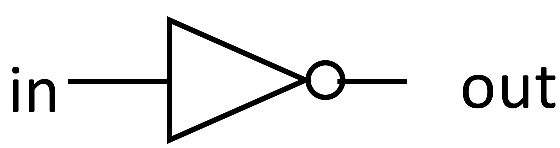
Big Picture: Abstraction

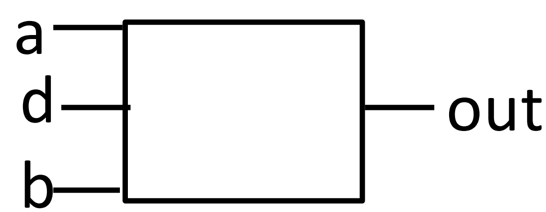
Hide complexity through simple abstractions

- Simplicity
  - Box diagram represents inputs and outputs
- Complexity
  - Hides underlying NMOS- and PMOS-transistors and atomic interactions









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- From Truth Tables to Circuits (Sum of Products)

Logic Circuit Minimization

- Algebraic Manipulations
- Karnaugh Maps

Logic Gates

- Digital circuit that either allows signal to pass through it or not
- Used to build logic functions
- Seven basic logic gates:

  AND,

  OR,

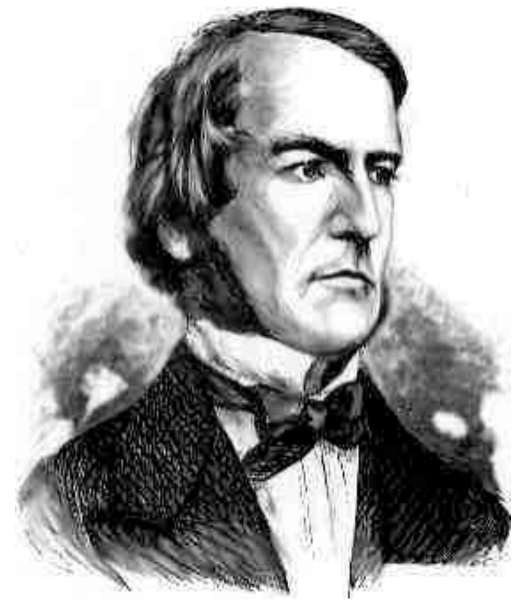
  NOT,

  NAND (not AND),

  NOR (not OR),

  XOR

  XNOR (not XOR)



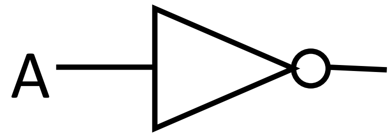
George Boole,(1815-1864)

Did you know?

George Boole Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.

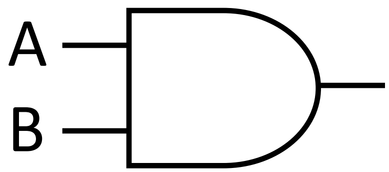
Logic Gates: Names, Symbols, Truth Tables

NOT:



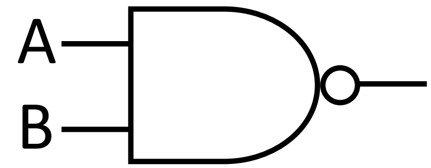
A	Out
0	1
1	0

AND:



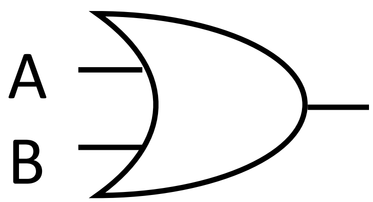
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

NAND:



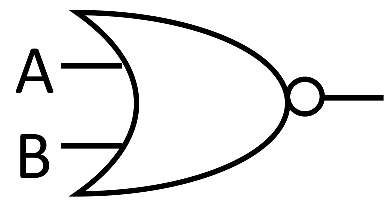
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

OR:



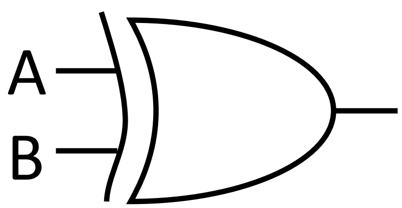
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

NOR:



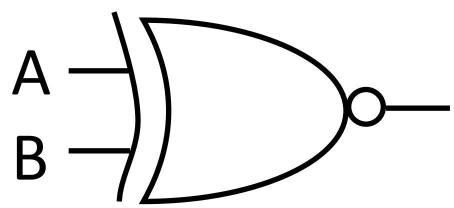
A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

XOR:.



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

XNOR:



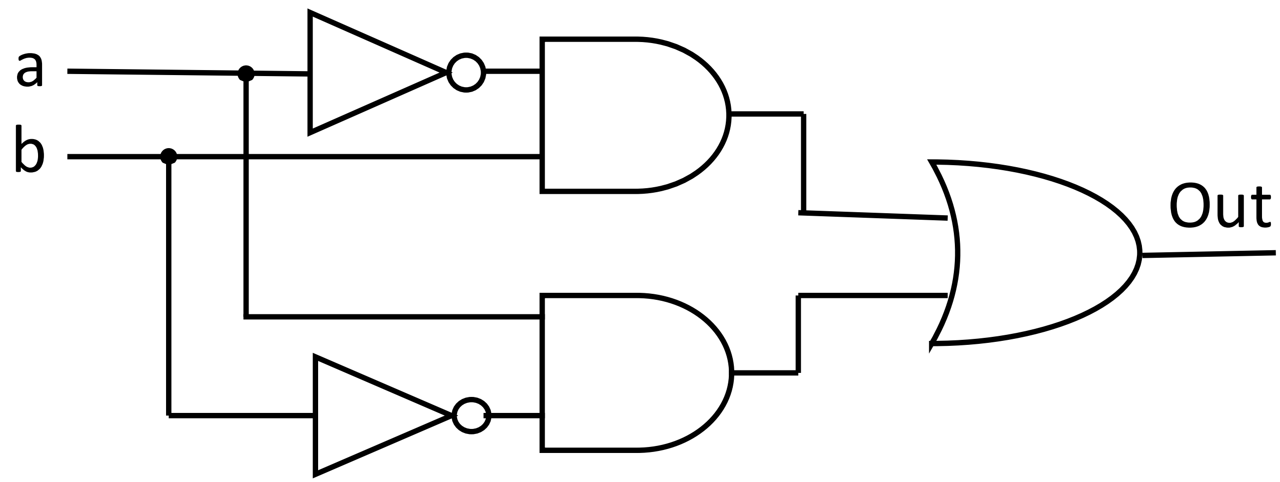
A	B	Out
0	0	1
0	1	0
1	0	0
1	1	1

Activity #2: Logic Gates

Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.

What does the logic circuit do?

a	**b**	**Out**

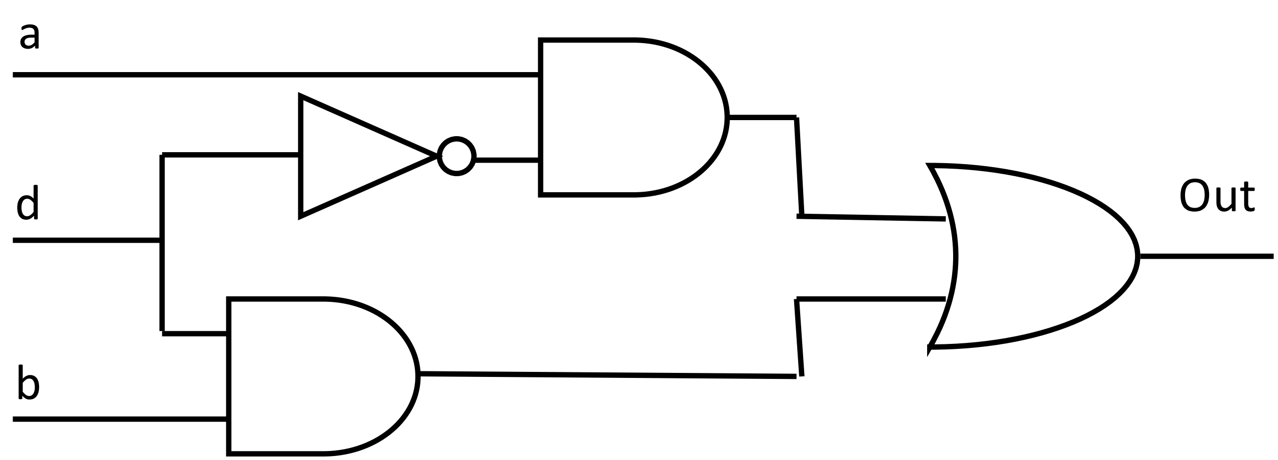


Activity #3: Logic Gates

Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.

What does the logic circuit do?

a	b	d	Out
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



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Next Goal

Given a Logic function → create a Logic Circuit that implements the Logic Function…

Logic Implementation

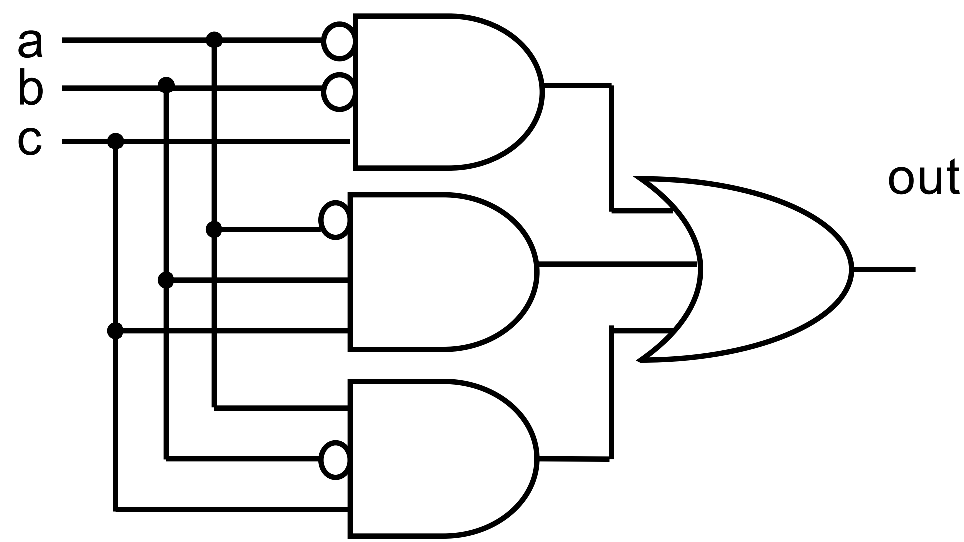
How to implement a desired logic function?

a	b	c	out	minterm
0	**0**	**0**	**0**	$\overline{a}\,\overline{b}\,\overline{c}$
0	**0**	**1**	**1**	$\overline{a}\,\overline{b}\,c$
0	**1**	**0**	**0**	$\overline{a}\,b\,\overline{c}$
0	**1**	**1**	**1**	$\overline{a}\,b\,c$
1	**0**	**0**	**0**	$a\,\overline{b}\,\overline{c}$
1	**0**	**1**	**1**	$a\,\overline{b}\,c$
1	**1**	**0**	**0**	$a\,b\,\overline{c}$
1	**1**	**1**	**0**	$a\,b\,c$

1) Write minterms
2) Write sum of products:

   OR of all minterms where out=1

$$out = \overline{a}\overline{b}c + \overline{a}bc + a\overline{b}c$$



Any combinational circuit *can be* implemented in two levels of logic (ignoring inverters)

Logic Equations

NOT: $= \bar{a}$ $= !a$ $= \neg a$

AND: $= a \cdot b$ $= a \,\&\, b$ $= a \wedge b$

OR: $= a + b$ $= a \mid b$ $= a \vee b$

XOR: $= a \oplus b = a\bar{b} + \bar{a}b$

NAND: $\overline{(a \bullet b)} = !(a \,\&\, b) = \neg (a \wedge b)$

NOR: $\overline{(a + b)} = !(a \mid b) = \neg (a \vee b)$

XNOR: $\overline{(a \oplus b)} = \overline{a\bar{b} + \bar{a}b}$

Logic Equations

- Constants: true = 1, false = 0
- Variables: a, b, out, …
- Operators (above): AND, OR, NOT, etc.

Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$

Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

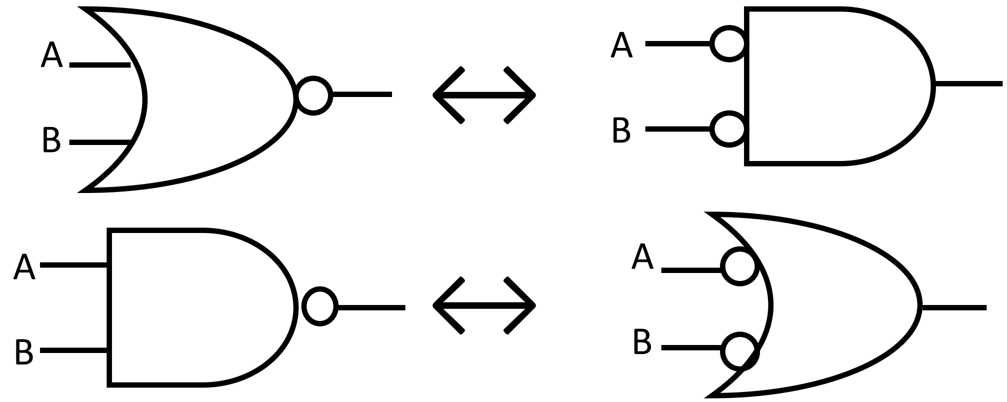
$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

a + a b = a

a (b+c) = ab + ac

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$



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Next Goal

Implement the Logic Function…

with the minimum number of logic gates

Fewer gates: A cheaper ($$$) circuit!

How to standardize this minimizing?

Activity #4: Logic Minimization

$a + 0 = a$

$a + 1 = 1$

$a + \bar{a} = 1$

$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot \bar{a} = 0$

$a + a\,b = a$

$a\,(b+c) = ab + ac$

$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$

Minimize this logic equation:

$(a+b)(a+c) =$

Checking Equality w/Truth Tables

circuits ↔ truth tables ↔equations

- Example: (a+b)(a+c) = a + bc

a	b	c					
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Minimization in Practice

How does one find the most efficient equation?

– Manipulate algebraically until…?
– Use Karnaugh Maps (optimize visually)
– Use a software optimizer

For large circuits

– Decomposition & reuse of building blocks

Building a Karnaugh Map

a	b	c	out
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Sum of minterms yields

out =

$$\overline{a}\overline{b}c + \overline{a}bc + a\overline{b}\overline{c} + a\overline{b}c$$

c \ ab	00	01	11	10
0	0	0	0	1
1	1	1	0	1

K-maps identify which inputs are relevant to the output

Minimization with K-Maps

c \ ab	00	01	11	10
0	0	0	0	1
1	1	1	0	1

(1) Circle the 1's (see below)

(2) Each circle is a logical component of the final equation

$$= a\bar{b} + \bar{a}c$$

Rules:

- Use fewest circles necessary to cover all 1's
- Circles must cover *only* 1's
- Circles span rectangles of size power of 2 (1, 2, 4, 8…)
- Circles should be as large as possible (all circles of 1?)
- Circles may wrap around edges of K-Map
- 1 may be circled multiple times *if* that means fewer circles

Karnaugh Minimization Tricks (1)

c \ ab	00	01	11	10
0	0	1	1	1
1	0	0	1	0

Minterms can overlap

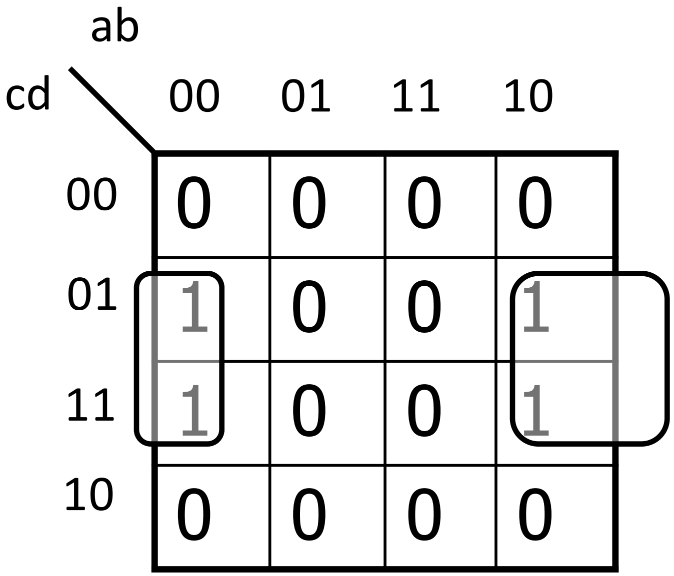
$$out = b\overline{c} + a\overline{c} + ab$$

c \ ab	00	01	11	10
0	1	1	1	1
1	0	0	1	0

Minterms can span 2, 4, 8 or more cells

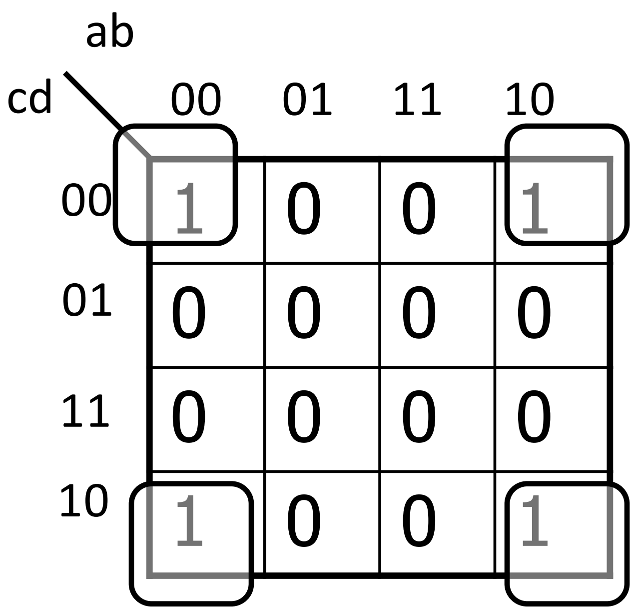
$$out = \overline{c} + ab$$

Karnaugh Minimization Tricks (2)



The map wraps around

$\text{out} = \bar{b}d$



$\text{out} = \bar{b}\,\bar{d}$

Don’t Cares

cd \ ab	00	01	11	10
00	0	0	0	0
01	1	x	x	x
11	1	x	x	1
10	0	0	0	0

“Don’t care” values can be interpreted individually in whatever way is convenient

- assume all x’s = 1

→ out = d

cd \ ab	00	01	11	10
00	1	0	0	x
01	0	x	x	0
11	0	x	x	0
10	1	0	0	1

- assume middle x’s = 0 (ignore them)
- assume 4^{th} column x = 1

→ out = $\overline{b}\ \overline{d}$

Takeaway

- Binary —two symbols: true and false—is the basis of Logic Design

- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.

- Any logic function can be implemented as “sum of products”. Karnaugh Maps minimize number of gates.

Summary

Most modern devices made of billions of transistors

- You will build a processor in this course!
- Modern transistors made from semiconductor materials
- Transistors used to make logic gates and logic circuits

We can now implement any logic circuit

- Use P- & N-transistors to implement NAND/NOR gates
- Use NAND or NOR gates to implement the logic circuit
- *Efficiently*: use K-maps to find required minimal terms