Circuits & Numbers

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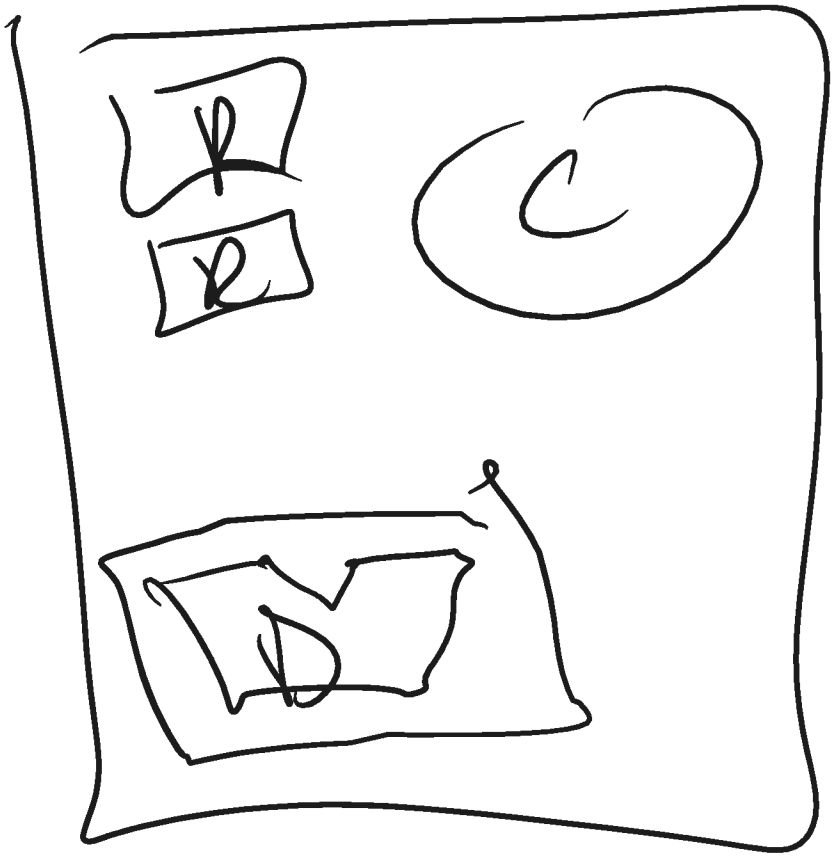
CS 3410, Spring 2010
Computer Science
Cornell University

See: P&H Chapter 2.4, 2.5, 3.2, C.5

Office Hours

HW1

CSUGLab

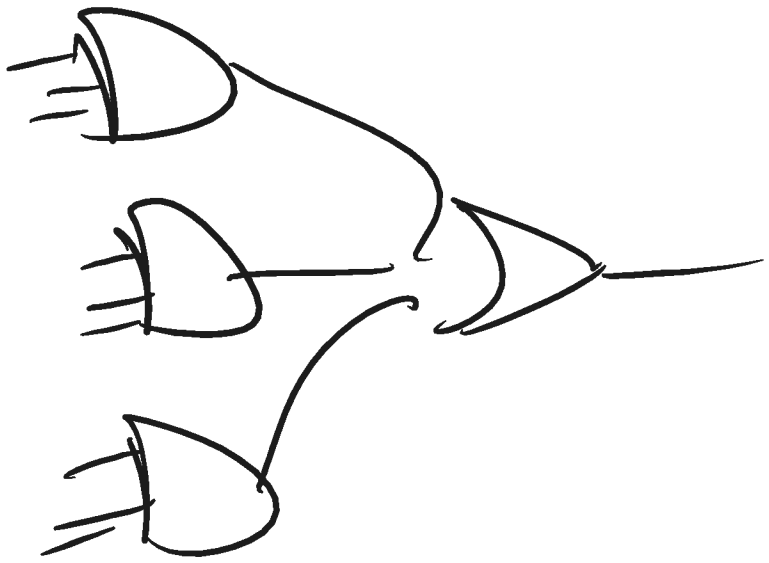


How to implement a desired function?

a	b	c	out	minterm
0	0	0	0	$\bar{a}\,\bar{b}\,\bar{c}$
0	0	1	1	$\bar{a}\,\bar{b}\,c$
0	1	0	0	$\bar{a}\,b\,\bar{c}$
0	1	1	1	$\bar{a}\,b\,c$
1	0	0	0	$a\,\bar{b}\,\bar{c}$
1	0	1	1	$a\,\bar{b}\,c$
1	1	0	0	$a\,b\,\bar{c}$
1	1	1	0	$a\,b\,c$

sum of products:

- OR of all minterms where out=1



corollary: *any* combinational circuit *can be* implemented in two levels of logic (ignoring inverters)

How does one find the most efficient equation?

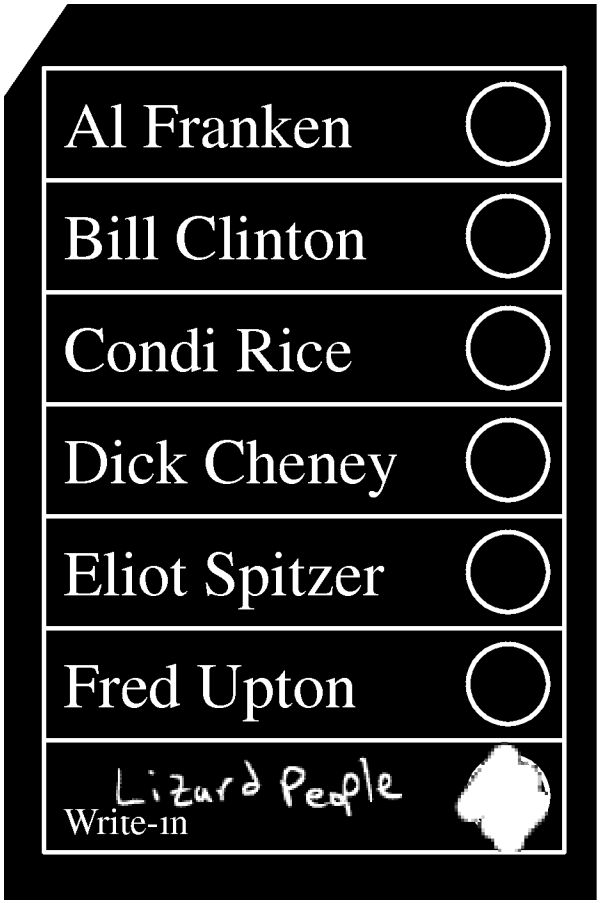
- Manipulate algebraically until…?
- Use Karnaugh maps (optimize visually)
- Use a software optimizer

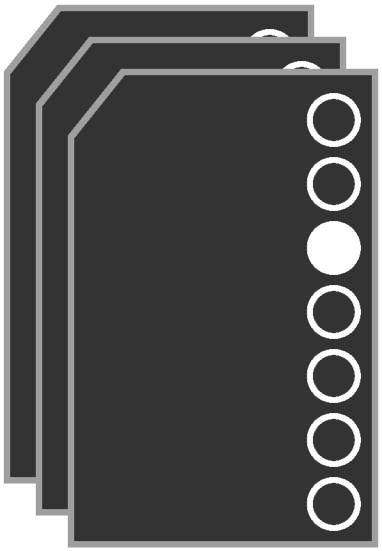
For large circuits

- Decomposition & reuse of building blocks

- Voting Machine!
  - optical scan (thanks FL)

- Assume:
  - vote is recorded on paper by filling a circle
  - fixed number of choices
  - don't worry about "invalids"





Ballots

The 3410 optical scan vote ~~counter~~ reader machine

5 Essential Components?

- Input: paper with at exactly one mark
- Datapath: process current ballot
- Output: a number the supervisor can record
- Memory & control: none for now

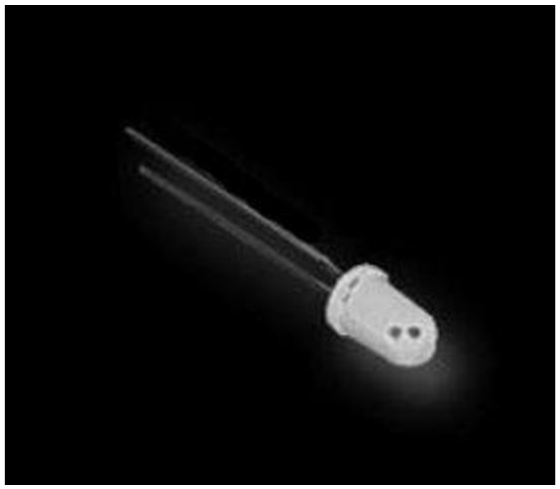
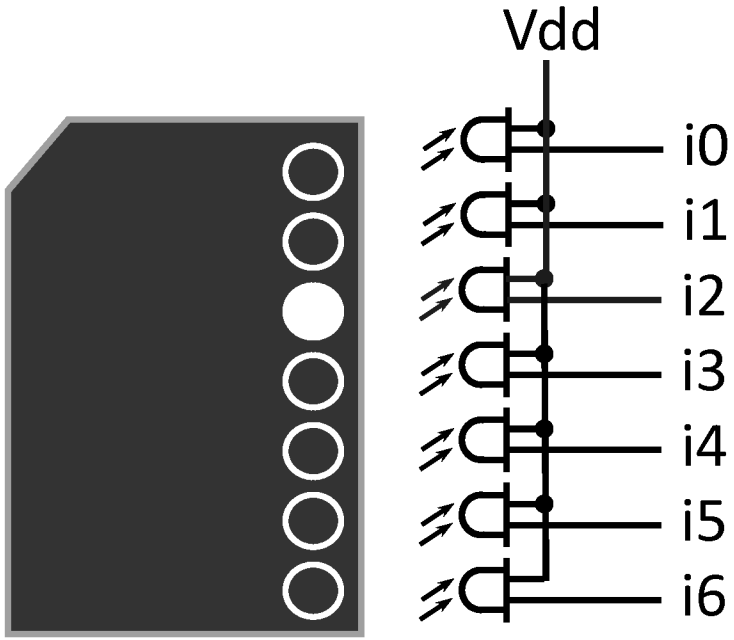


Photo-sensitive transistor

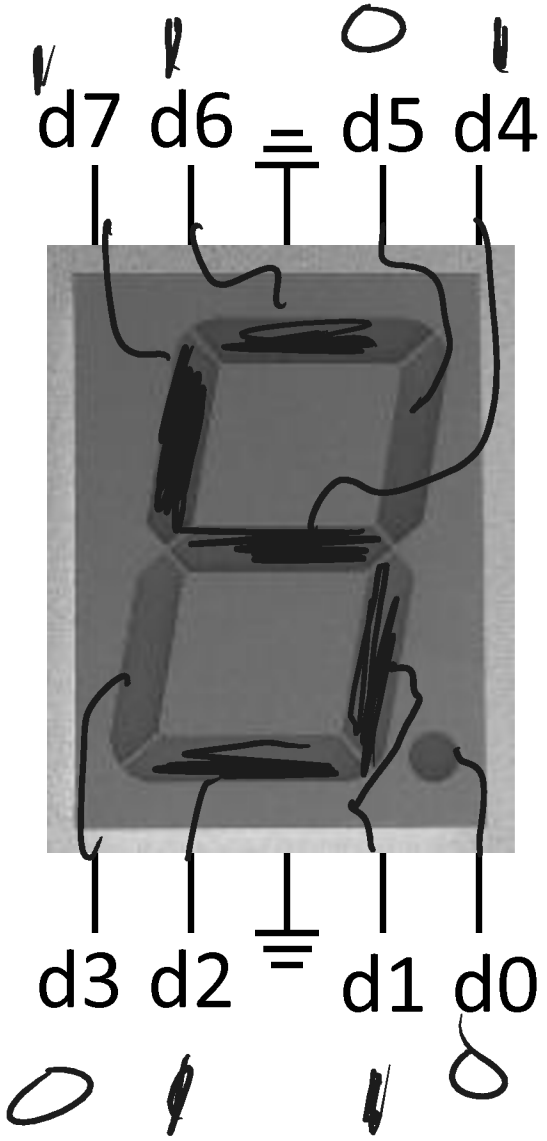
- photons replenish gate depletion region
- can distinguish dark and light spots on paper

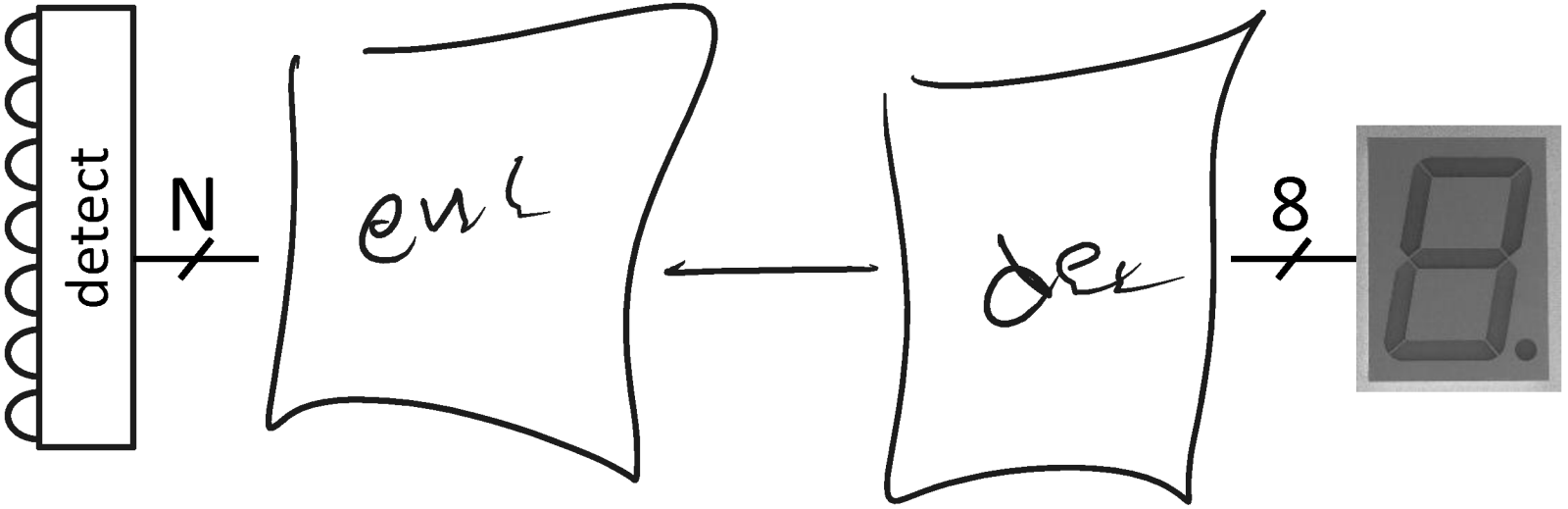


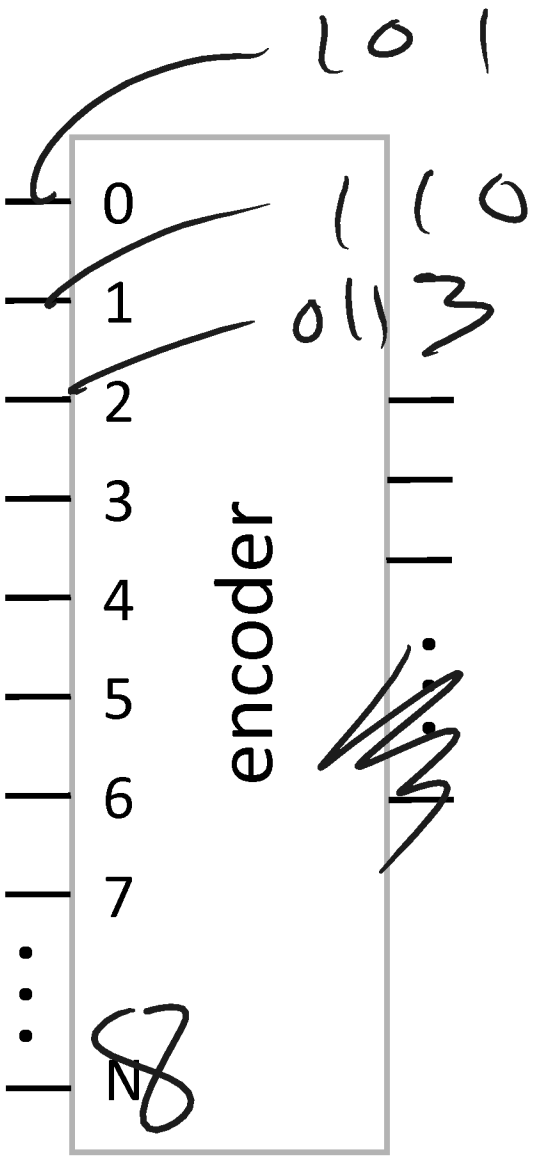
- Use array of N sensors for voting machine input

7-Segment LED

- photons emitted when electrons fall into holes




detect
N
enc
dec
8



N might be large

- Routing wires is expensive

More efficient encoding?

$\log_2(N)$

- Base 10 - Decimal

$$\underline{6}\ \underline{3}\ \underline{7}$$

$$10^2\ 10^1\ 10^0$$

- Just as easily use other bases
  - Base 2 - Binary
  - Base 8 - Octal
  - Base 16 - Hexadecimal

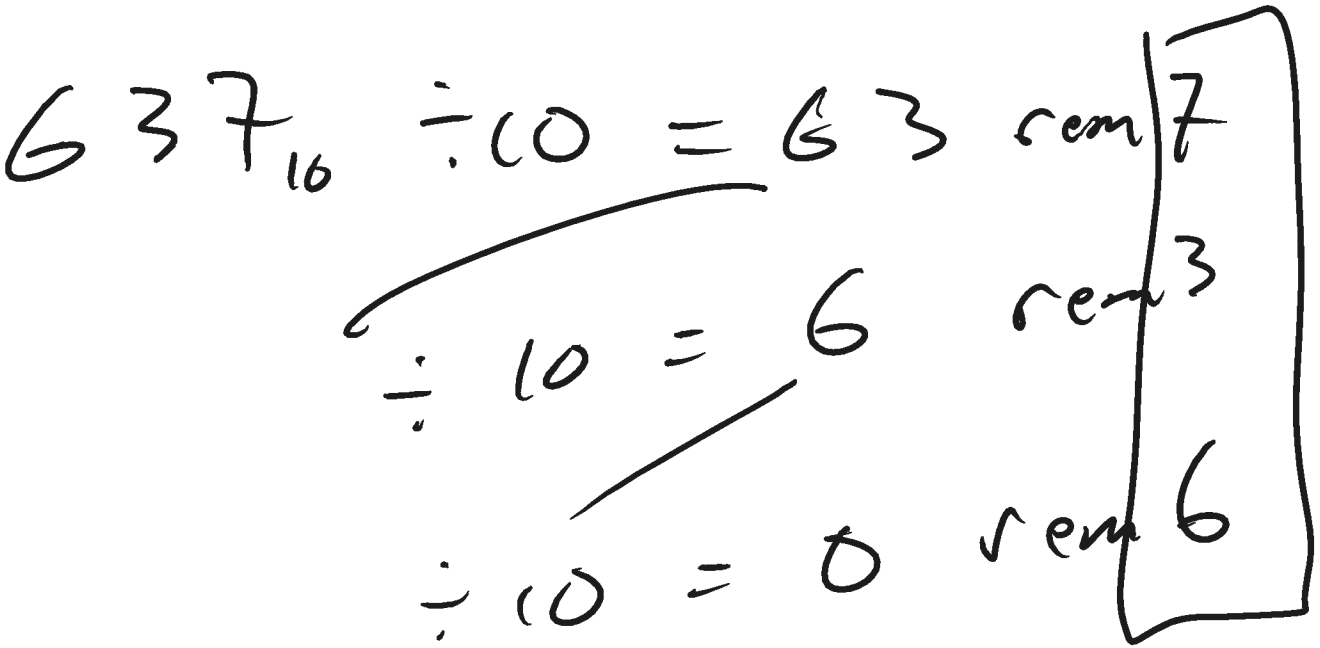
$$10110$$

$$= 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$$

Counting

	Octal
0	0
1	1
2	3
3	4
⋮	⋮
9	7
10	10
⋮	⋮
99	77
100	100
⋮	⋮

- Base conversion via repetitive division
  - Divide by base, write remainder, move left with quotient



- Base conversion via repetitive division
  - Divide by base, write remainder, move left with quotient

637 ÷ 2 = 318 rem 1
÷ 2 = 159 rem 0
÷ 2 = 79 rem 1
÷ 2 = 39 rem 1
÷ 2 = 19 rem 1
÷ 2 = 9 rem 1
÷ 2 = 4 rem 1
÷ 2 = 2 rem 0
2 rem 0
0 rem 1

1001111101_2

$27d_{16}$

- Base conversion via repetitive division
  - Divide by base, write remainder, move left with quotient

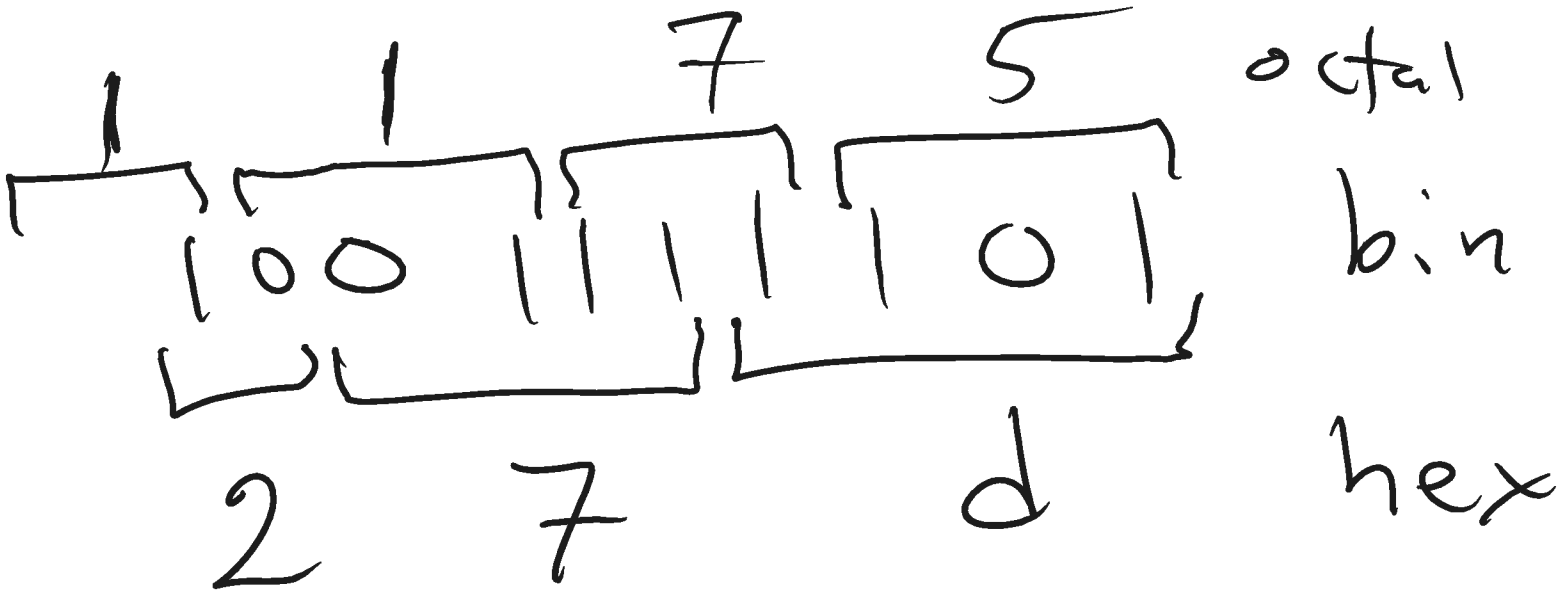
$637 \div 16 = 39$ rem 13

$\div 16 = 2$ rem 7

$\div 16 = 0$ rem 2

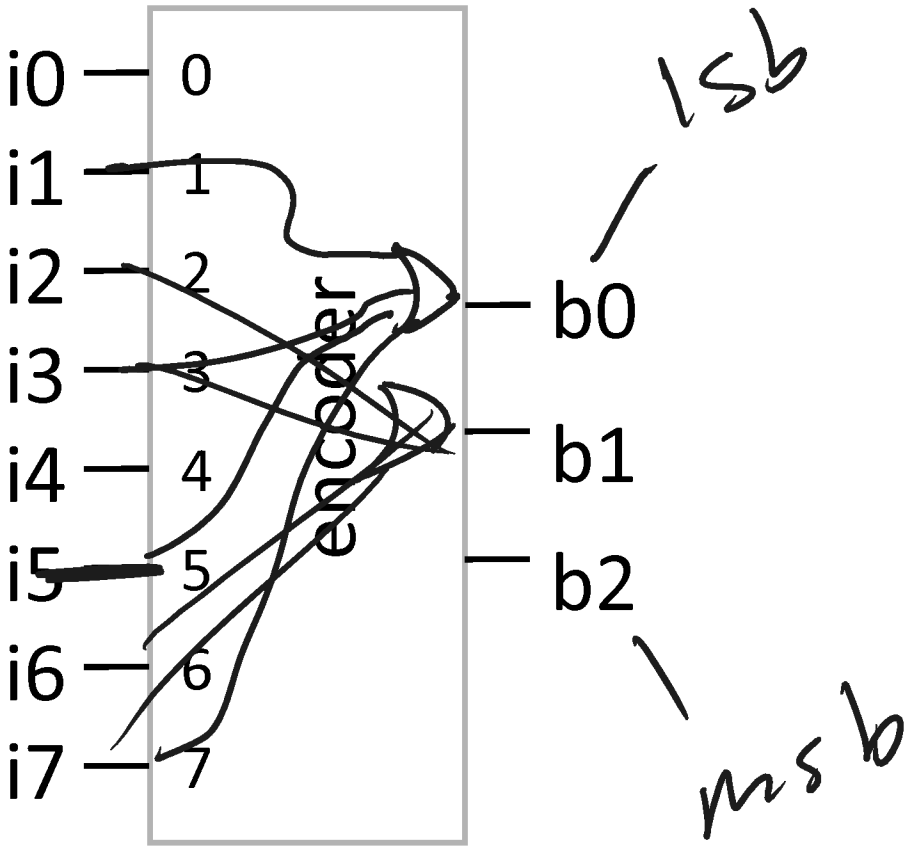
$637_{10} = 27d_{16}$

$= 0x27d$


1 1 7 5 octal
1 0 0 1 1 1 1 1 0 1 bin
2 7 d hex

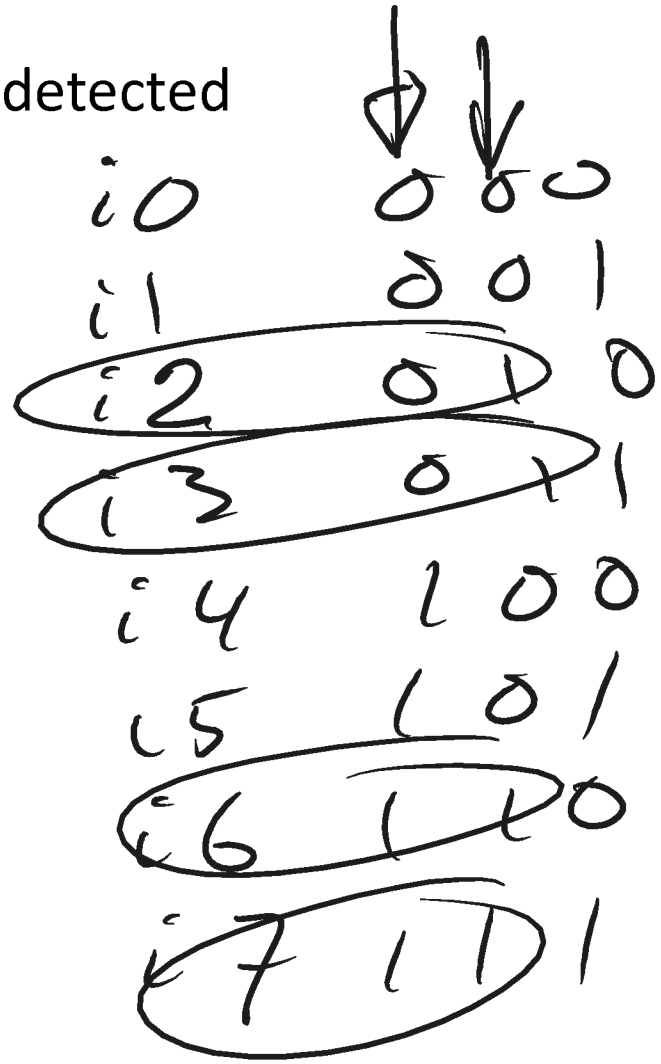
Implementation . . .

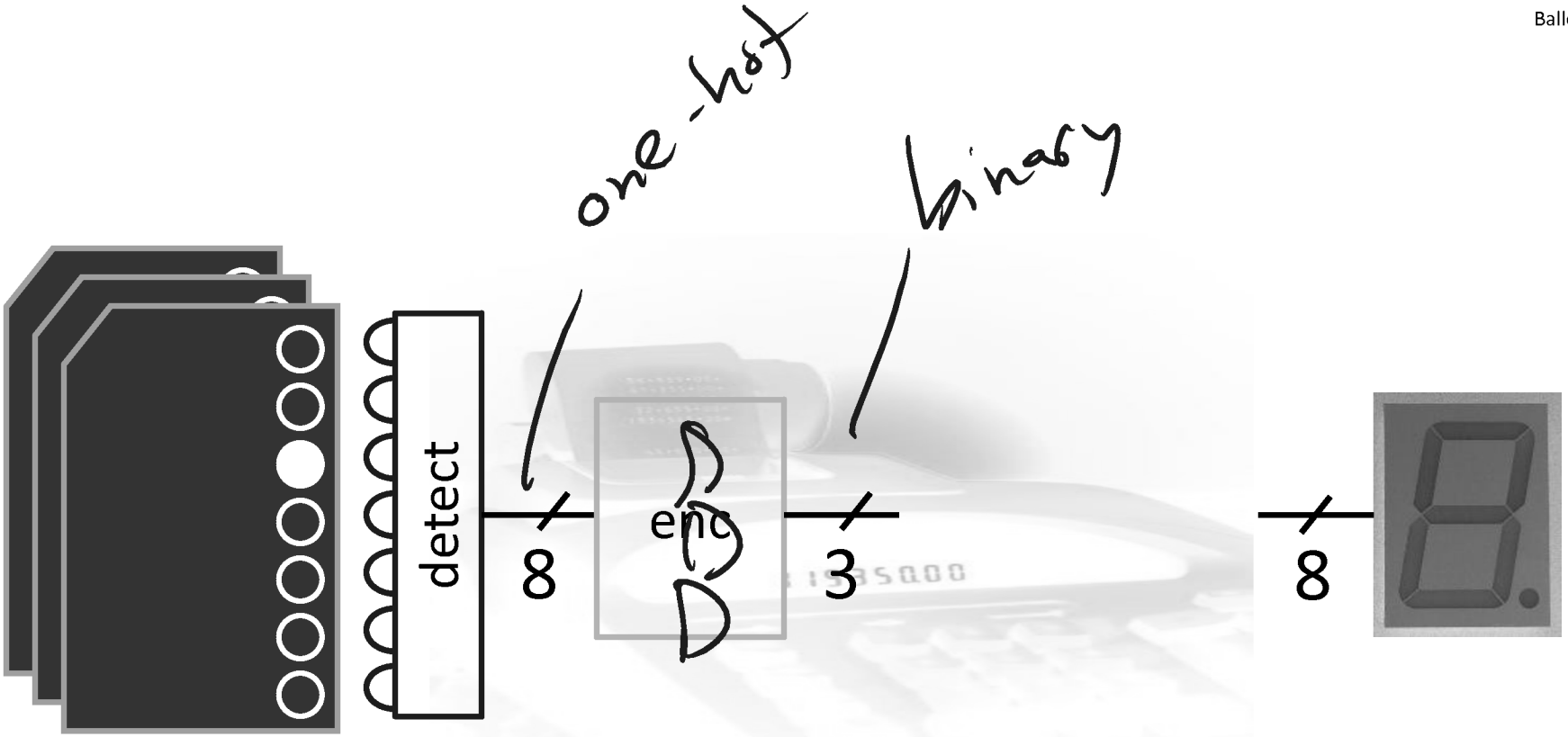
– assume 8 choices, exactly one mark detected

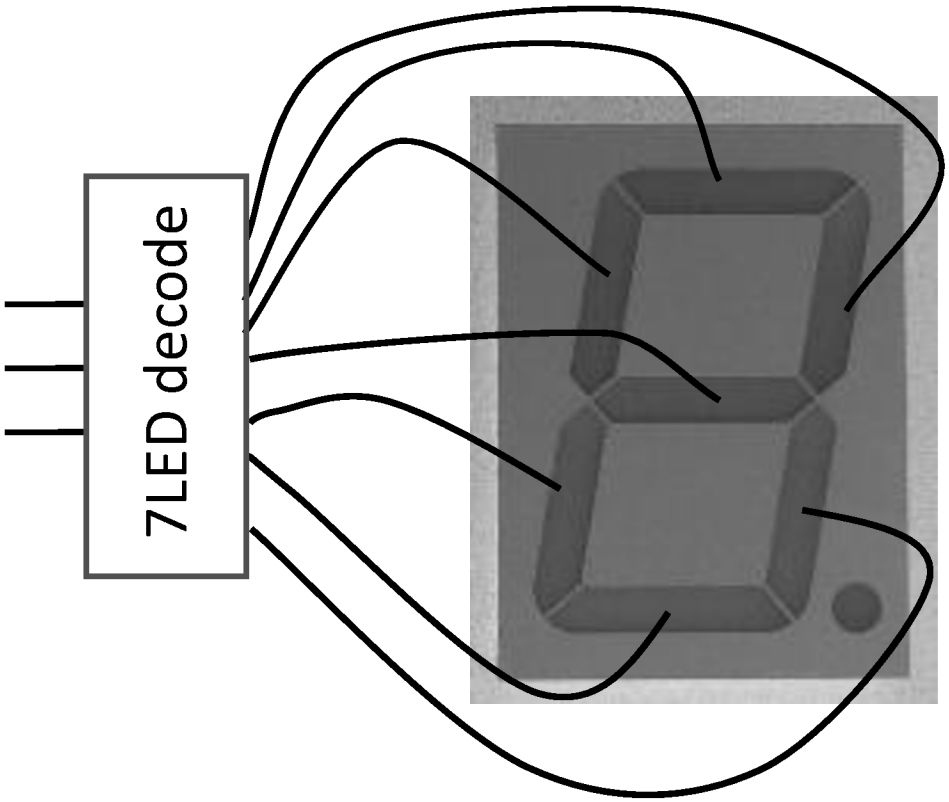


3-bit encoder

(8-to-3)




one-hot
binary
detect
8
enc
3
8



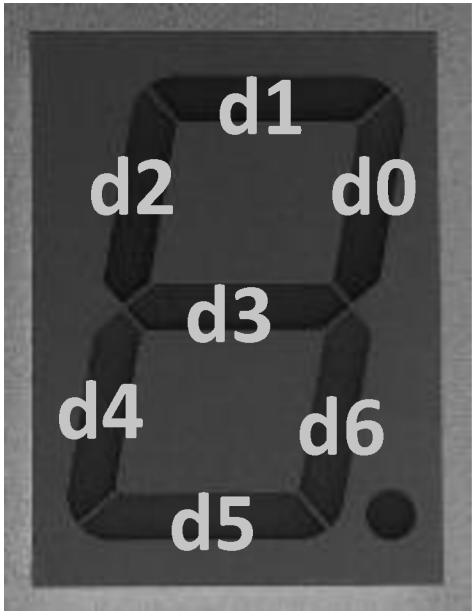
3 inputs

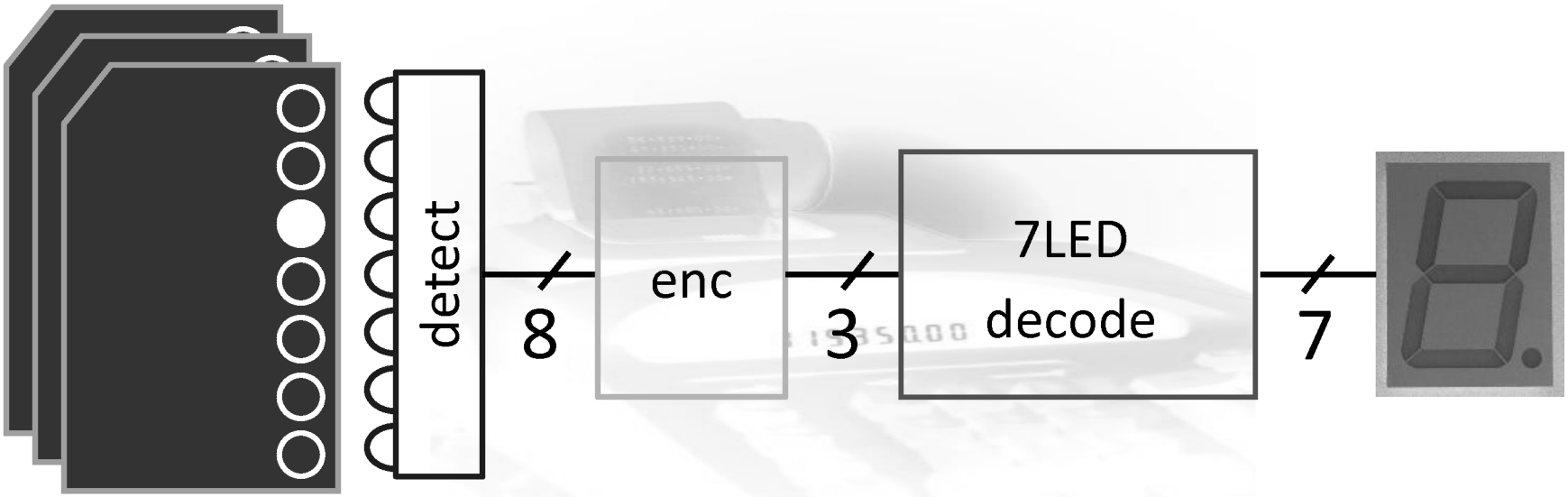
- encode 0 – 7 in binary

7 outputs

- one for each LED

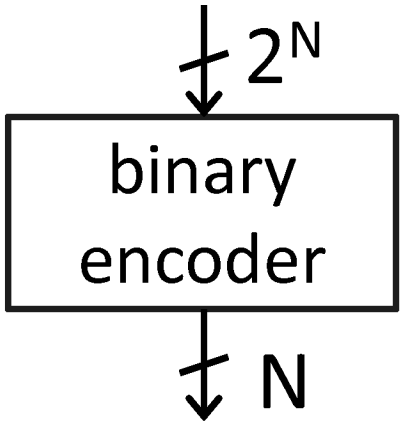
b2	b1	b0	d6	d5	d4	d3	d2	d1	d0
0	0	0	1	1	1	0	1	1	1
0	0	1	1	0	0	0	0	0	1
0	1	0	0	1	1	1	0	1	1
0	1	1	1	1	0	1	0	1	1
1	0	0	1	0	0	1	1	0	1
1	0	1	1	1	0	1	1	1	0
1	1	0	1	1	1	1	1	1	0
1	1	1	1	0	0	0	0	1	1

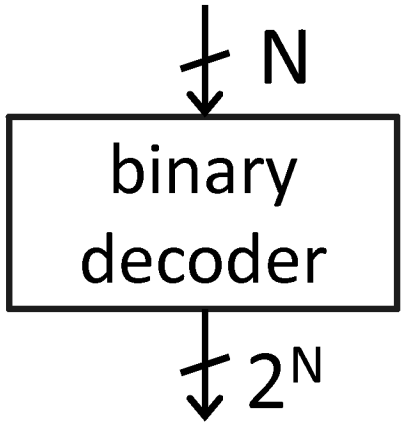


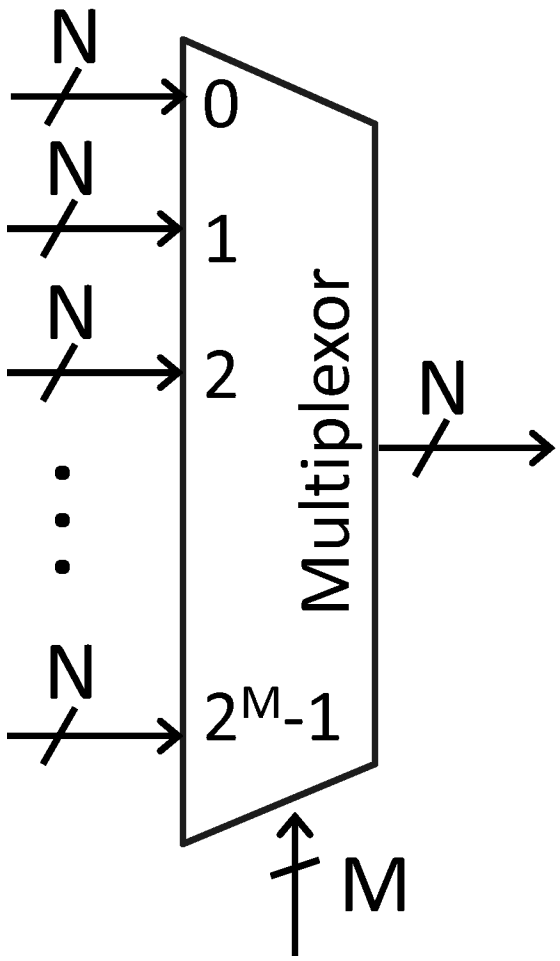


Ballots

The 3410 optical scan vote ~~counter~~ reader machine


2^N
binary
encoder
N


N
binary
decoder
2^N

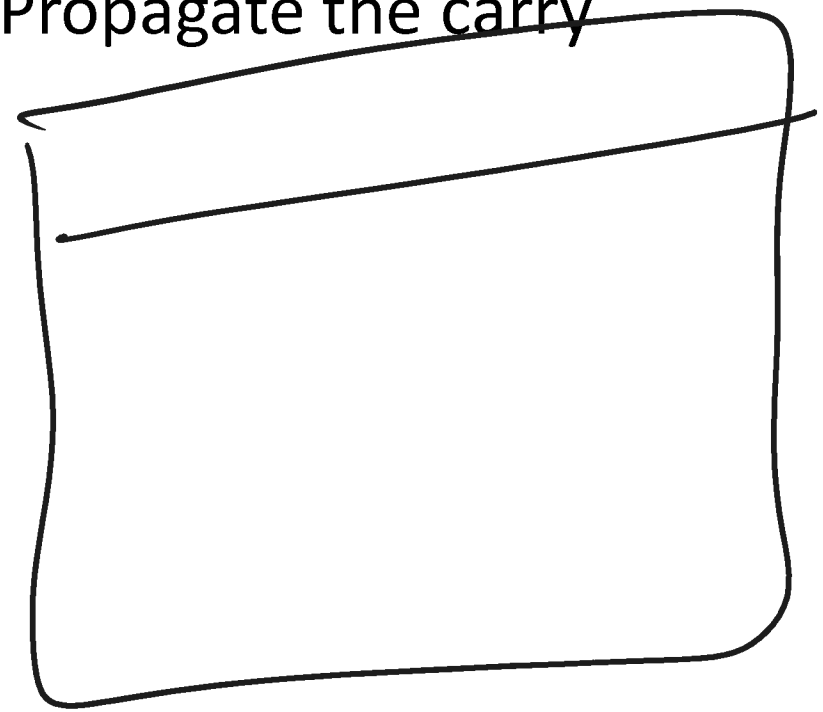

N
0
N
1
N
2
N
2^M-1
Multiplexor
N
M

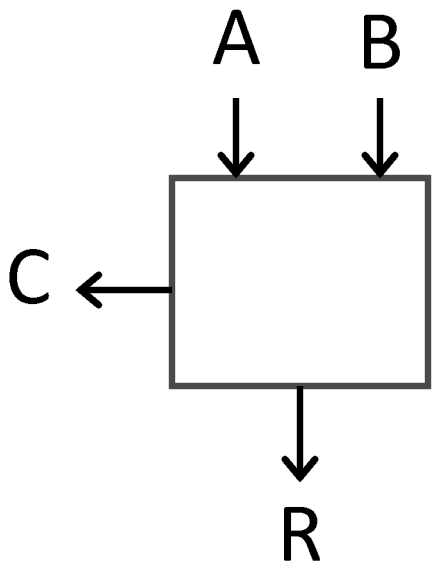
```
  183
+ 254
-----
  437
```

decimal

```
  001110
+ 011100
--------
  000010
```

- Addition works the same way regardless of base
  - Add the digits in each position
  - Propagate the carry

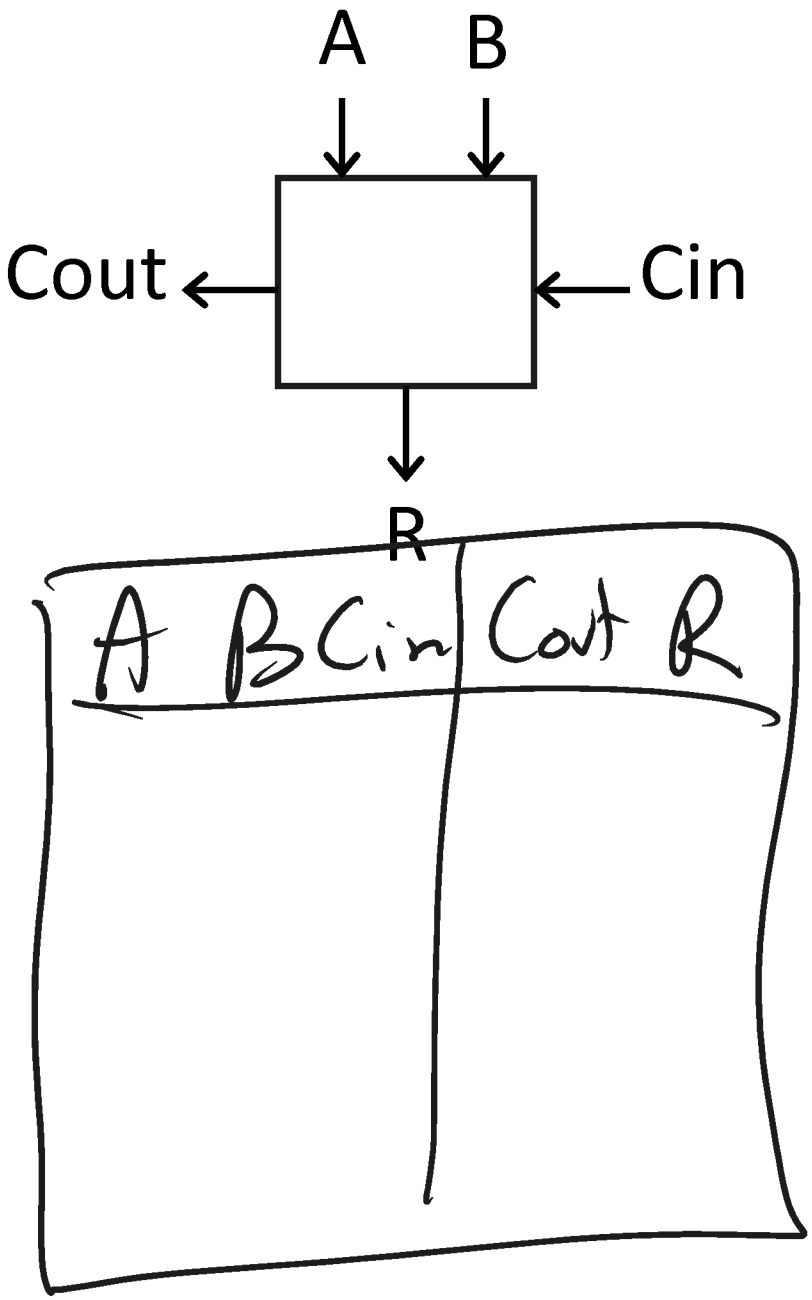




Half Adder

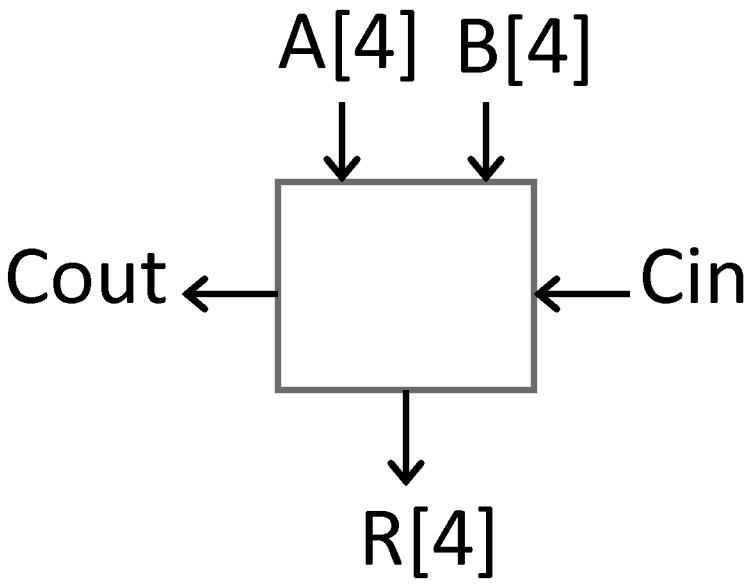
- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry

A	B	C	R
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



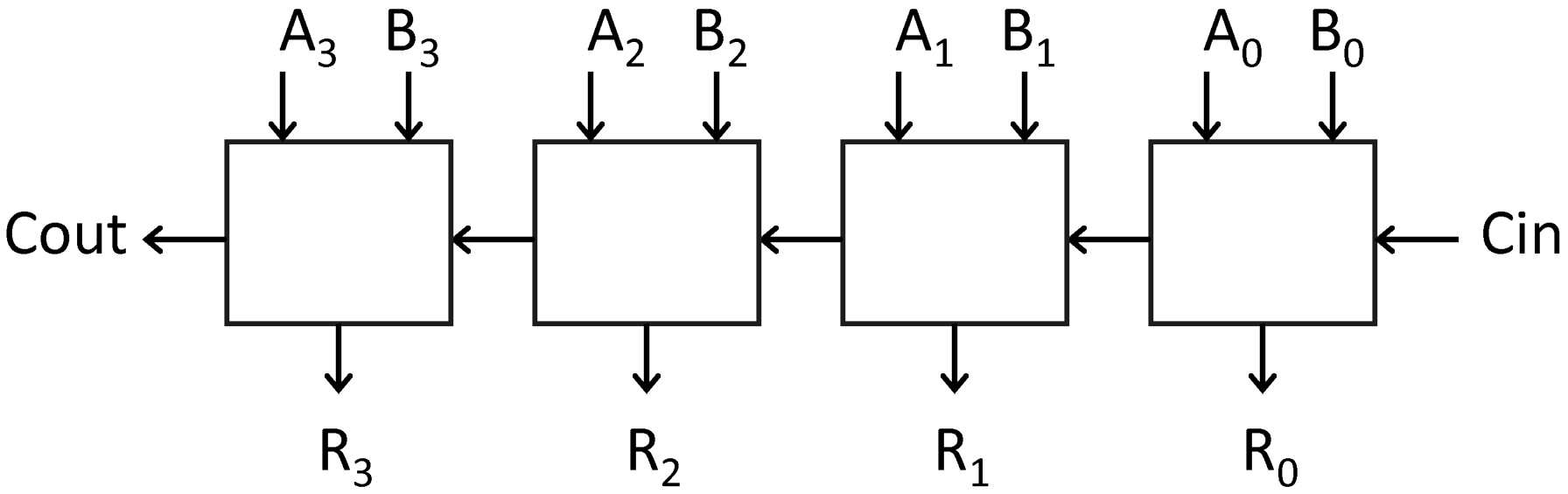
Full Adder

- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry
- Can be cascaded



4-Bit Full Adder

- Adds two 4-bit numbers and carry in
- Computes 4-bit result and carry out
- Can be cascaded


A3
B3
A2
B2
A1
B1
A0
B0
Cout
Cin
R3
R2
R1
R0

We can now implement any combinational (combinatorial) logic circuit

- Decompose large circuit into manageable blocks
  - Encoders, Decoders, Multiplexors, Adders, ...
- Design each block
  - Binary encoded numbers for compactness
- Can implement circuits using NAND or NOR gates
- Can implement gates using use P- and N-transistors