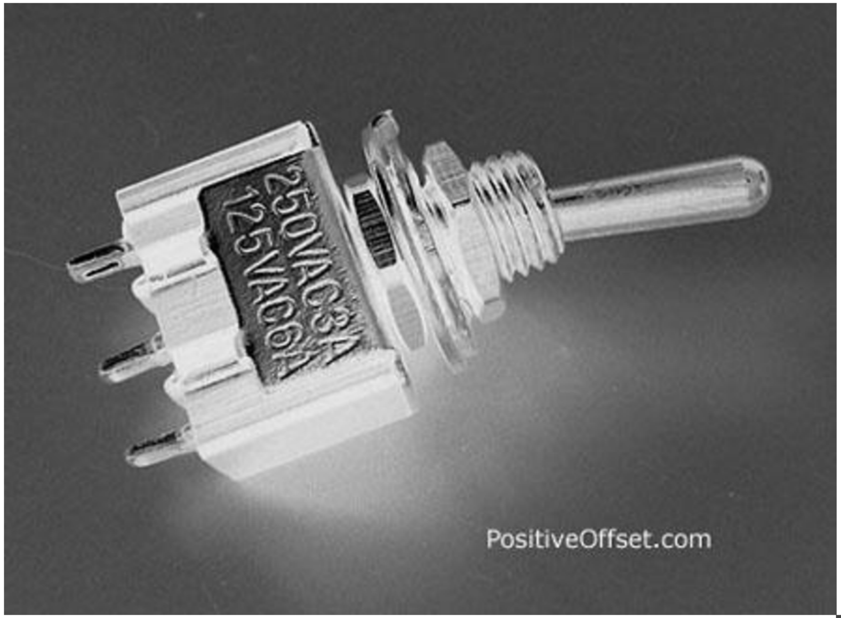
xkcd.com/74/

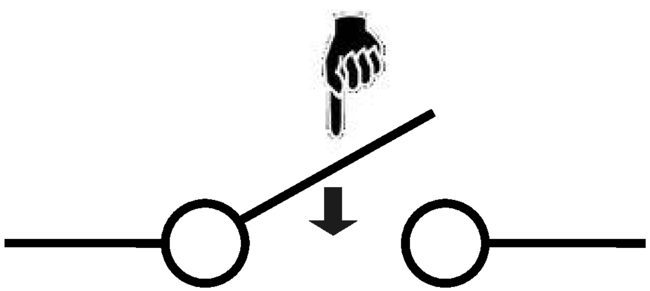
Gates and Logic

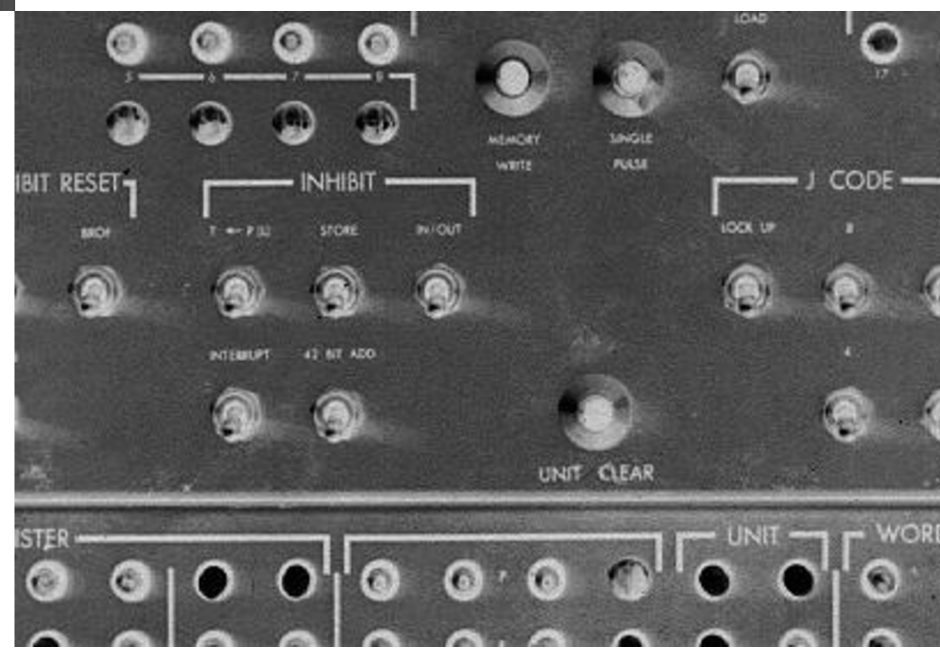
See: P&H Appendix C.2, C.3

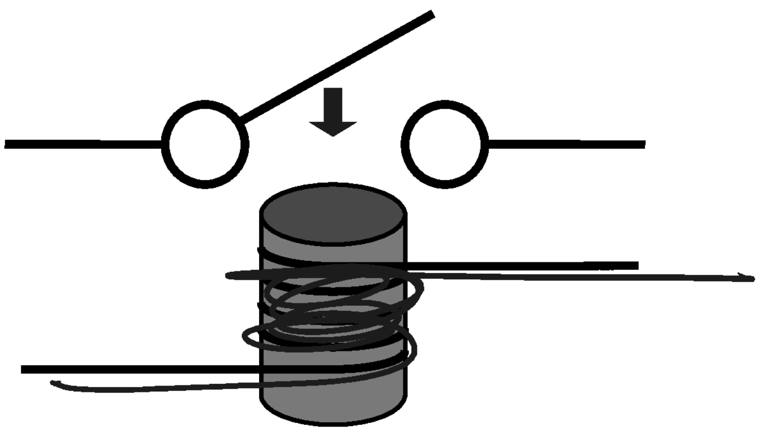


- Acts as a *conductor* or *insulator*

- Can be used to build amazing things…

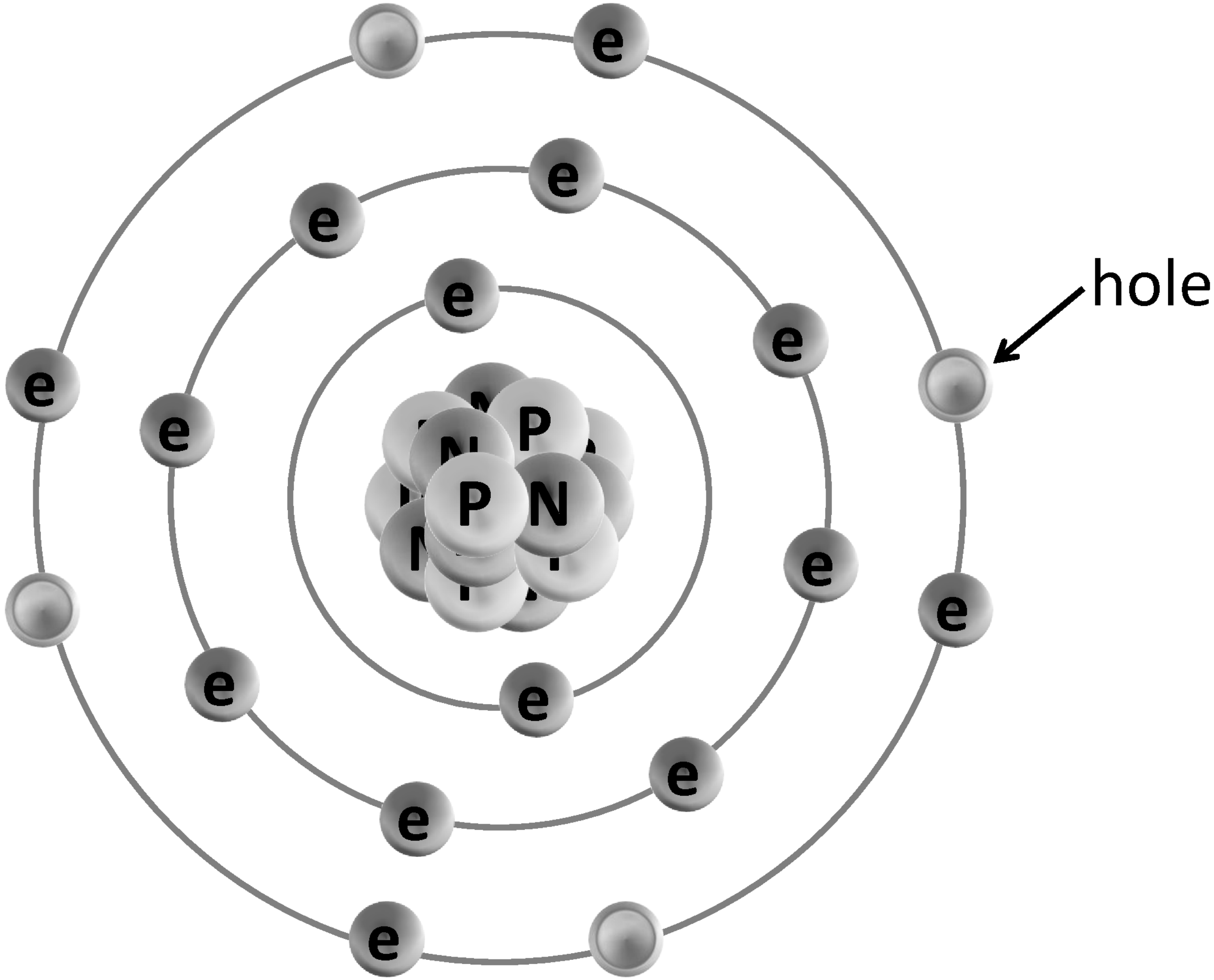


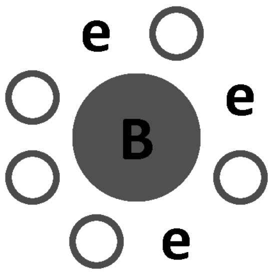




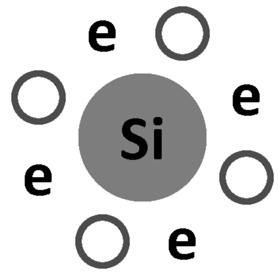
- One current controls another (larger) current

- Static Power:
  – Keeps consuming power when in the *ON* state
- Dynamic Power:
  – Jump in power consumption when switching

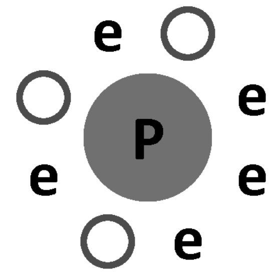

hole



Boron

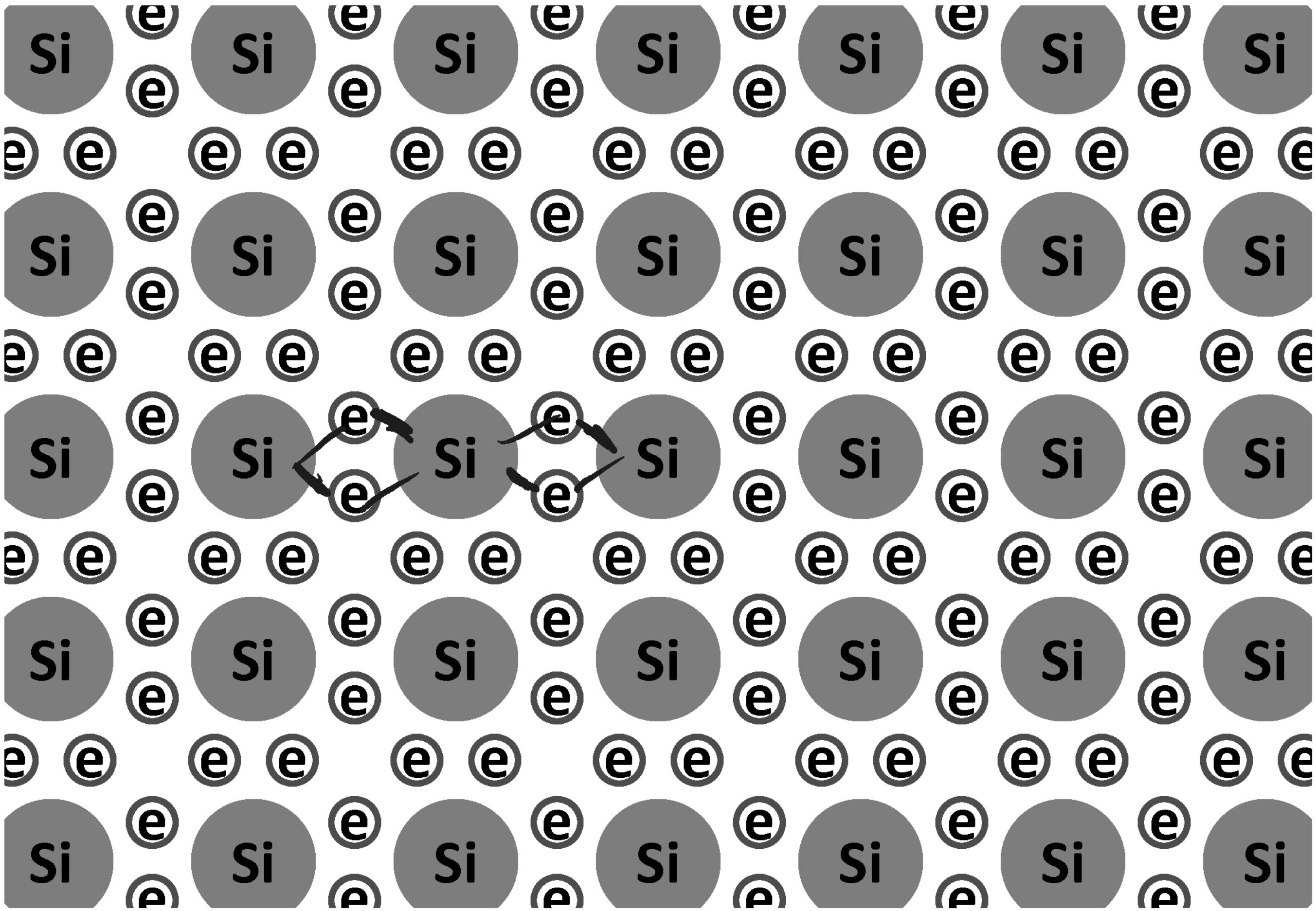


Silicon

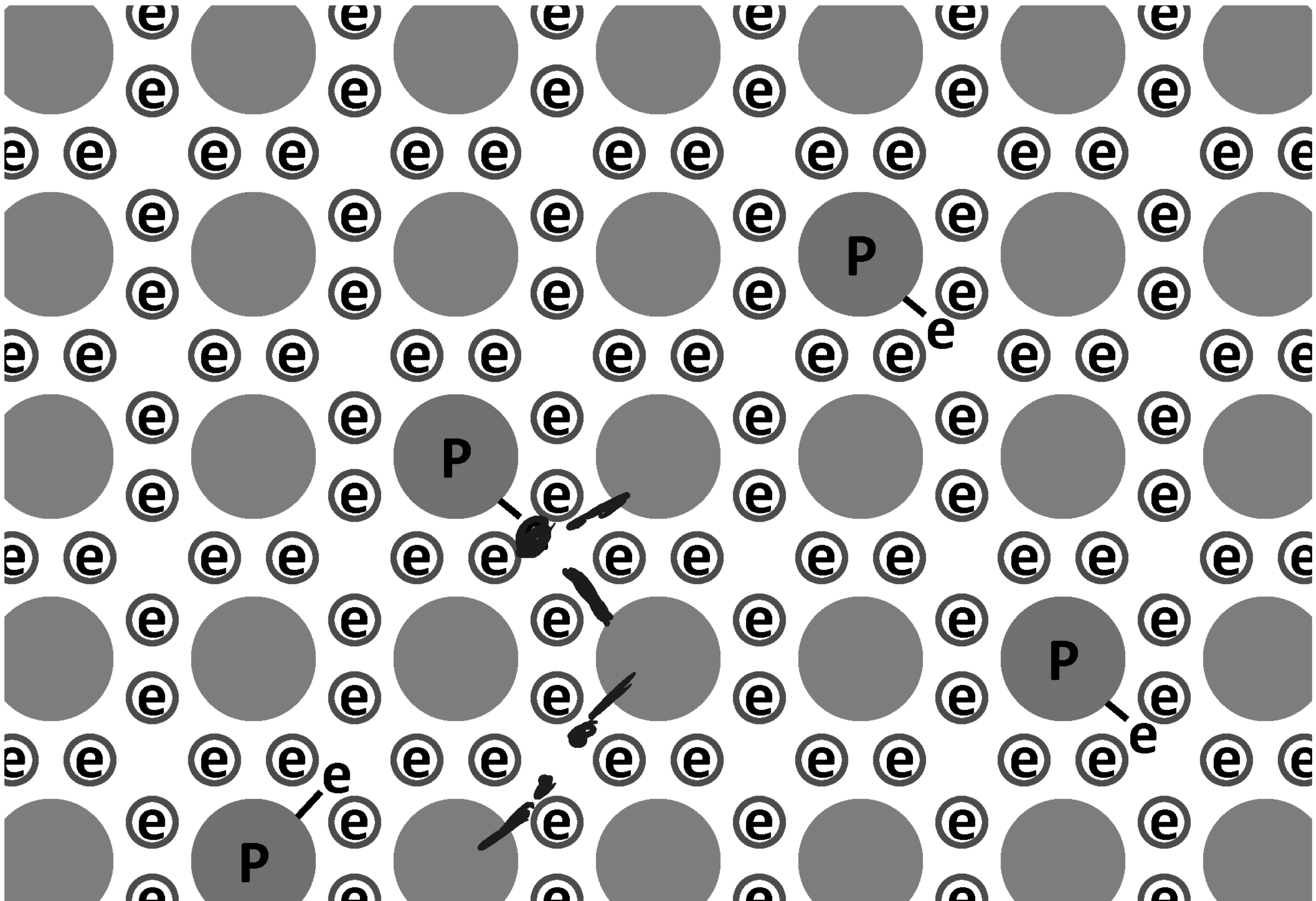


Phosphorus

Silicon



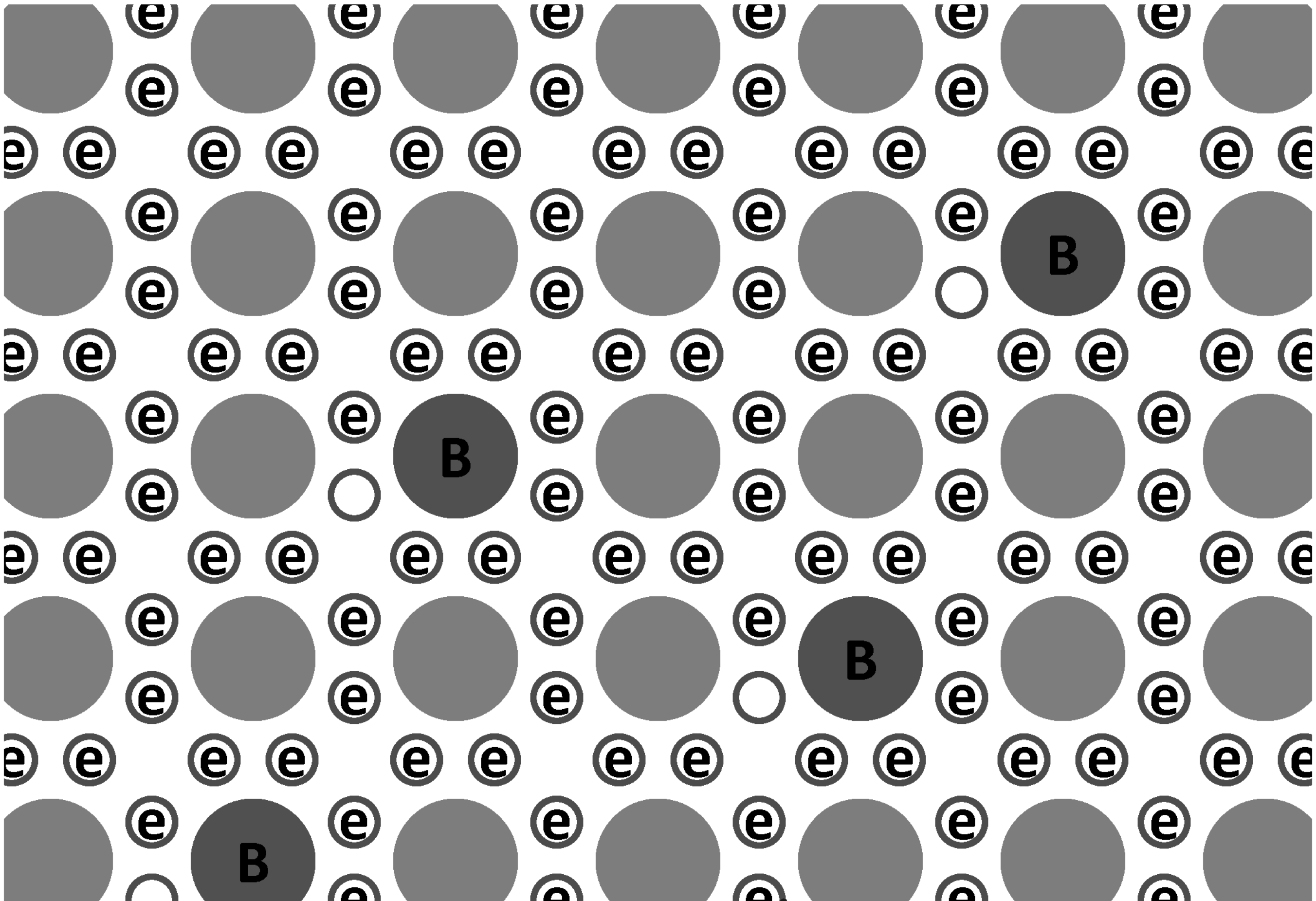
N-Type: Silicon + Phosphorus



mobile electrons $\Rightarrow$ conductor

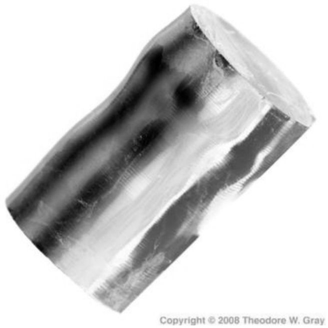
h. + v. $\Rightarrow$ depleted $\Rightarrow$ Ohms.

P-Type: Silicon + Boron



mobile holes ⇒ cond.

⇒ depleted ⇒ ins.



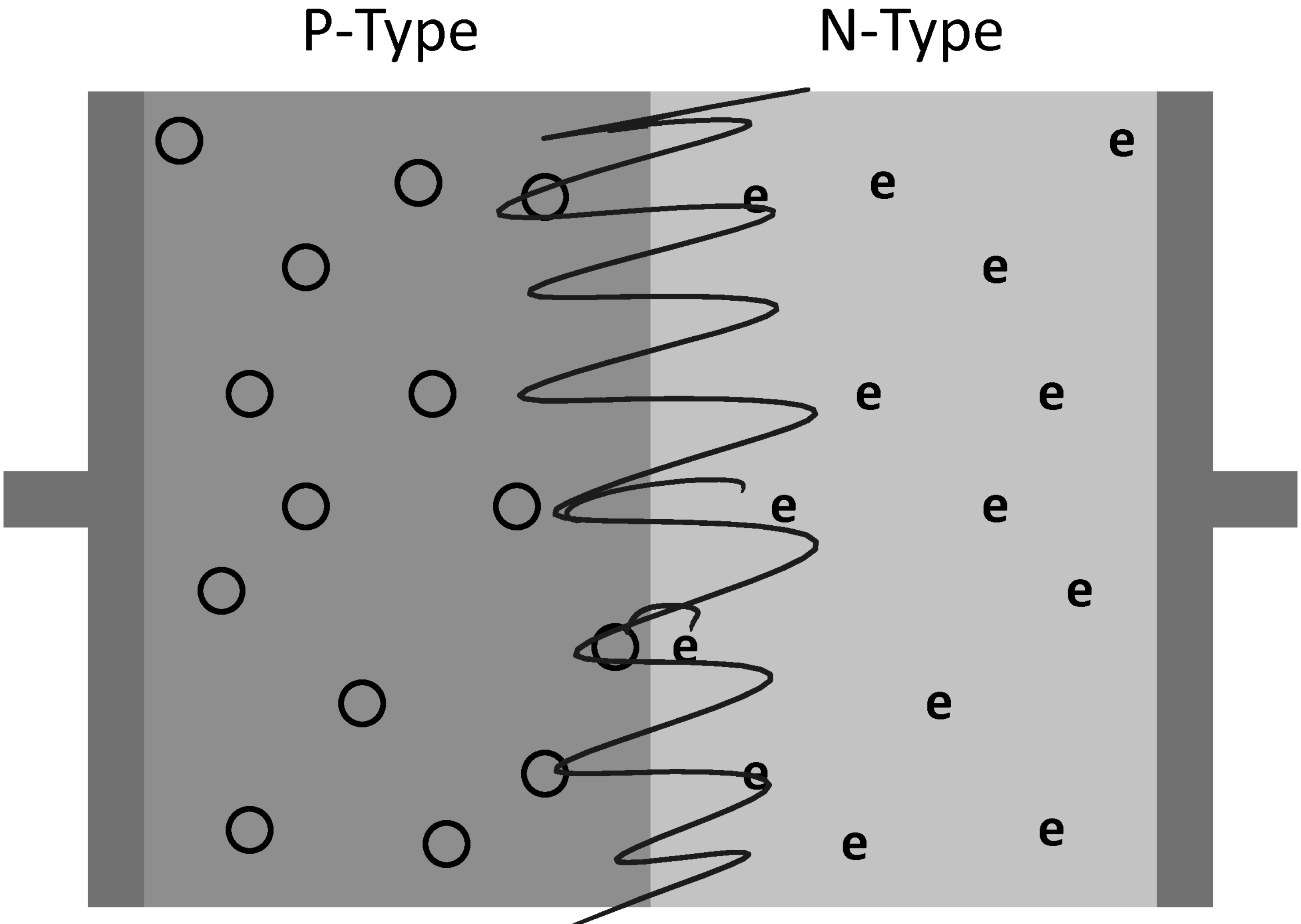
Insulator

p-type (Si+Boron)
has mobile holes:

low voltage (depleted)
→ insulator
high voltage (mobile holes)
→ conductor

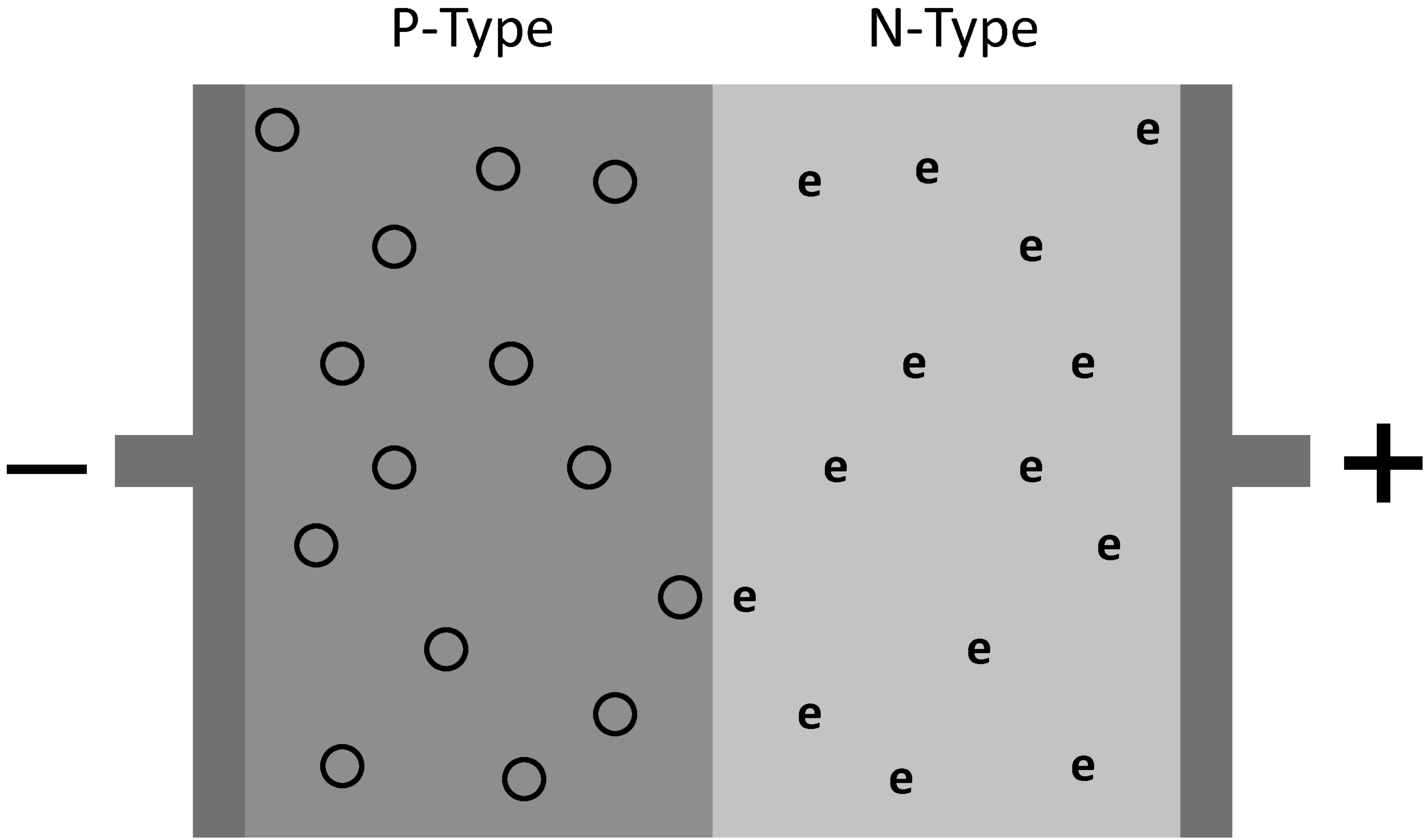
n-type (Si+Phosphorus)
has mobile electrons:

low voltage (mobile electrons)
→ conductor
high voltage (depleted)
→ insulator



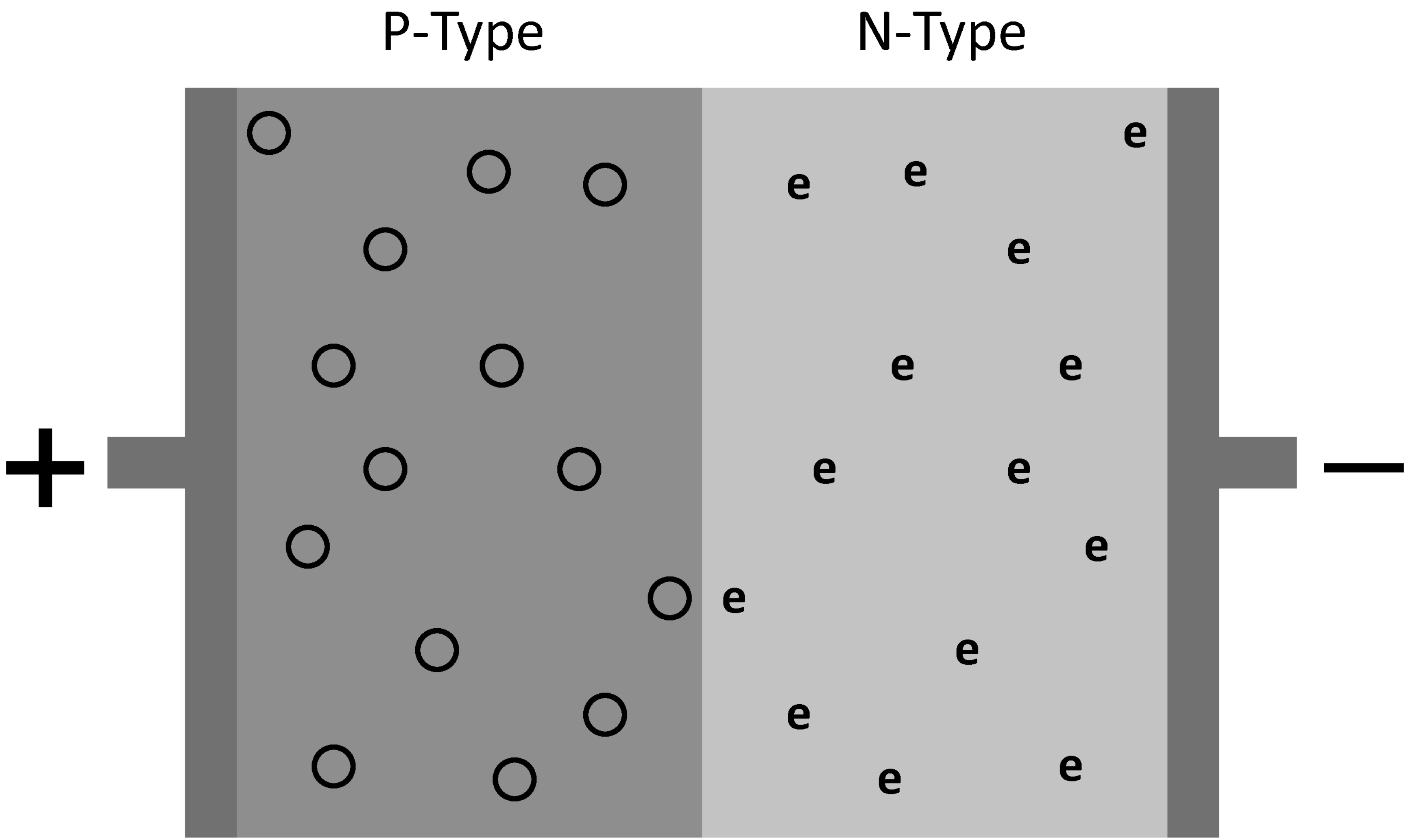
low v → insulator
high v → conductor

low v → conductor
high v → insulator



low v → insulator
high v → conductor

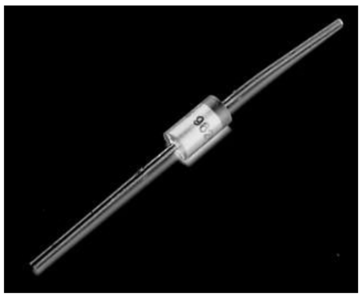
low v → conductor
high v → insulator

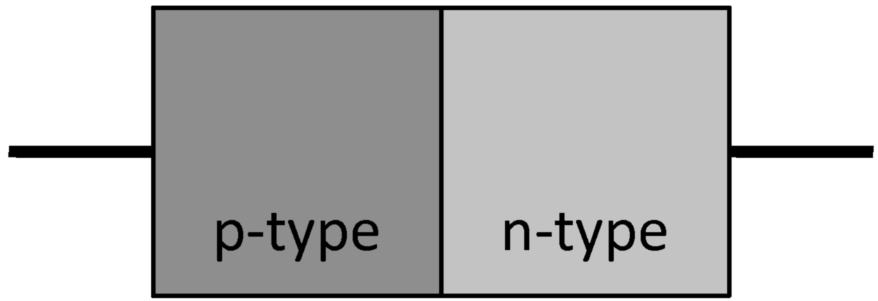


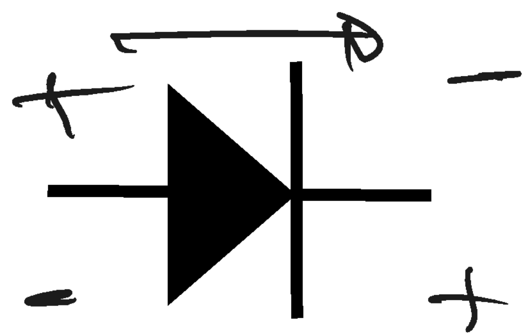
low v → insulator
high v → conductor

low v → conductor
high v → insulator

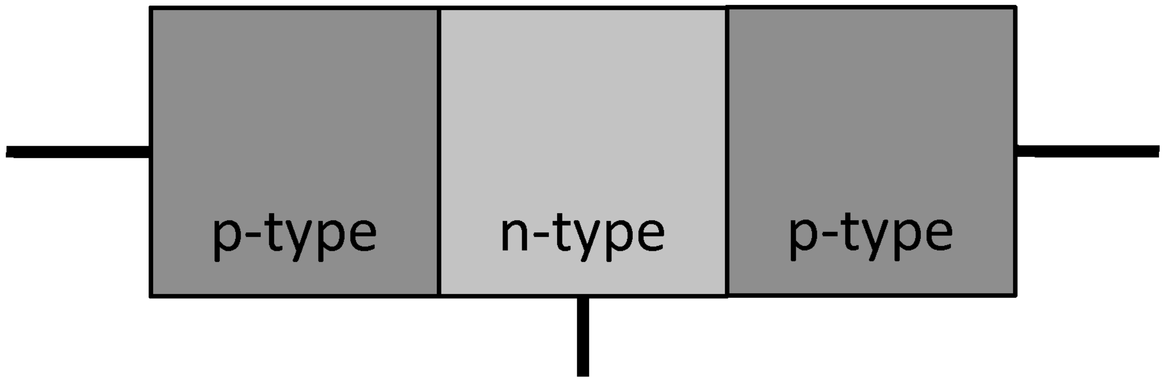
PN Junction "Diode"

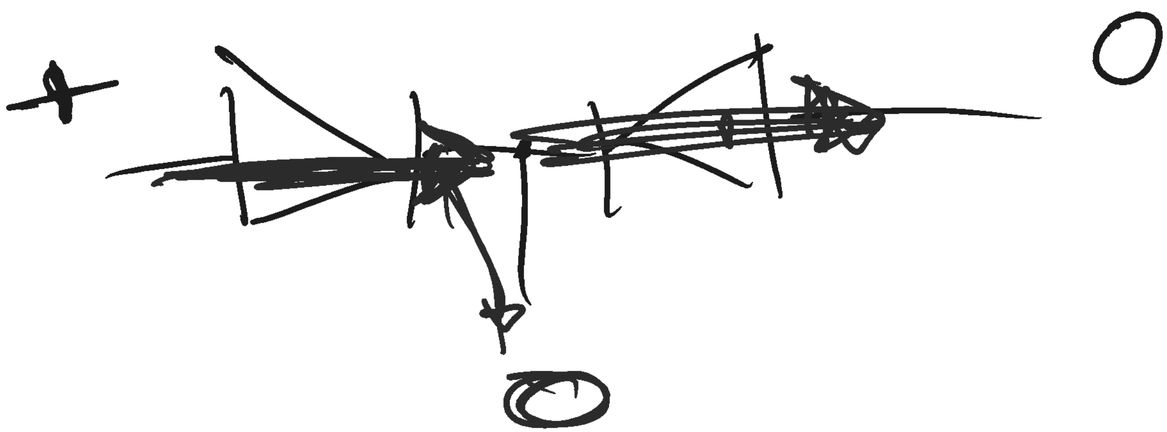


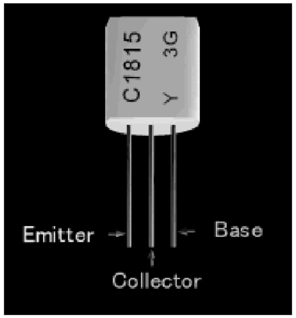




Conventions:
vdd = vcc = +1.2v = +5v = hi
vss = vee = 0v = gnd


p-type
n-type
p-type

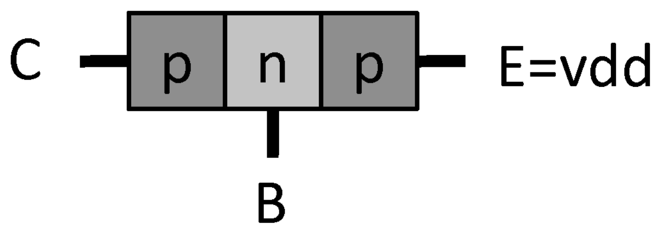




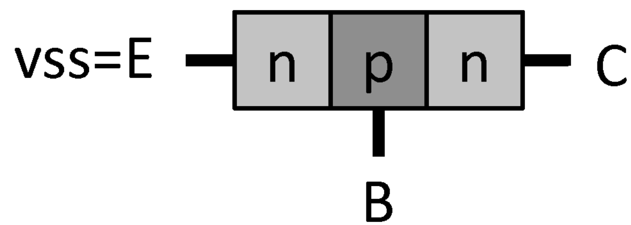
- Solid-state switch: The most amazing invention of the 1900s

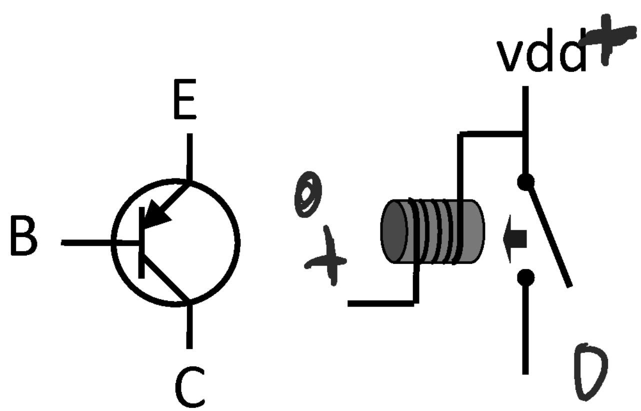
Emitter = “input”, Base = “switch”, Collector = “output”

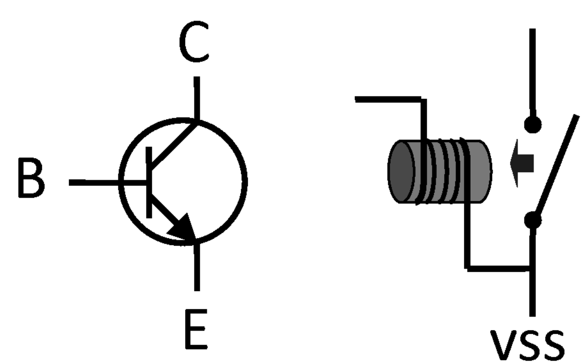
PNP Transistor



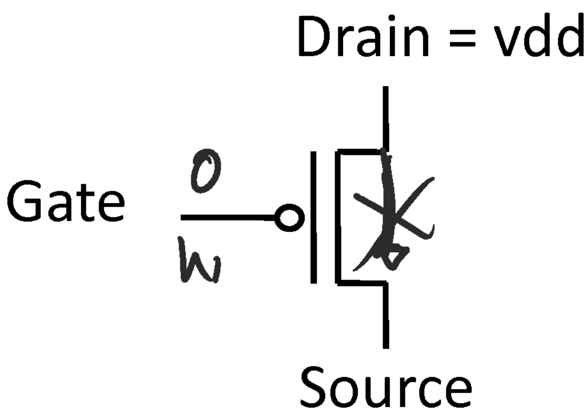
NPN Transistor





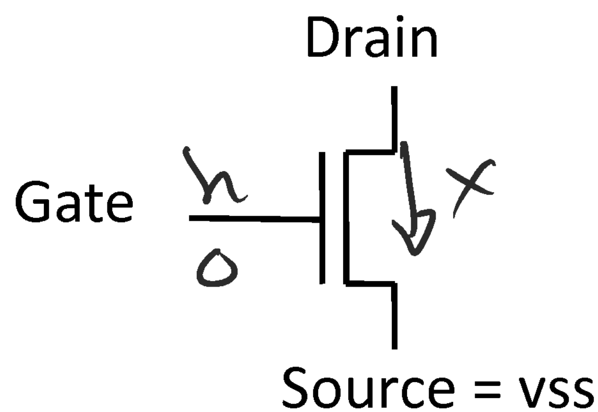


P-type FET

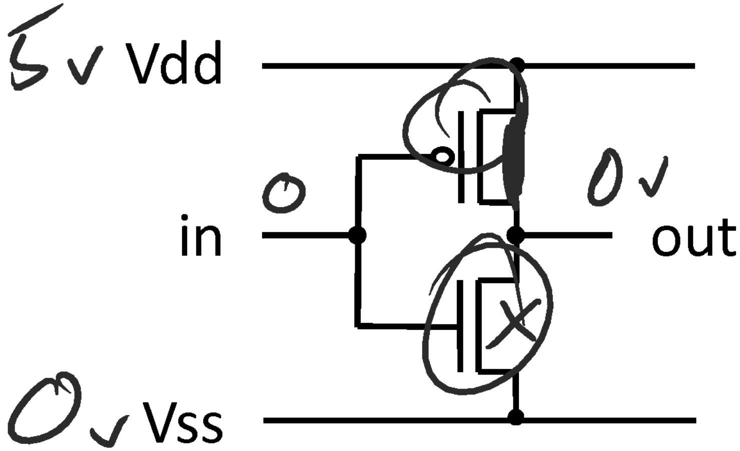


- Connect Source to Drain when Gate = lo
- Drain must be vdd, or connected to source of another P-type transistor

N-type FET

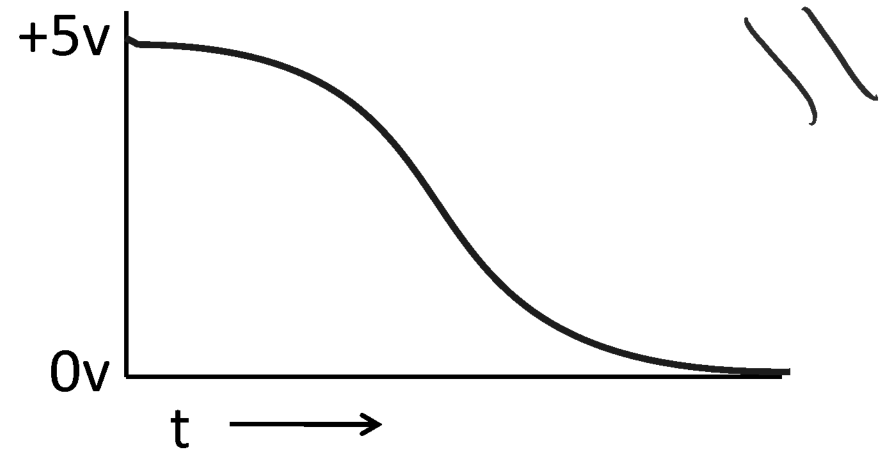


- Connect Source to Drain when Gate = hi
- Source must be vss, or connected to drain of another N-type transistor



In	Out
5v	0v
0v	5v

voltage



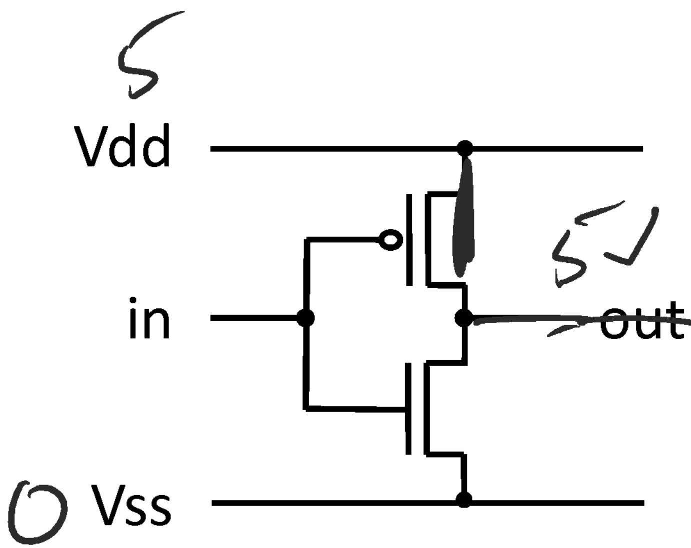
Gate delay

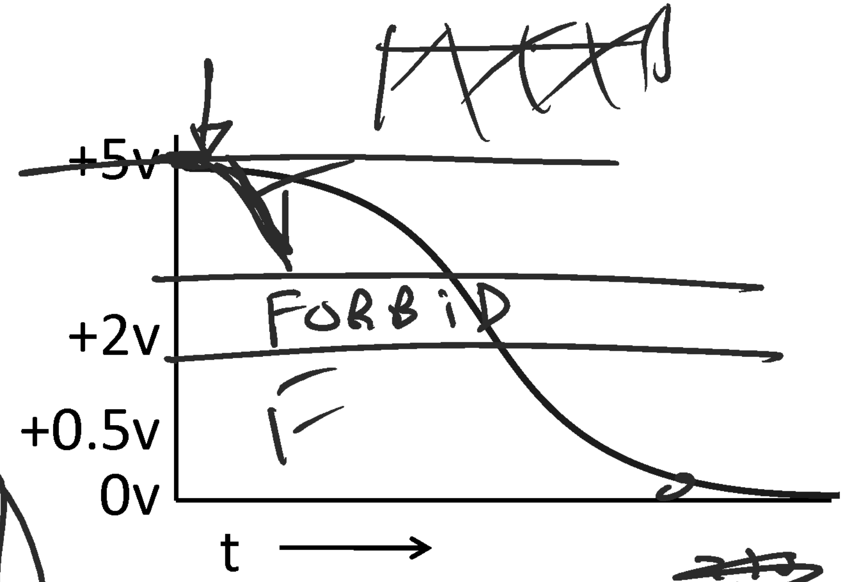
- transistor switching time
- voltage, propagation, fanout, temperature, …

CMOS design

(complementary-symmetry metal–oxide–semiconductor)

- Power consumption = dynamic + leakage





In	Out
+5v	0v
0v	+5v

voltage

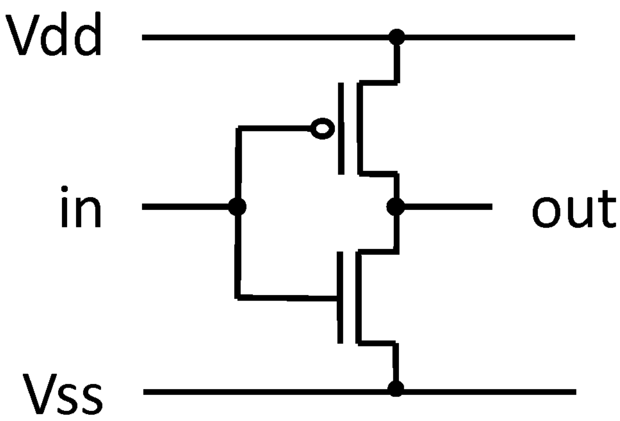
In	Out
T	F
F	T

truth table

Conventions:

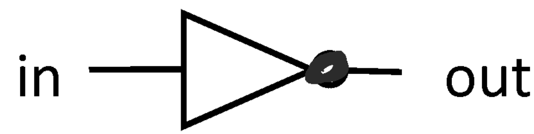
vdd = vcc = +1.2v = +5v = hi = true = 1

vss = vee = 0v = gnd = false = 0



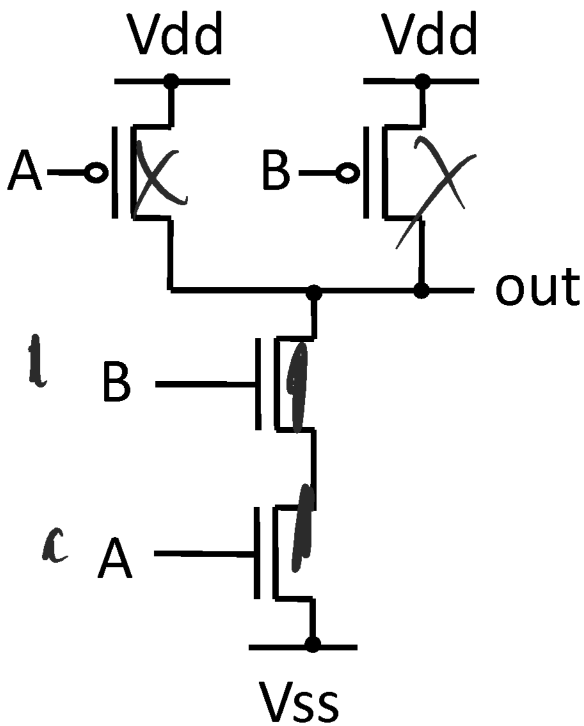
Function: NOT

- Symbol:



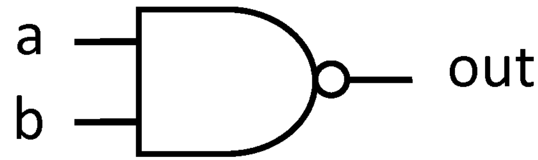
In	Out
0	**1**
1	**0**

Truth table

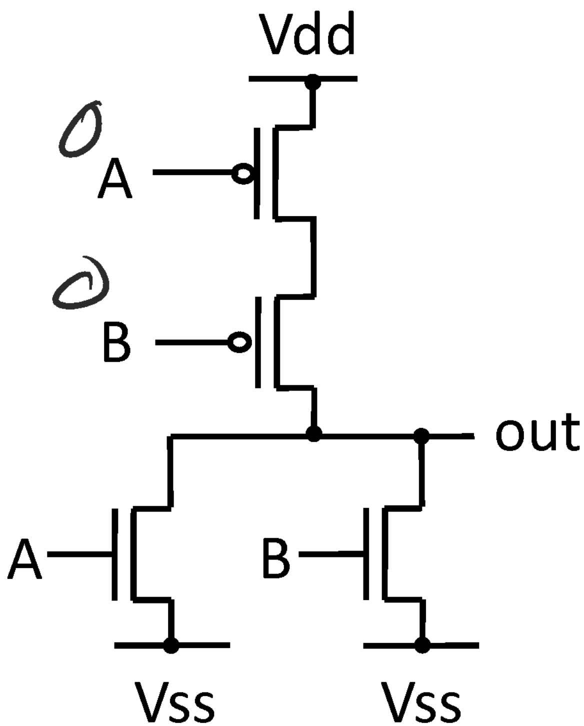


Function: NAND

- Symbol:

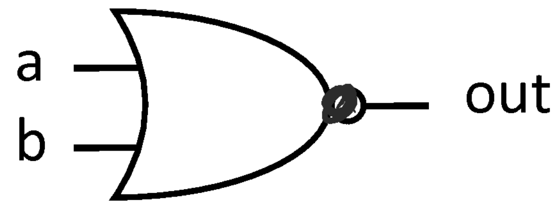


A	B	out
0	0	1
0	1	1
1	0	1
1	1	0



Function: NOR

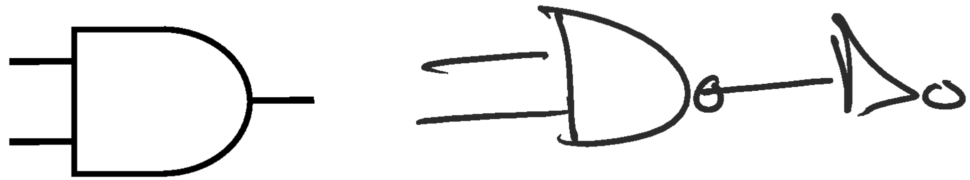
- Symbol:



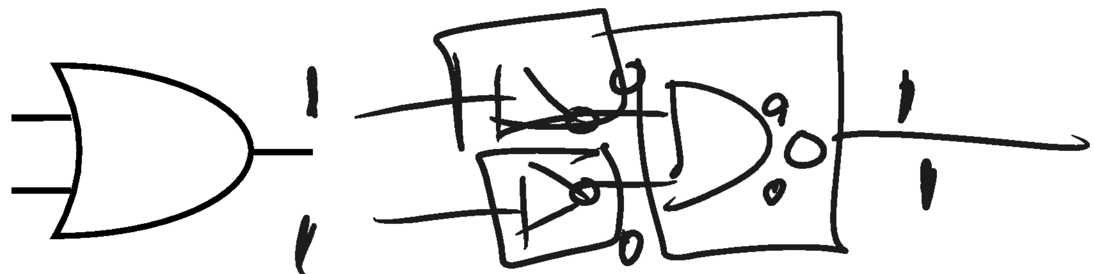
A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

NOT (~~A~~ and ~~B~~) = (~~NOT~~ ~~A~~) or (~~NOT~~ ~~B~~)
NOT A NOT B ~~NOT A~~ ~~NOT~~ NOT B

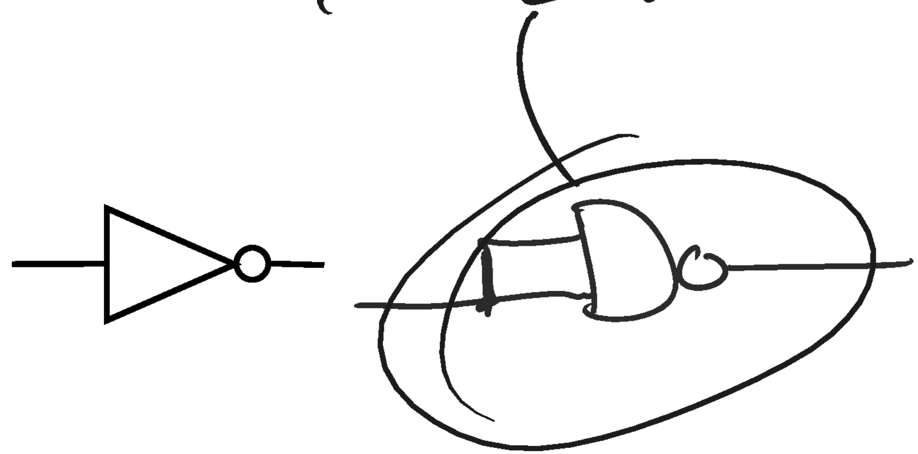
- AND:



- OR:



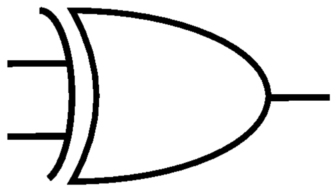
- NOT:

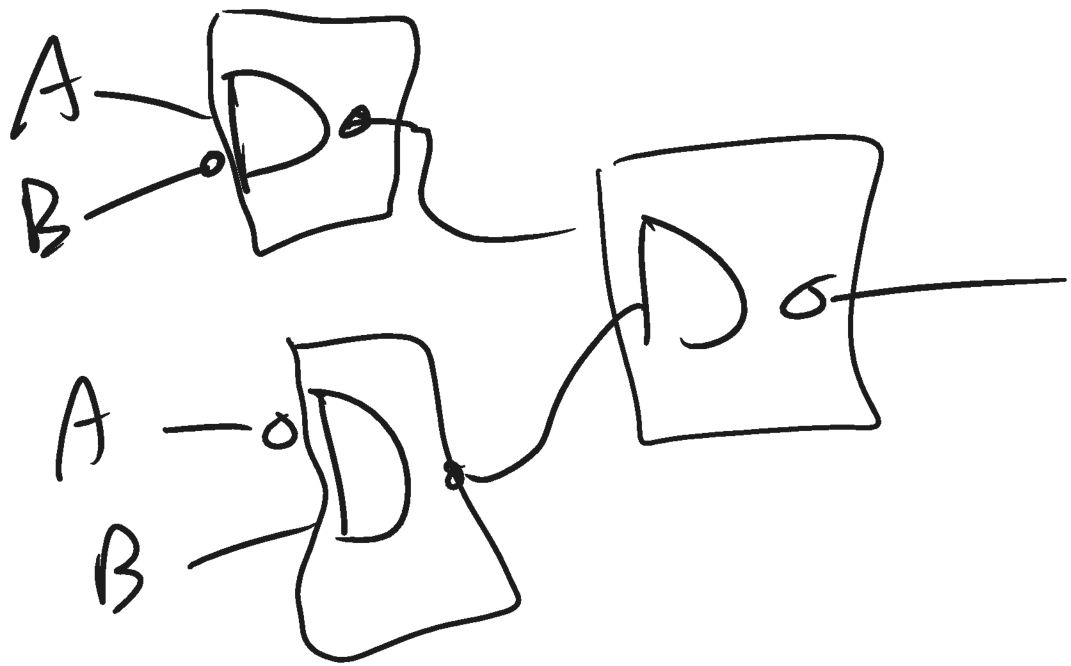


NAND is universal (so is NOR)

- Can implement any function with just NAND gates
  - De Morgan's laws are helpful (pushing bubbles)
- useful for manufacturing

E.g.: XOR (A, B) = A or B but not both ("exclusive or")





Proof: ?

Some notation:

- constants: true = 1, false = 0
- variables: a, b, out, …
- operators:

1 1

0 0

• AND(a, b)	= a b	= a & b	= $a \wedge b$
• OR(a, b)	= a + b	= a | b	= $a \vee b$
• NOT(a)	= $\bar{a}$	= !a	= $\neg a$

$0\,0 = 0$ $\quad$ $0 + 0 = 0$

$0 \cdot 1 = 0$ $\quad$ $0 + 1 = 1$

$1 \cdot 0 = 0$ $\quad$ $1 + 0 = 1$

$1 \cdot 1 = 1$ $\quad$ $1 + 1 = 1$

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a\,0 = 0$$

$$a\,1 = a$$

$$a\,\bar{a} = 0$$

$$\overline{(a + b)} = \bar{a}\,\bar{b}$$

$$\overline{(a\,b)} = \bar{a} + \bar{b}$$

$$a + a\,b = a$$

$$a(b+c) = ab + ac$$

$$\overline{a(b+c)} = \bar{a} + \overline{bc}$$

- functions: gates ↔ truth tables ↔ equations
- Example: (a+b)(a+c) = a + bc

a	b	c	a+b	a+c	LHS	bc	RHS
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

$$(a+b)(a+c)$$

$$\underbrace{\underline{a}\cdot a}_{a\cdot 1} + \underline{a}\cdot c + b\cdot \underline{a} + b\cdot c$$

$$a(\underbrace{\underbrace{1 + c}_{1} + b}_{1}) + b\cdot c$$

$$\underbrace{a\cdot 1}_{a} + bc$$

$$a + bc$$