CS 3220: Prelim 1 example questions

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Whats in this document

These questions are intended to be somewhat representative of the type of questions that could arise on the first prelim.

Question 1

Here, we consider some properties of $\|A\|_F$. For what follows $A \in \mathbb{R}^{n\times n}$

(a) Prove that

$$\|A\|_F^2 = \sum_{i=1}^{n} \|A(:,i)\|_2^2.$$

(b) For any two orthogonal matrices $Q_1 \in \mathbb{R}^{n\times n}$ and $Q_2 \in \mathbb{R}^{n\times n}$ prove that

$$\|Q_1 A Q_2\|_F = \|A\|_F$$

(c) Prove that

$$\|A\|_F = \sqrt{\sum_{i=1}^{n} \sigma_i^2},$$

where $\sigma_1, \ldots, \sigma_n$ are the singular values of A.

Question 2

Here, we ask you to interpret the condition number of a 2×2 matrix geometrically. (Hint: pictures are useful here!)

1. We saw that the SVD of a 2×2 matrix allows us to view any matrix A as mapping a circle to an ellipse. If A becomes increasing ill-conditioned what is geometrically happening to the ellipse?
2. Geometrically argue why for an ill-conditioned matrix a small relative change in b can result in a big relative change in x.

Question 3

Say you are given a symmetric matrix A and tasked with computing the algebraically smallest eigenvalue. Using only the power method (applied to A or matrices related to A), how might you go about doing this? (Hint: think about how the eigenvalues/vectors of $A - \gamma I$ relate to those of A for any scalar $\gamma \in \mathbb{R}$.)