CS 3220: PRELIM 1 EXAMPLE QUESTIONS
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WHAT'S IN THIS DOCUMENT
These questions are intended to be somewhat representative of the type of questions that could arise on the first prelim.

QUESTION 1
Here, we consider some properties of \( \| A \|_F \). For what follows \( A \in \mathbb{R}^{n \times n} \)

(a) Prove that
\[
\| A \|_F^2 = \sum_{i=1}^{n} \| A(:,i) \|_2^2.
\]

(b) For any two orthogonal matrices \( Q_1 \in \mathbb{R}^{n \times n} \) and \( Q_2 \in \mathbb{R}^{n \times n} \) prove that
\[
\| Q_1 A Q_2 \|_F = \| A \|_F
\]

(c) Prove that
\[
\| A \|_F = \sqrt{\sum_{i=1}^{n} \sigma_i^2},
\]
where \( \sigma_1, \ldots, \sigma_n \) are the singular values of \( A \).

QUESTION 2
Here, we ask you to interpret the condition number of a \( 2 \times 2 \) matrix geometrically. (Hint: pictures are useful here!)

1. We saw that the SVD of a \( 2 \times 2 \) matrix allows us to view any matrix \( A \) as mapping a circle to an ellipse. If \( A \) becomes increasingly ill-conditioned what is geometrically happening to the ellipse?

2. Geometrically argue why for an ill-conditioned matrix a small relative change in \( b \) can result in a big relative change in \( x \).

QUESTION 3
Say you are given a symmetric matrix \( A \) and tasked with computing the algebraically smallest eigenvalue. Using only the power method (applied to \( A \) or matrices related to \( A \)), how might you go about doing this? (Hint: think about how the eigenvalues/vectors of \( A - \gamma I \) relate to those of \( A \) for any scalar \( \gamma \in \mathbb{R} \).)