

CS322 Homework #8

Due: Wed April 16, 2008 (in class)

Prof. Doug James

1. **Recursive Trapezoid Formula:** In this computer problem, you will implement the Recursive Trapezoid Formula estimates, $R(n,0)$ (see equation (10) on page 197). For each integral, report the estimates $R(n,0)$ for each approximation level $n=0, 1, 2, \dots, N$. Estimate each integral to **six significant figures**. What maximum level N , and approximately how many functions evaluations, are required to achieve such accuracy in each case?

Consider numerical integration of the following two integrals:

- *The Bessel function of order 0* at $x = 10$ defined by:

$$J_0(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin \theta) d\theta$$

- *The Gaussian/probability integral* with a suggested change of variables to make the range of integration finite:

$$\int_0^\infty e^{-x^2} dx, \quad \text{using } x = -\ln t.$$

2. **Romberg Algorithm:** Now numerically evaluate the same two integrals using the Romberg Algorithm and your implementations of $R(n,0)$ from the previous question. How many function evaluations are required to obtain six significant figures now?
3. **QR Factorization:** In this question you will manually construct a modified Gram-Schmidt QR factorization (as discussed in class) for the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Verify that $A=QR$.