5. Consider the matrix

$$A = \begin{bmatrix} a & 0 & 0 & z \\ 0 & b & 0 & 0 \\ 0 & x & c & 0 \\ w & 0 & y & d \end{bmatrix}$$

[a] **a.** Determine a unit lower triangular matrix $\boldsymbol{M}$ and an upper triangular matrix $\boldsymbol{U}$ such that $\boldsymbol{MA} = \boldsymbol{U}$.

[a] **b.** Determine a lower triangular matrix $\boldsymbol{L}'$ and a unit upper triangular matrix $\boldsymbol{U}'$ such that $\boldsymbol{A} = \boldsymbol{L}'\boldsymbol{U}'$.

11. Consider the system of equations

$$\begin{cases} 6x_1 = 12 \\ 6x_2 + 3x_1 = -12 \\ 7x_3 - 2x_2 + 4x_1 = 14 \\ 21x_4 + 9x_3 - 3x_2 + 5x_1 = -2 \end{cases}$$

a. Solve for x_1, x_2, x_3, and x_4 (in order) by forward substitution.

b. Write this system in matrix notation $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{x} = [x_1, x_2, x_3, x_4]^T$. Determine the $\boldsymbol{LU}$ factorization $\boldsymbol{A} = \boldsymbol{LU}$, where $\boldsymbol{L}$ is unit lower triangular and $\boldsymbol{U}$ is upper triangular.

19. a. Prove that the product of two lower triangular matrices is lower triangular.

b. Prove that the product of two unit lower triangular matrices is unit lower triangular.

c. Prove that the inverse of a unit lower triangular matrix is unit lower triangular.

d. By using the transpose operation, prove that all of the preceding results are true for upper triangular matrices.

22. Find the Cholesky factorization of

$$A = \begin{bmatrix} 4 & 6 & 10 \\ 6 & 25 & 19 \\ 10 & 19 & 62 \end{bmatrix}$$