

1. Jacobian is

$$J(p) = \begin{pmatrix} e^{p_2 x_1} & x_1 p_1 e^{p_2 x_1} \\ e^{p_2 x_2} & x_2 p_1 e^{p_2 x_2} \\ e^{p_2 x_3} & x_3 p_1 e^{p_2 x_3} \\ e^{p_2 x_4} & x_4 p_1 e^{p_2 x_4} \end{pmatrix}.$$

The code and plot are in separate files on the website. Convergence is not guaranteed, but if it converges, it seems to converge to the same solution. An example of non-convergence is using $p^0 = [5, 5]^T$.

2. The minimum is where the gradient is zero and the Hessian is positive definite. The gradient is

$$f'(x) = \begin{pmatrix} 2x_1^3 - 2x_1x_2 + x_1 - 1 \\ x_2 - x_1^2 \end{pmatrix},$$

which, by inspection, is zero at $x_1 = x_2 = 1$. Its Hessian is

$$f''(x) = \begin{pmatrix} 6x_1^2 - 2x_2 + 1 & -2x_1 \\ -2x_1 & 1 \end{pmatrix}.$$

So $f''([1, 1]^T) = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \equiv H$. Since $H = LL^T$ where $L = \begin{pmatrix} 2.2361 & 0 \\ -0.8944 & 0.4472 \end{pmatrix}$, that is, it has Cholesky factor, it is positive definite.

For Newton method, $f''([2, 2]^T) = \begin{pmatrix} 24 & -4 \\ -4 & 1 \end{pmatrix}$ and $f'([2, 2]^T) = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$. Hence, $\Delta x = [-0.2, 1.2]^T$. So $x_1 = [1.8, 3.2]^T$.

3. The given system is equivalent to

$$\begin{aligned} \frac{dx}{dt} &= v \\ \frac{dy}{dt} &= w \\ \frac{dv}{dt} &= \frac{-x}{(x^2 + y^2)^{1.5}} \\ \frac{dw}{dt} &= \frac{-y}{(x^2 + y^2)^{1.5}} \end{aligned}$$

Codes and plots given separately. The trajectories are not the expected ellipses, but look like ellipse with revolving semimajor and semiminor axes. From energy plots, it is seen that in AB1, energy oscillates but increasing overall while in AB2, energy changes is little.