CS 322: Introduction to Scientific Computing
Spring 2003
Prelim 2

Handed out: Thursday, April 10.

PLEASE DO NOT OPEN THIS BOOKLET UNTIL THE SIGNAL IS GIVEN. This examination lasts 75 minutes and has 75 points total. It is closed book and closed note, but you are permitted to use an $8\frac{1}{2}'' \times 11''$ crib-sheet with notes written on both sides. This exam counts for 25% of your final course grade. There are 8 questions total spread over pages 2–4 of this booklet. Write your answers in this booklet. You can separate the pages since we have staplers available at the front of the room. You can write continuations on the last page if one of your answers is too long to fit in the space.

Name: | first name(s) | last name(s) |

CU ID number:

NetID:

Section #

Note: Section 1 is Thurs, 12:20, section 2 is Thurs, 3:35, section 3 is F 2:30 and section 4 is F 3:35.

DO NOT DISCUSS THE QUESTIONS WITH ANYONE UNTIL SATURDAY, SINCE THE MAKEUP PRELIM WILL ALSO HAVE SOME OF THESE SAME QUESTIONS.

"Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination."

Sign in the box →

1. **[5 points]** Name one difference between Gaussian quadrature and Newton-Cotes quadrature.

   Answer here:

2. **[5 points]** How many flops, accurate to the leading term, are required to perform Gaussian elimination with partial pivoting on an $n \times n$ matrix?

   Answer here:

3. **[5 points]** What is the definition of the vector ∞-norm?

   Answer here:

4. **[5 points]** What does it mean for a matrix to be orthogonal?

   Answer here:

5. **[10 points]** Let A be an $m \times n$ matrix of rank n and let $\mathbf{b} = A(:, 1)$. What is the solution $\mathbf{x}$ of the least-squares problem $\min \|A\mathbf{x} - \mathbf{b}\|_2$?

   Answer here:

6. **[15 points]** Consider the following three-point quadrature rule for integrating functions on the interval $[0, 1]$

$$\int_0^1 f(x)\,dx \approx \sum_{i=1}^{3} w_i f(x_i)$$

   where $x_1 = 0$, $x_2 = 0.5$, $x_3 = 1$; $w_1 = -10^{17} + 0.5$, $w_2 = 2 \cdot 10^{17}$, $w_3 = -10^{17} + 0.5$.

   (a) Use algebra to show that this rule is exact for constant and linear functions.

   (b) Explain why this rule is undesirable from the standpoint of stability.

   Answer here:

7. **[15 points]** Suppose that plain Gaussian elimination is applied to an $n \times n$ matrix A all of whose rows are equal to one another (i.e., $A(1,:) = A(2,:) = \cdots = A(n,:)$) and whose $(1,1)$ entry is nonzero. Explain what will happen with both plain GE and with partial pivoting applied to this matrix. (Hint: neither will get past the first column.) Then explain that, despite the fact that plain Gaussian elimination is not able to run to completion, a factorization $A = LU$ can be deduced from the partial output.

   Answer here:

8. **[15 points]** Given an vector $\mathbf{b} \in \mathbf{R}^{10}$, consider the problem of finding two scalars s, t such $[s; s+t; s; s+t; s; s+t; s; s+t; s; s+t]$ is as close as possible to $\mathbf{b}$ in the 2-norm. Set this up as a linear least-squares problem of the form $\min \|A\mathbf{x} - \mathbf{b}\|_2$.

   Answer here:

Write continuations to your answers on this page. Label the continuations.

Please do not write in this area.
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