CS 322: Introduction to Scientific Computing
Spring 2003
Prelim 1

Handed out: Tuesday, February 25.

PLEASE DO NOT OPEN THIS BOOKLET UNTIL THE SIGNAL IS GIVEN. This examination lasts 75 minutes and has 75 points total. It is closed book and closed note, but you are permitted to use an $8\frac{1}{2}'' \times 11''$ crib-sheet with notes written on both sides. This exam counts for 25% of your final course grade. There are 7 questions total spread over pages 2–4 of this booklet. Write your answers in this booklet. You can separate the pages since we have staplers available at the front of the room. You can write continuations on the last page if one of your answers is too long to fit in the space.

Name: | first name(s) | last name(s) |

CU ID number:

NetID:

Section #

Note: Section 1 is Thurs, 12:20, section 2 is Thurs, 3:35, section 3 is F 2:30 and section 4 is F 3:35.

DO NOT DISCUSS THE QUESTIONS WITH ANYONE UNTIL SATURDAY, SINCE THE MAKEUP PRELIM WILL ALSO HAVE SOME OF THESE SAME QUESTIONS.

"Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination."

Sign in the box →

1. **[5 points]** What is meant by "overflow" in floating-point arithmetic?

   Answer here:

2. **[5 points]** What is the "not-a-knot" end condition for a cubic spline?

   Answer here:

3. **[10 points]** The Newton form of a degree-n polynomial is

$$p(x) = c_1 + c_2(x - x_1) + c_3(x - x_1)(x - x_2) + \cdots + c_n(x - x_1)\cdots(x - x_{n-1}).$$

   Suppose it is known that x_1 is actually a root of $p(x)$. What information does this give us about $c_1, \ldots, c_n$?

   Answer here:

4. **[10 points]** How many flops are required to evaluate an Hermite cubic polynomial at a point x^*, assuming it is in the form given in lecture?

$$p(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^2(x - x_{i+1})$$

Note: use an efficient evaluation procedure that does not waste flops.

Answer here:

5. **[15 points]** In lecture, it was proved that given n data points (x_1, y_1), ..., (x_n, y_n) such that $x_1 < x_2 < \cdots < x_n$, there exists a unique polynomial p of degree at most $n - 1$ such that $p(x_i) = y_i$, $i = 1, \ldots, n$. Suppose that the x-coordinates in the data instead have the property $x_1 = x_2 < x_3 < x_4 \cdots < x_n$. Does p still exist? Is it unique? Explain your answers. If your answer is, "it depends", then explain what it depends on.

Answer here:

6. **[15 points]** (a) Explain why direct evaluation of the following formula is prone to cancellation error when $x \approx 0$:

$$y = (1 + x)^{-1/2} - 1.$$

(b) Determine a different way to evaluate y when $x \approx 0$ that does not suffer from cancellation as much as direct evaluation. [Hint: multiply and divide by the same factor to clear the square root.]

Answer here:

7. **[15 points]** The following Matlab fragment was presented in lecture for computing the coefficients of the standard-form representation of the polynomial $p(x) = (x - x_1) \cdots (x - x_n)$.

```
b(1) = 1;
for k = 1 : n
  b(1:k+1) = [0;b(1:k)] - x(k) * [b(1:k);0];
end
```

Explain what it would compute if the inner statement were replaced by:

```
b(1:k+1) = 2*[0;b(1:k)] - x(k) * [b(1:k);0];
```

Answer here:

Write continuations to your answers on this page. Label the continuations.

Please do not write in this area.
1.
2.
3.
4.
5.
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7.
total