CS 322: Introduction to Scientific Computing
Spring 2003
Practice Prelim 2

Handed out: Saturday, April 5 (on the web).

This examination lasted 75 minutes and had 75 points total. Students were permitted to use an $8\frac{1}{2}'' \times 11''$ crib-sheet with notes written on both sides (same as this year). The course was somewhat farther along when this exam was given.

1. **[5 points]** Is the 2×2 matrix
$$\begin{pmatrix} .9 & .1 \\ -.1 & .9 \end{pmatrix}$$
a Givens rotation? Why or why not?

2. **[5 points]** Let C be a symmetric positive definite matrix whose Cholesky factorization is $C = GG^T$. What kind of matrix is G? Be as specific as possible.

3. **[5 points]** Consider finding a root of $f(x) = \sin x - x^2 - .5$ using Newton's method. Write down the formula for $x^{(k+1)}$ in terms of $x^{(k)}$ for this particular f.

4. **[5 points]** Write down an example of a 2×2 matrix that is simultaneously symmetric and orthogonal.

5. **[10 points]** Consider finding roots of the function $f(x) = x^2 - 9$. Which of the following intervals would work best as the starting interval if you were planning to use the bisection method: $[-2, 0]$, $[0, 2]$, or $[2, 4]$? Briefly explain why.

6. **[15 points]** Let U be an $(n + 1) \times n$ matrix that is upper triangular except for the the last row. (In other words, $U(i, j) = 0$ for all (i, j) satisfying $j < i \leq n$.) How many flops, accurate to the leading term, are required to reduce this matrix to upper triangular form using Givens rotations?

7. **[15 points]** For the matrix U in the last question, suppose one wants to solve $\min \|U\mathbf{x} - \mathbf{b}\|_2$ for some vector $\mathbf{b}$ using the method of normal equations. Determine how many flops (accurate to the leading term) are required for this computation.

8. **[15 points]** Let U be an $n \times n$ matrix that is upper triangular except for a single entry in the lower triangular portion at position (i, j) (so $i > j$). Can this matrix be reduced to upper triangular with a single Givens rotation? (In other words, is there an $n \times n$ Givens rotation G such that GU is upper triangular?) Explain your answer. Most likely your answer will depend on i and j.