CS 322: Introduction to Scientific Computing
Spring 2003
Practice Prelim 1

Handed out: Wed., Feb. 19 (web only).

This exam had 80 points and lasted 80 minutes. Students were allowed to use a crib-sheet.

1. [**5 points**] Describe an advantage of cubic spline interpolation over Hermite interpolation.

2. [**5 points**] Consider using compound QNC(3) (that is, Simpson's rule) to integrate $f(x)$ over $[0,1]$ using 10 subintervals, i.e., writing

$$\int_0^1 f(x)\,dx = \int_0^{0.1} f(x)\,dx + \cdots + \int_{0.9}^1 f(x)\,dx$$

and then approximating each integral on the right-hand side with QNC(3). How many times does f have to be evaluated for this computation?

3. [**5 points**] Describe a fast way to tell whether an upper triangular matrix is nonsingular.

4. [**10 points**] The following Matlab fragment computes the product C of an $n \times n$ matrix A with an $n \times n$ upper triangular matrix U. The inner loop is an inner product. Determine accurate to the leading term how many floating-point operations are used by this fragment.

```
for i = 1 : n
  for j = 1 : n
    C(i,j) = A(i,1:j) * U(1:j,j);
  end
end
```

5. [**10 points**] Rewrite the fragment in the preceding question with better vectorization. In particular, use only one loop. But be sure not to increase the number of flops required.

6. [**10 points**] Let $R(x)$ be a cubic spline function, with breakpoints located at $x_1, \ldots, x_n$ such that $x_1 < x_2 < \cdots < x_n$. What kind of function is R''? Characterize R'' as specifically as possible.

7. [**15 points**] Suppose you are given data points $(x_1, y_1), \ldots, (x_n, y_n)$ such that $x_1 < x_2 < \cdots < x_n$ and $y_1 = y_2 = \cdots = y_n = K$, where K is a constant. Suppose Newton's interpolation method is applied to these data points to find a degree-$(n-1)$ polynomial interpolant. How will the coefficients $c_1, \ldots, c_n$ depend on K?

8. **[15 points]** The following identity holds for any continuous function f defined on $[0, 1000]$:

$$\int_0^1 f(x)\,dx = \int_0^{1000} f(x)\,dx - \int_1^{1000} f(x)\,dx.$$

One way to approximate the integral on the left-hand side would be to compute approximations via Gauss quadrature to the two integrals on the right-hand side and then subtract. Give two reasons why this is probably a bad algorithm for approximating the integral of the left-hand side.