CS 322: Prelim 1 Review Questions

Solutions posted on the course website at noon, Monday, March 11. These problems are not necessarily typical exam problems. The exam syllabus is defined by S1-S7, P1-P3, and all readings through §6.2.

1. Assume that M is a positive integer and that M and $M+2$ are floating point numbers in a base-2 floating point number system with t-bit mantissas. If $M+1$ is *not* a floating point number then what can you say about t?

2. Complete the following function so that it performs as advertised. Be sure to vectorize.

```
  function t = Prod(x)
% x is a column n-vector and n > 3.
% y = A*x where A is the n-by-n matrix defined by
%
%           0                 if |i-j|>2
% A(i,j) =
%           cos(|i-j)|*pi/n)  if |i-j|<=2
```

3. Complete the following function so that it performs as advertised. Be sure to vectorize.

```
  function m = MaxJump(x,y)
% x and y are column n-vectors and x(1) < x(2) <...< x(n).
% Let S be the cubic spline produced by spline(x,y).
% m is the maximum value of f(z) on the interval [x(2) , x(n-1)] where
% f(z) is the limit of |S'''(z+delta) - S'''(z-delta)| as delta goes to zero.
```

4. Complete the following function so that it performs as advertised

```
function c = Convert(a,alfa,beta)
% a is a column 4-vector and alfa and beta are scalars.
% c is a column 4-vector so that the cubic polynomials
%
%       p(x) = a(1) + a(2)(x-alfa) + a(3)(x-alfa)^2 + a(4)(x-alfa)^3
%
%       q(x) = c(1) + c(2)(x-beta) + c(3)(x-beta)^2 + c(4)(x-beta)^3
```

are identical.

5. Modify the following function so that it takes *full* advantage of A's banded structure.

```
  function [L,U] = SpecLU(A,p)
% A is an n-by-n matrix with lower bandwidth 1 and upper bandwidth p.
% Assume that A has an LU factorization.
% Computes the factorization A = LU where L is an n-by-n unit lower unit
% triangular matrix and U is an n-by-n upper triangular.

[n,n] = size(A); v = zeros(n,1);
for k=1:n-1
   v(k+1) = A(k+1,k)/A(k,k);
   A(k+1,k:n) = A(k+1,k:n) - v(k+1)*A(k,k:n);
end
L = diag(v(2:n),-1); U = triu(A);
```

6. Assume that the function `y = f(x)` is available and that f has period π. How would you use `QUAD` to compute

$$I = \int_a^b f(x)dx$$

with absolute error $\leq 10^{-4}$?