CS321: Numerical Methods in Comp Mol Bio

Homework 7

Due: Thursday, Oct 27 2005 at the begining of the section

Problem 1

Suppose we are sampling from a Uniform distribution on the interval (0,10). Assume we aren't aware of the underlying distribution and wish to use the CLT to determine the mean of this distribution. We build a 95% confidence intrval using 50 samples using the formula

$$0.95 = P(-t_{.475} \leq \frac{\overline{X} - \hat{\mu}}{\hat{\sigma}/\sqrt{n}} \leq t_{.475}) = P(\overline{X} - \frac{\hat{\sigma}t_{.475}}{\sqrt{n}} \leq \hat{\mu} \leq \overline{X} + \frac{\hat{\sigma}t_{.475}}{\sqrt{n}})$$

to get the $CI = [\overline{X} - \frac{\hat{\sigma}t_{.475}}{\sqrt{n}}, \overline{X} + \frac{\hat{\sigma}t_{.475}}{\sqrt{n}}]$
Say that for a specific sample we got the confidence interval $CI = (5.5, 6.1)$, what is the probability that $\mu \in CI$?
How would the expected length of the confidence intrval change if we build it using 100 samples? Why?
How would the expected length of the confidence intrval change if the distribution was U(0,12)? Why?
How would the expected length of the confidence intrval change if the distribution was U(2,12)? Why?
Say we repeat the process 1000 times. How many of these intervals would you expect to not include the value 5?
Show how you would build a one-sided confidence interval for the mean assuming we don't mind underestimating μ.

Problem 2

Build a 99% confidence interval for the the mean of a distribution given samples:
{2.9787 6.2289 6.0155 8.3849 6.1826 3.7128 5.7607 2.9818 4.9610 4.9036}
How would you change your confidence interval if you knew that the varience of the underlying distribution is Normal with $\sigma = 2$?
How would you change your confidence interval if you knew that the mean of the underlying distribution is $\mu = 5$?
Improve your CI from above by adding 30 more samples and assuming $\sigma = 2$:
{5.0001 4.3643 7.1900 1.2520 5.8564 6.7913 6.4619 6.1557 5.0806 6.3542
6.1378 4.4887 4.2451 4.4082 2.0497 4.5320 5.2369 5.6296 7.8870 4.2981
6.2465 6.5981 6.8818 3.0158 5.4241 5.4758 2.9845 3.5159 7.1646 4.7370}
Build a one sided CI for the larger sample, assume we rather overestimate the mean than underestimate it.

Problem 3

Find the Likelihood function of an exponential distribution with p.d.f. $\lambda e^{-\lambda x}$.
Find the MLE for the mean ($\mu = \frac{1}{\lambda}$) for this distribution by setting the above likelihood function's derivative to zero.
Show that this is an unbiased estimator for $\frac{1}{\lambda}$.
Find the MLE for the varience ($\sigma^2 = (\frac{1}{\lambda})^2$) for this distribution.
Show that this is a biased estimator for $(\frac{1}{\lambda})^2$.
Find an unbiased estimator for $(\frac{1}{\lambda})^2$.