CS321: Numerical Methods in Comp Mol Bio

Homework 3

Due: Thursday, Sept 15 2005 at the beginning of the section

Problem 1

Using the Uniform random variable U on the interval (0,1), how would you ex-

press
$$W$$
, a random variable with the c.d.f. $F(w) = \begin{cases} 0 & w \le 0 \\ \frac{w^2}{2} & 0 \le w \le 1 \\ \frac{2 - (w - 2)^2}{2} & 1 \le w \le 2 \\ 1 & 2 < w \end{cases}$

Can you express W as a linear combination of X and Y, 2 independant Uniform random variables each on the interval (0,1)?

(hint: you have seen this c.d.f. lately.)

Problem 2

Suppose that X and Y are independent random variables, each uniformly distributed on the interval (0,1). Let W=X+Y.

Calculate the mean of W via linear combination of the means of X and Y.

Calculate the variance of W via linear combination of the variances of X and Y.

Calculate the mean of W directly by the definition (i.e. first calculate $f_W(w)$). Calculate the variance W directly by the definition (as above).

Problem 3

The formula for the p.d.f. of the distribution of a normal random variable X is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Use the change of variable technique with $u = \frac{x-\mu}{\sqrt{2}\sigma}$ to find the mean of the normal distribution.

Assuming that the X has mean μ and variance σ^2 , how would you express the normal random variable Z with mean 0 and variance 1 using linear operators on X?