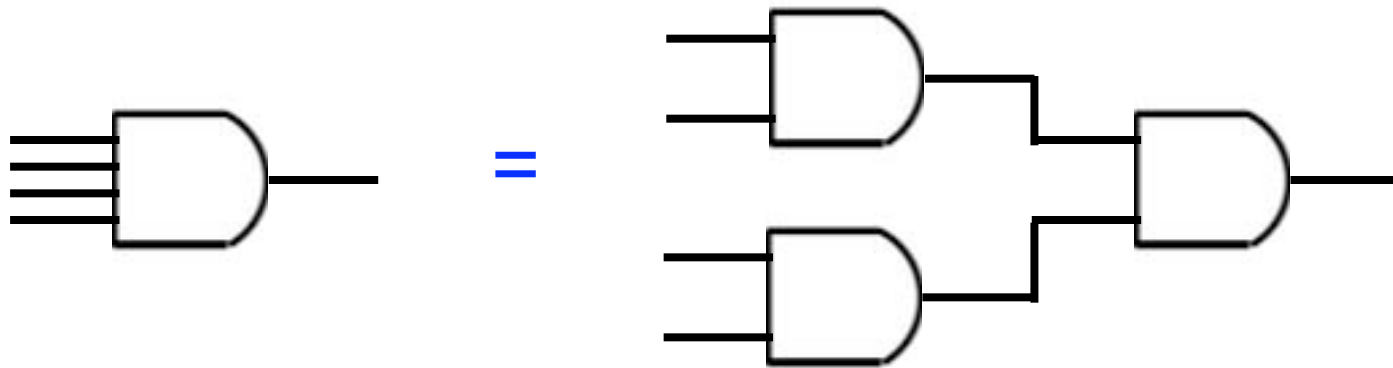


# Recursion: N-input AND gate

---

A  $2N$ -input AND gate is built from a pair of  $N$ -input AND gates and a single 2-input AND



# Expressing this in CAST

```
define AndN(int N)(node[N] in; node out)
{
  [N=1 -> out = in[0];]
  [N=2 -> And2()(in[0], in[1], out);]
  [N>2 ->
    int N2 = N/2;
    And2 a2(.,out);
    AndN(N2)(in[0..N2-1], a2.a);
    AndN(N-N2)(in[N2..N-1], a2.b);
  ]
}
```

CAST conditions

(cases for both  $N=1$  and  $N=2$   
allow use of  $\text{AndN}(X)$  even  
if  $X$  is not a power of 2)



# What if N is not a power of 2?

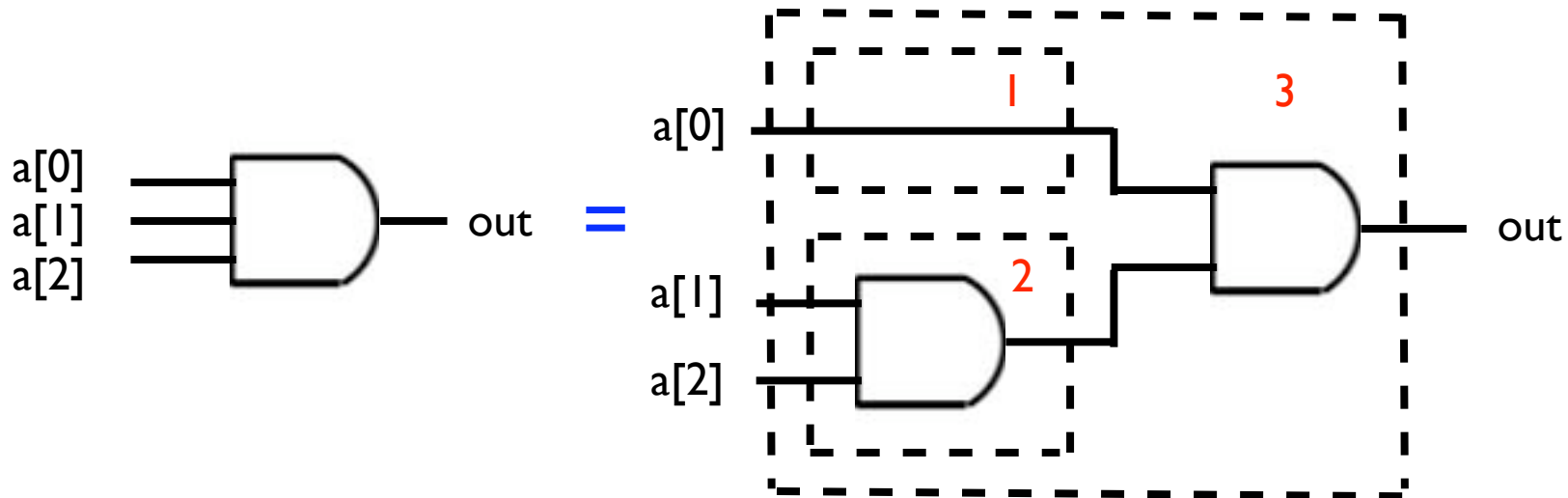
Example:

AndN(3)(...)

Note  $N2 = 3/2 = 1$  and  $N - N2 = 3 - 1 = 2$

So this instantiates

AndN(1)(...) and AndN(2)(...)



# A More Sophisticated Example

---

An L-bit decoder with N outputs ...

Inputs:

node[L] in

node enb

L-bit input

output is 0 unless enb is TRUE

Outputs:

node[N] out

N-bit output:

out[i] is set iff  $i = \text{value of in}$

$\Rightarrow L = \log(N)$

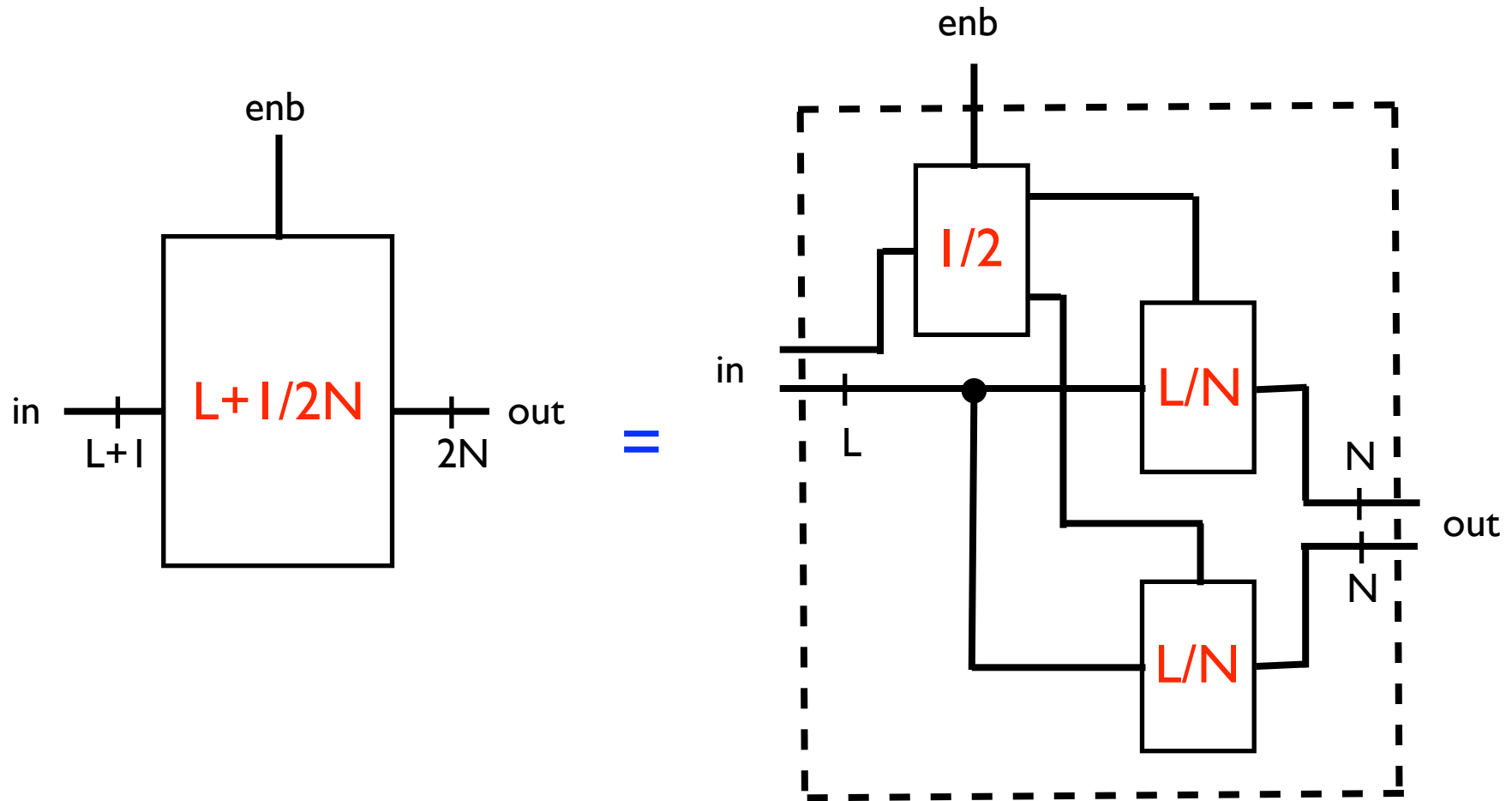
Note this makes no sense for  $(L=0, N=1)$ !

```
define DLN(int L; int N)(node[L] in; node enb; node[N] out)
{ ... }
```

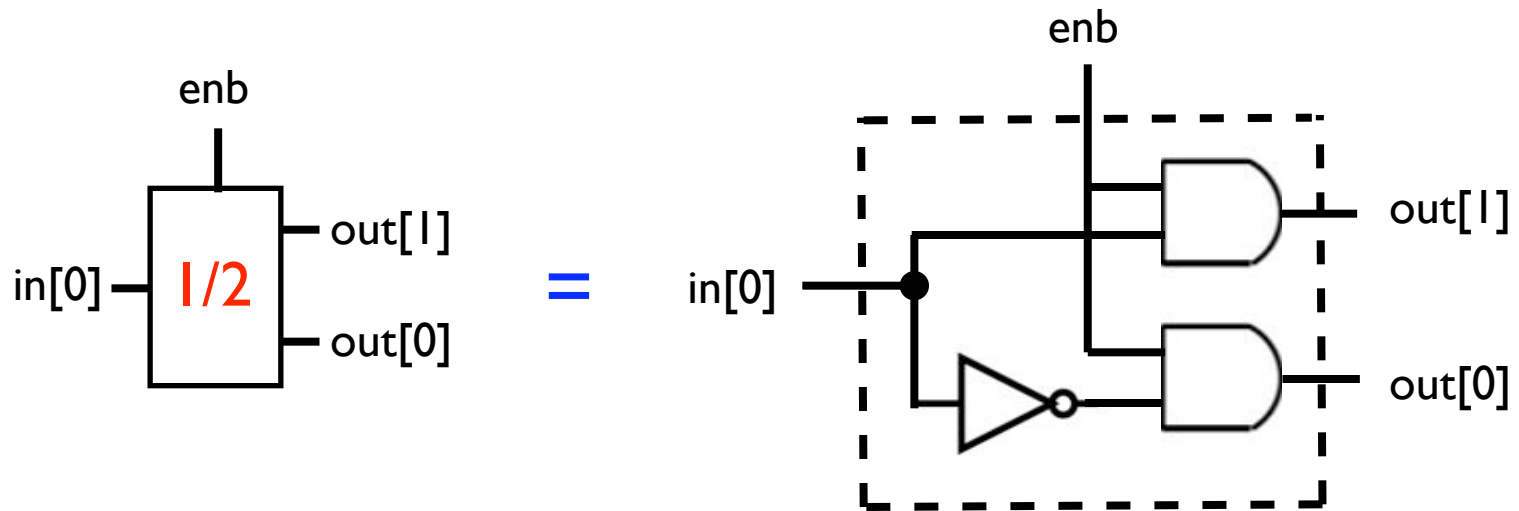


# Recursive Decoder Definition

Text



# Decoder Definition - Basis



# Expressing this in CAST

---

This assumes  $N$  is exactly  $2^{**}L$  ...

```
define DLN(int L; int N)(node[L] in; node enb; node[N] out)
{
  [L=1 ->
    node _in0; Inv()(in[0], _in0);
    And2()(enb, _in0, out[0]);
    And2()(enb, in0, out[1]);
  ]
  [L>1 ->
    DLN(1,2) sel(in[L-1], enb,);
    DLN(L-1,N/2)(in[0..L-2], sel.out[0], out[0..N/2-1]);
    DLN(L-1,N/2)(in[0..L-2], sel.out[1], out[N/2..N-1]);
  ]
}
```



# The Bottom Line ...

---

Use recursion in CAST to describe large circuits if they can be defined by “divide-and-conquer” or as tree-like structures.

Examples:

N-input And, OR, ...

N-input Mux, N-output decoder

N-bit Carry-lookahead adder, incrementer, ...

Fast N-bit zero or equality test

N-bit shift register with variable shift amount

... and a host of others!

