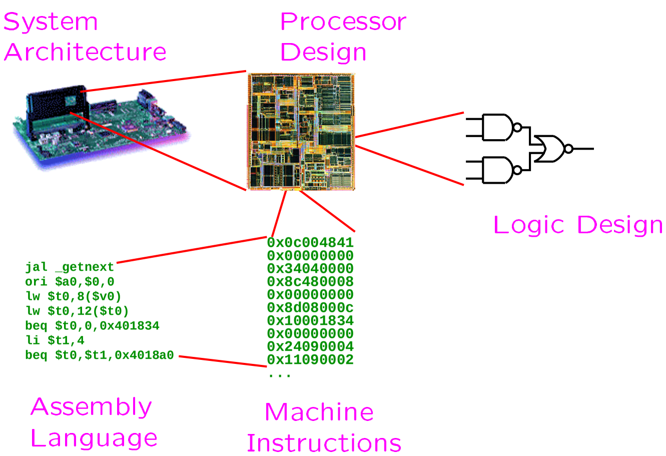
What 314 Is About



Building A Computer

Information is encoded with bits: 0's and 1's.
(we've already seen 2's complement numbers)

These are encoded using *voltages*...
+ well understood
+ easy to generate, detect
- affected by environment

But why 1's and 0's only?

Digital Representation

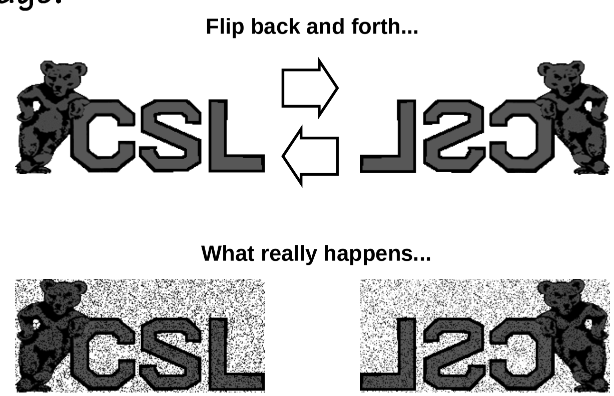
Example: representing a B&W picture:
- Black = 0 V
- White = 1 V
- 80% grey = 0.8 V
- ...

Represent by scanning picture in fixed order.

Let's try doing some computation with the voltages...

Digital Representation

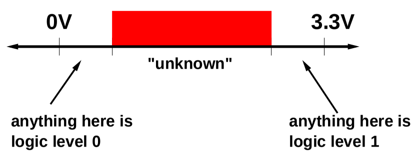
Flip image:



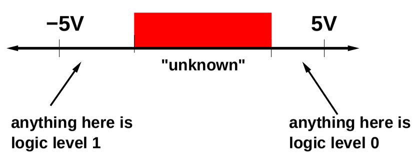
Have to build system to tolerate some error (noise).

Logic Levels

- Store just one bit on a wire...
- Gain reliability



Different conventions are possible

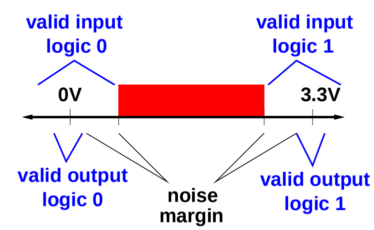


Combinational Devices

A **combinational device:**

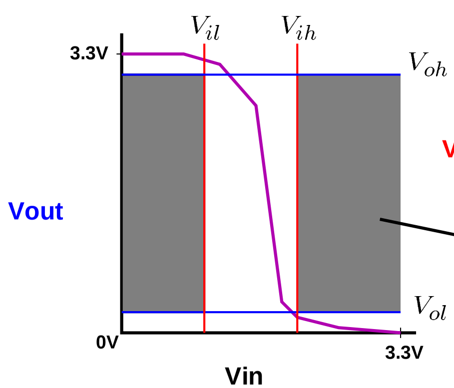
- Output is a function of inputs only ("memoryless")
- Takes input to valid, stable outputs

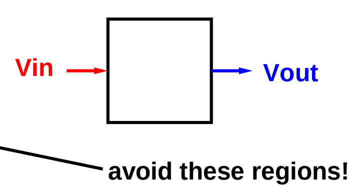
Combinational devices **restore** marginally valid signals!



Example

- Input: logic 0 if $< V_{il}$, logic 1 if $> V_{ih}$
- Output logic 0 if $< V_{ol}$, logic 1 if $> V_{oh}$





Digital View

- If input is 0, output is 1
- If input is 1, output is 0

Normally written in a table, like this:

In	Out
0	1
1	0

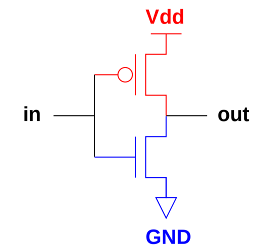
Called a "truth-table".

Implementation: Switching Networks

Lots of ways to build switches...

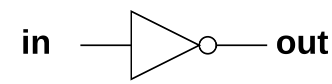
- relays
- vacuum tubes
- transistors
- ...

Switching Networks: Inverter

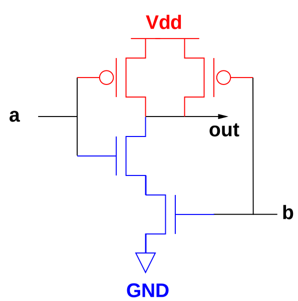


in	out
0	1
1	0

- Function: NOT
- Called an inverter
- Symbol:

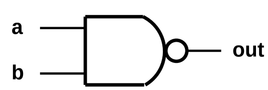


Switching Networks: NAND

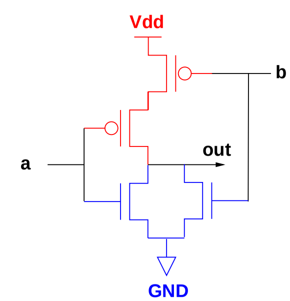


a	b	out
0	0	1
1	0	1
0	1	1
1	1	0

- Function: NAND
- Symbol:

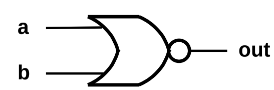


Switching Networks: NOR



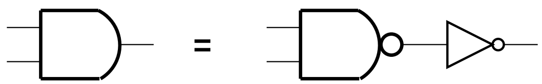
a	b	out
0	0	1
1	0	0
0	1	0
1	1	0

- Function: NOR
- Symbol:

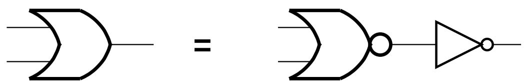


Building Functions From Gates

- AND:



- OR:



Can specify function by describing gates, truth table, or logic equations.

Logic Equations

AND:

$$out = a \cdot b$$
$$out = ab$$
$$out = a \wedge b$$

OR:

$$out = a + b$$
$$out = a \vee b$$

NOT:

$$out = \neg in$$
$$out = \overline{in}$$

Logic Equations

Fun with identities:

$a + \overline{a} = 1$
$a + 0 = a$
$a + 1 = 1$

$a\overline{a} = 0$
$a \cdot 0 = 0$
$a \cdot 1 = a$

$a(b + c) = ab + ac$
$\overline{(a + b)} = \overline{a} \cdot \overline{b}$
$\overline{(a \cdot b)} = \overline{a} + \overline{b}$
$a + \overline{a}b = a + b$

Check by writing truth tables, or by manipulating logic equations.

Let's Build An Adder

Write down function:

- Two 1-bit inputs, a and b
- Two 1-bit outputs, sum and $carry$

Truth-table:

a	b	carry	sum
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	0

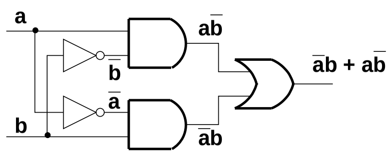
Let's Build An Adder

Sum output:

a	b	sum	Logic term
0	0	0	$\overline{a} \cdot \overline{b}$
1	0	1	$a \cdot \overline{b}$
0	1	1	$\overline{a} \cdot b$
1	1	0	$a \cdot b$

Logic equation: $a \cdot \overline{b} + \overline{a} \cdot b$

Circuit:



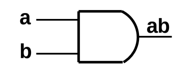
Let's Build An Adder

Carry output:

a	b	carry	Logic term
0	0	0	$\overline{a} \cdot \overline{b}$
1	0	0	$a \cdot \overline{b}$
0	1	0	$\overline{a} \cdot b$
1	1	1	$a \cdot b$

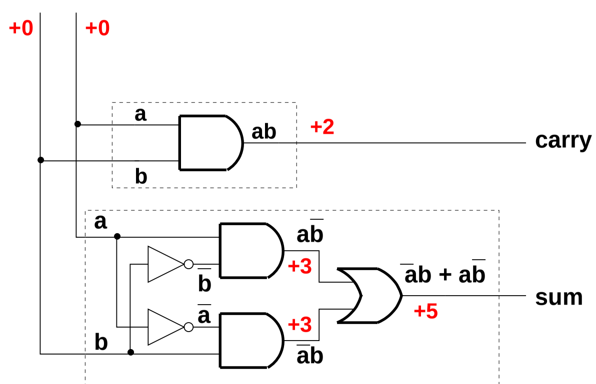
Logic equation: $a \cdot b$

Circuit:



Let's Build An Adder

Final Circuit:



Numbers indicate the number of sequential steps from input to output (worst-case).