

CS 312	Data Structures and Functional Programming
	Fall 2005
<b>Prelim II</b>	11/15/2005

NAME:\_\_\_\_\_

CU ID:\_\_\_\_\_ Net ID:\_\_\_\_\_

Section instructor\_\_\_\_\_

You have one and a half hours to do this exam.

All programs in this exam must be written in SML.

Problem	Score	Grader
1 (25 pts)		
2 (12 pts)		
3 (8 pts)		
4 (10 pts)		
5 (20 pts)		
6 (25 pts)		
Total (100 pts)		

1. (Type inference in ML, 25 points)

- (a) (5 points) Consider the ML function shown below.

```
fun foldr f a l =  
  case l of  
    nil => a  
  | (h::t) => f(h,foldr f a t)
```

- i. (3 points) Write down a set of type equations for this function. For each equation, write two or three words to explain where the equation comes from.
- ii. (2 points) Solve this set of equations to determine the most general type for this function.

- (b) (10 points) Consider the ML function shown below.

```
fun scanr f z = foldr (fn(e,l) => f(e, hd l)::l) [z]
```

- i. (5 points) Using the type signature you computed for `foldr`, write down a set of type equations for this function. For each equation, write two or three words to explain where the equation comes from.
- ii. (5 points) Solve this set of equations to determine the most general type for this function.

- (c) (5 points) Consider the ML functions shown below. You can assume that the type of `id` is  $'a \rightarrow 'a$ .

```
fun id x = x
```

```
fun foo f x = (id f) (id x)
```

- i. (3 points) Using the type signature of `id`, write down a set of type equations for `foo`. For each equation, write two or three words to explain where the equation comes from.
- ii. (2 points) Solve this set of equations to determine the most general type for `foo`.

- (d) (5 points) Write down a one-line function for which the ML type inference system is unable to compute a type. Justify your answer briefly.

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2. (Binary search trees and splay trees, 12 points)

Emil is learning about binary search trees and splay trees.

- (a) (8 points) Suppose Emil has a splay tree. Draw his splay tree after each step of the following sequence of operations: (insert 1), (insert 5), (insert 3), (insert 2), (insert 4).
- (b) (4 points) Emil has an empty splay tree while Emily has an empty binary search tree. You will ask them to insert the integers 1 through 5 in some order into their trees. For example, one possible order you may give them is "Start, insert 4, insert 3, insert 1, insert 2, insert 5, stop."

Is there an order of these insertions for which their trees are identical after each insertion in the sequence of five insertions?

If not, is there an order of these insertions for which their trees are identical after all insertions are completed?

Justify your answer.

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3. (Scope rules, 8 points)

Consider the following code:

```
val x = ref 0
fun inc(y) = (x := !x + 1; x)
val x = ref 1
fun f(x) = if (!x)*(!x) < 2 then !x else f(x)
```

- (a) In a variant of ML that is eager/static (like SML), what (if anything) is the value of  $f(\text{inc}(x))$ ?
- (b) In a variant of ML that is eager/dynamic, what (if anything) is the value of  $f(\text{inc}(x))$ ?
- (c) In a variant of ML that is lazy/static, what (if anything) is the value of  $f(\text{inc}(x))$ ?
- (d) In a variant of ML that is lazy/dynamic, what (if anything) is the value of  $f(\text{inc}(x))$ ?

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4. (Asymptotic complexity, 10 points)

- (a) Suppose that the function  $M$  is defined for all powers of 2 and is described by the following recurrence equation and base case.

$$M(n) = n-1+2M(n/2)$$

$$M(1) = 0$$

- i. Write down the exact solution for  $M$  when  $n$  is a power of 2.
- ii. What is the asymptotic order of  $M(n)$ ?

- (b) Find the asymptotic order of the solution to the following recurrence equation.

$$T(n) = 2T(n/2) + cn^2$$

$$T(1) = 1$$

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5. (Short answers, 20 points)

- (a) (5 points) The following outline indicates that  $P$  is an invariant of the while loop, while  $Q$  and  $R$  are the pre- and post-conditions. State in words the four steps involved in proving a loop like the one shown below correct.

```
{Q}
  initialization;
  {invariant P}
  while ( B ) {
    S
  }
{R}
```

- (b) (5 points) Write down the type and value that the following SML fragment evaluates to.

```
let
  datatype 'a t = A of 'a t list
  val a = A [A [A [], A []], A [A [], A []]]
  val m = foldr (fn (x,y) => if x > y then x else y) 0
  val rec d = fn A c => 1 + m (map d c)
in d a
end
```

- (c) (5 points) Explain the difference between polymorphism of Java methods and polymorphism of SML functions.
- (d) (5 points) Using the environment model diagrams shown in class, draw the environment immediately after each invocation of `fib` in the following code. Your diagrams must show the static and dynamic links clearly.

```
let
  fun fib n = if (n < 2) then 1 else fib(n-1) + fib(n-2)
in fib(3) end
```

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6. (B-trees, 25 points)

The picture below shows a B-tree B0 whose minimum degree is 3.

- (a) Draw the B-tree B1 after the key B is inserted into B0.
- (b) Draw the B-tree B2 after the key Q is inserted into B1.
- (c) Draw the B-tree B3 after the key L is inserted into B2.
- (d) Draw the B-tree B4 after the key F is inserted into B3.

