CS 3110

Monads

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Spring 2019

Today’s music: *Vámanos Pal Monte* by Eddie Palmieri

Review

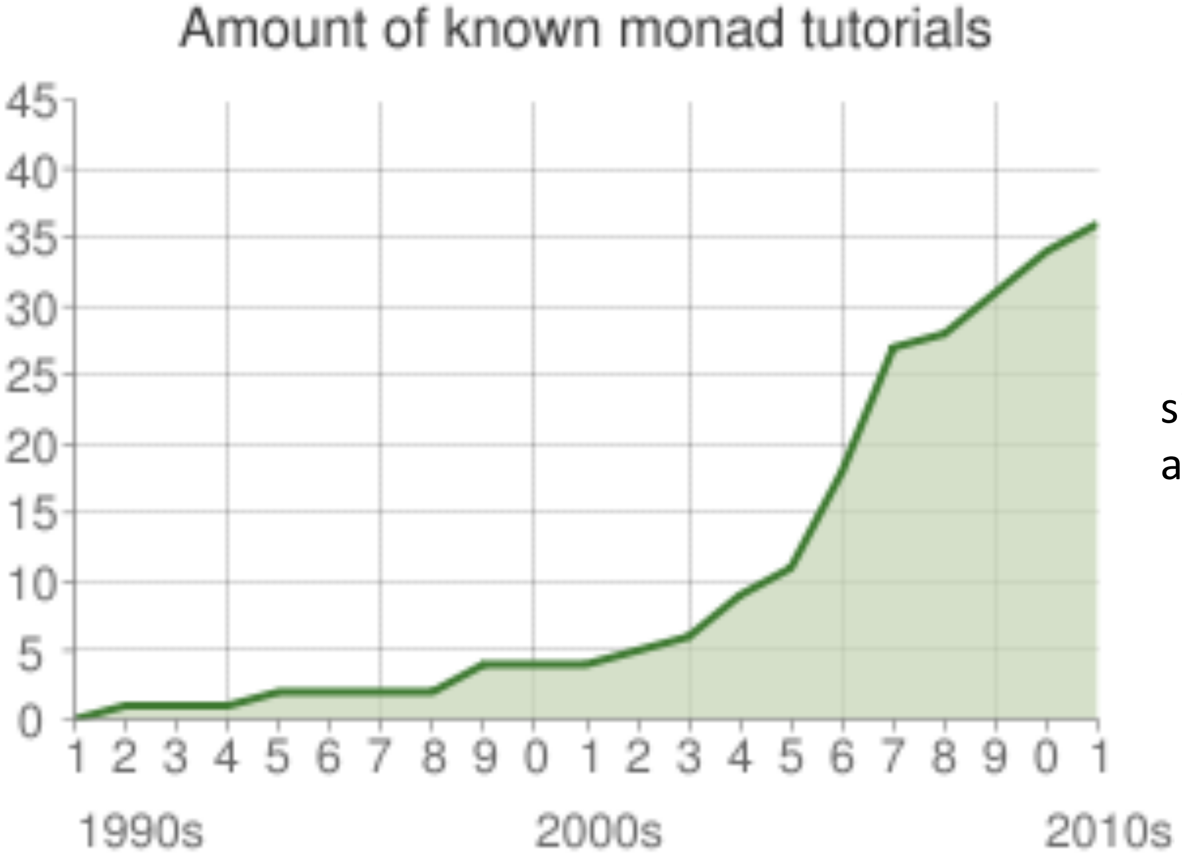
Currently in 3110: Advanced data structures

- Streams
- Balanced trees
- Mutability
- Promises

Today:

- Monads

Monad tutorials



since 2011:
another 34 at least

source: https://wiki.haskell.org/Monad_tutorials_timeline

Monad tutorials

"A monad is a monoid object in a category of endofunctors....It might be helpful to see a monad as a lax functor from a terminal bicategory."

Monad tutor

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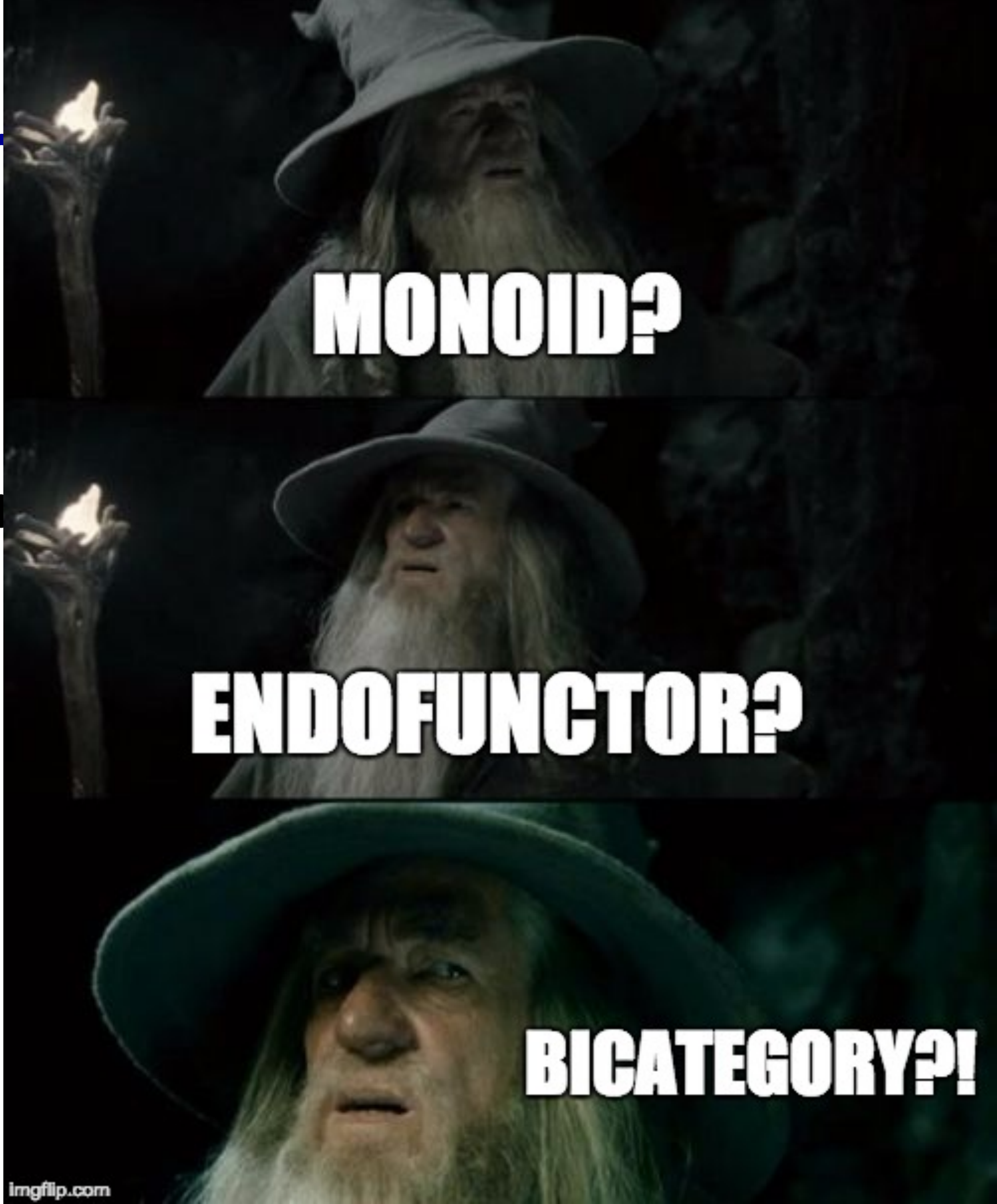
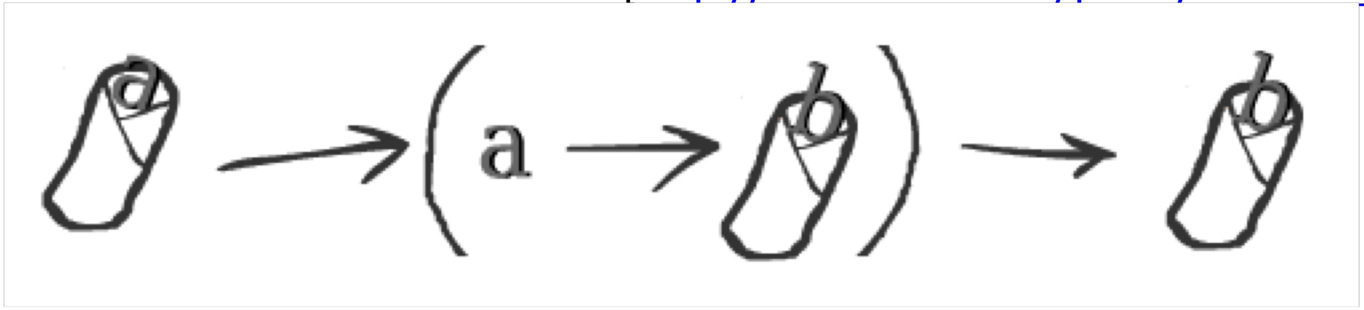


Monad tutorials

"A monad is a monoid object in a category of endofunctors....It might be helpful to see a monad as a lax functor from a terminal bicategory."

"Monads are burritos." [http://chrisdone.com/posts/monads-are-burritos]



Monad

For our purposes:

```
module type Monad = sig
  type 'a t
  val bind   : 'a t -> ('a -> 'b t) -> 'b t
  val return : 'a -> 'a t
end
```

Any structure that implements the **Monad** signature is a **monad**.

What's the big deal???

LOGGABLE FUNCTIONS

Demo

Question

```
let inc_log x =
  (x+1, "incremented " ^ string_of_int x ^ "; ")
let dec_log x =
  (x-1, "decremented " ^ string_of_int x ^ "; ")
```

```
let id_log = inc_log >> dec_log
```

Why doesn’t that definition work?

A. It doesn't type check

B. It computes the wrong integer

C. It computes the wrong log message

D. Both B and C

LOGGABLE FUNCTIONS

Demo

Upgrading a function

What if we could upgrade a loggable function to accept the input from another loggable function?

```
upgrade f_log
: int*string -> int*string
```

Discussion: how could you implement that?

Another kind of upgrade

- Given `f : int -> int`
- How to make it loggable, but with empty log message?
- Need to "lift" a function
  from `int -> int`
  to `int -> int*string`

Demo

Types

Consider the types:

```
val upgrade :
     (int          -> int * string)
  ->  int * string -> int * string

val trivial :
     int -> (int * string)
```

Types

Another way of writing those types:

```
type 'a t = 'a * string

val upgrade :
      (int    -> int t)
  ->   int t -> int t

val trivial :
     int -> int t
```

Have you seen those types before???

Types

Let’s swap the argument order of upgrade...

```
val upgrade :
  (int -> int t)
  -> int t
  -> int t

let upgrade' x f = upgrade f x

val upgrade’ :
 int t
 -> (int -> int t)
 -> int t
```

Types

```
type 'a t  = 'a * string

val upgrade' :
     int t
  -> (int -> int t)
  -> int t

val trivial :
     int -> int t
```

Have you seen those types before?

Rewriting types

```
type 'a t  = 'a * string

val bind :
      int t
  -> (int -> int t)
  -> int t

val return :
      int -> int t
```

```
module type Monad = sig
  type 'a t
  val bind :
       'a t
    -> ('a -> 'b t)
    -> 'b t
  val return :
     'a -> 'a t
end
```

Loggable is a monad

```
module Loggable : Monad = struct
  type 'a t = 'a * string
  let bind (x,s1) f =
    let (y,s2) = f x in
    (y,s1^s2)
  let return x = (x,"")
end
```

More often called the **writer** monad

Stepping back...

- We took functions
- We made them compute *something more*
  - A logging string
- We invented ways to pipeline them together
  - **`upgrade`**, **`trivial`**
- We discovered those ways correspond to the **`Monad`** signature

FUNCTIONS THAT PRODUCE ERRORS

Functions and errors

A *partial* function is undefined for some inputs

- e.g., **`max_list : int list -> int`**
- with that type, programmer probably intends to raise an exception on the empty list
  - could also produce an option
  - or could use variant to encode result...

Dem
o

What are the types?

```
type 'a t = Val of 'a | Err
val value : 'a -> 'a t
val (|>?) : 'a t -> ('a -> 'b t) -> 'b t
```

Have you seen those types before???

```
module type Monad = sig
  type 'a t
  val bind :
       'a t
    -> ('a -> 'b t)
    -> 'b t
  val return :
    'a -> 'a t
end
```

Error is a monad

```
module Error : Monad = struct
  type 'a t = Val of 'a | Err
  let return x = Val x
  let bind m f =
    match m with
    | Val x -> f x
    | Err -> Err
end
```

Option is a monad

```
module Option : Monad = struct
  type 'a t = Some of 'a | None
  let return x = Some x
  let bind m f =
    match m with
    | Some x -> f x
    | None -> None
end
```

Stepping back...

- We took functions
- We made them compute *something more*
  - A value or possibly an error
- We invented ways to pipeline them together
  - **`value`**, **`(|>?)`**
- We discovered those ways correspond to the **`Monad`** signature

LWT

Lwt is a monad

```
module Lwt : sig
  type 'a t
  val return : 'a -> 'a t
  val bind : 'a t -> ('a -> 'b t) -> 'b t
end
```

- `return` takes a value and returns an immediately resolved promise
- `bind` takes a promise, and a callback function, and returns a promise that results from applying the callback

Stepping back...

- We took functions
- The Lwt library made them compute *something more*
  - a promised result
- The Lwt library invented ways to pipeline them together
  - **`return`**, **`(>>=)`**
- Those ways correspond to the **`Monad`** signature
- So we call Lwt a *monadic concurrency library*

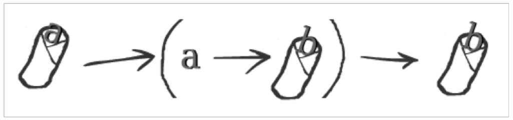
Another view of Monad

```
module type Monad = sig
  (* a "boxed" value of type 'a *)
  type 'a t

  (* [m >>= f] unboxes m,
   * passes the result to f,
   * which computes a new result,
   * and returns the boxed new result *)
  val (>>=) : 'a t -> ('a -> 'b t) -> 'b t

  (* box up a value *)
  val return : 'a -> 'a t
end
```

(equate "box" with "tortilla" and you have the burrito metaphor)



SO WHAT IS A MONAD?

Computations

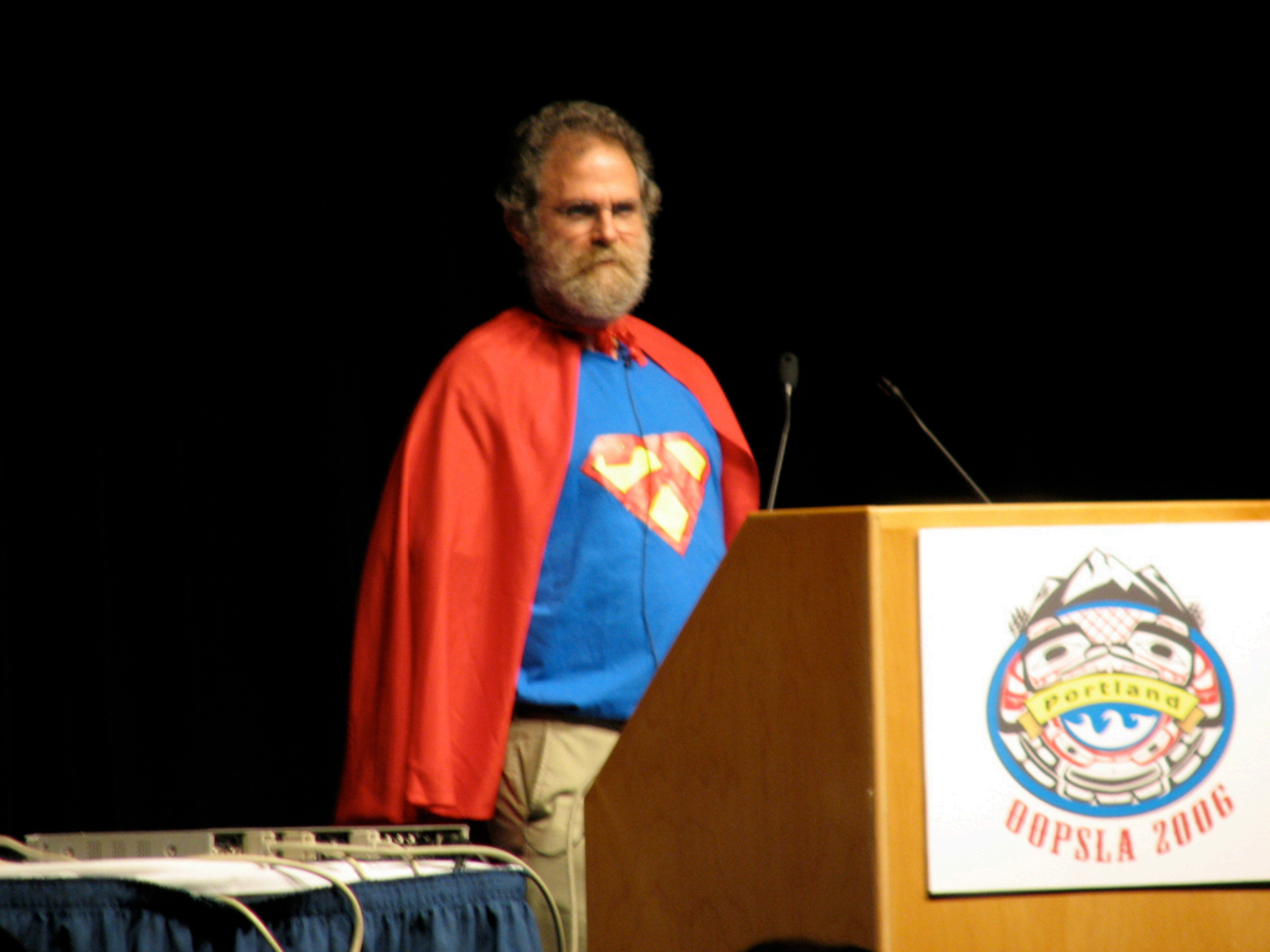
- A *function* maps an input to an output
- A *computation* does that and more: it has some *effect*
  - Loggable computation: effect is a string produced for logging
  - Error computation: effect is a possible error instead of a value
  - Option computation: effect is a possible None instead of a value
  - Promised computation: effect is delaying production of value until later
- A *monad* is a data type for computations
  - **`return`** has the trivial effect
  - **`(>>=)`** does the "plumbing" between effects

Phil Wadler

b. 1956

- A designer of Haskell
- Wrote *the* paper* on using monads for functional programming

* http://homepages.inf.ed.ac.uk/wadler/papers/marktoberdorf/baastad.pdf


portland
OOPSLA 2006

Other monads

- **State:** modifying the state is an effect
- **List:** producing a list of values instead of a single value can be seen as an effect
- **Random:** producing a random value can be seen as an effect
- …

Monad laws

- We expect data types to obey some algebraic laws
  - e.g., for stacks, **`peek (push x s) = x`**
  - We don't write them in OCaml types, but they're essential for expected behavior
- Monads must obey these laws:
  1. **`return x >>= f`** is equivalent to **`f x`**
  2. **`m >>= return`** is equivalent to **`m`**
  3. **`(m >>= f) >>= g`** is equivalent to
     `m >>= (fun x -> f x >>= g)`
- Why? The laws make sequencing of effects work the way you expect

Monad laws

1. `(return x >>= f) = f x`

   Doing the trivial effect then doing a computation **`f`** is the same as just doing the computation **`f`**

   (return is left identity of bind)

2. `(m >>= return) = m`

   Doing only a trivial effect is the same as not doing any effect

   (return is right identity of bind)

3.
   ```
   ((m >>= f) >>= g)
    = (m >>= (fun x -> f x >>= g))
   ```

   Doing **`f`** then doing **`g`** as two separate computations is the same as doing a single computation which is **`f`** followed by **`g`**

   (bind is associative)

Upcoming events

- [Today] Foster OH 1:15-2:15pm

This is effectful.

THIS IS 3110