CS3110 Spring 2017 Lecture 19: Binary Search Trees Continued

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	Date for	Due Date
PS6	Out on April 24	May 8 (day of last lecture)

1 More on Binary Search Trees

Recall from Lecture 18 that if elements are "randomly inserted" into a bst, we can expect the look up and addition of elements to be of order n log n time. If the elements are inserted in order, then we obtain a list not a tree.

The mistake that the class found in the tree example from last year suggests a "security issue" with a bst. If we manually change the tree by splicing in a node, we can destroy the properties and "hide an element" so that it is not found in the binary search. Yet when questioned about whether the element is in the tree, we can examine the tree and find that it is there. Such a situation could lead to mischief.

In lecture 18 we stressed the cases that arise in deleting elements. We looked at deleting a leaf, a node with one child, and a node with two children. The two children case can either use the **max-left** method or the **min-right** method. We also included examples from Dr. Bickford's verified operations on binary search trees developed in Nuprl.

2 Konig's Lemma

König's Lemma (Smullyan p.32 [1]): Every finitely generated tree T with an unbounded number of points contains at least one unbounded branch.

Proof (non-constructive)

1. Call a point of T *good* iff it has an unbounded number of descendants and *bad* otherwise.

2. We assume an unbounded number of nodes, so *the origin is good.*

3. If all the successors of a point in T are bad, then the point is bad, since it has only finitely many immediate successors. Thus a good point has at least one good successor.

   So the origin, say a_0, has at least one good successor a_1, and a_1 has a good a_2, etc.

   Thus $(a_0, a_1, a_2, \cdots)$ is the unbounded branch.

Qed.

If T has an unbounded number of points, they must be scattered at an unbounded number of levels because any finite level has only finitely many points.

A spread S is a *fan* (finitary tree) iff each node has only finitely many immediate descendants.

3 Brouwer's Fan Theorem

Brouwer's Fan Theorem (M. van Atten p.59, as he stated [2]): If to each element e of a finitary spread M a natural number β_e is associated, a natural number z can be (found) specified such that β_e is completely determined by the first z choices generating e.

Brouwer 1927 (see *From Frege to Gödel*, 1967) "On the domains of definition of functions" [3].

Note, in a *spread* a node can have an unbounded number of successors.

References

[1] R. M. Smullyan. *First–Order Logic.* Springer-Verlag, New York, 1968.

[2] Mark van Atten. *On Brouwer.* Wadsworth Philosophers Series. Thompson/Wadsworth, Toronto, Canada, 2004.

[3] J. van Heijenoort, editor. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931.* Harvard University Press, Cambridge, MA, 1967.