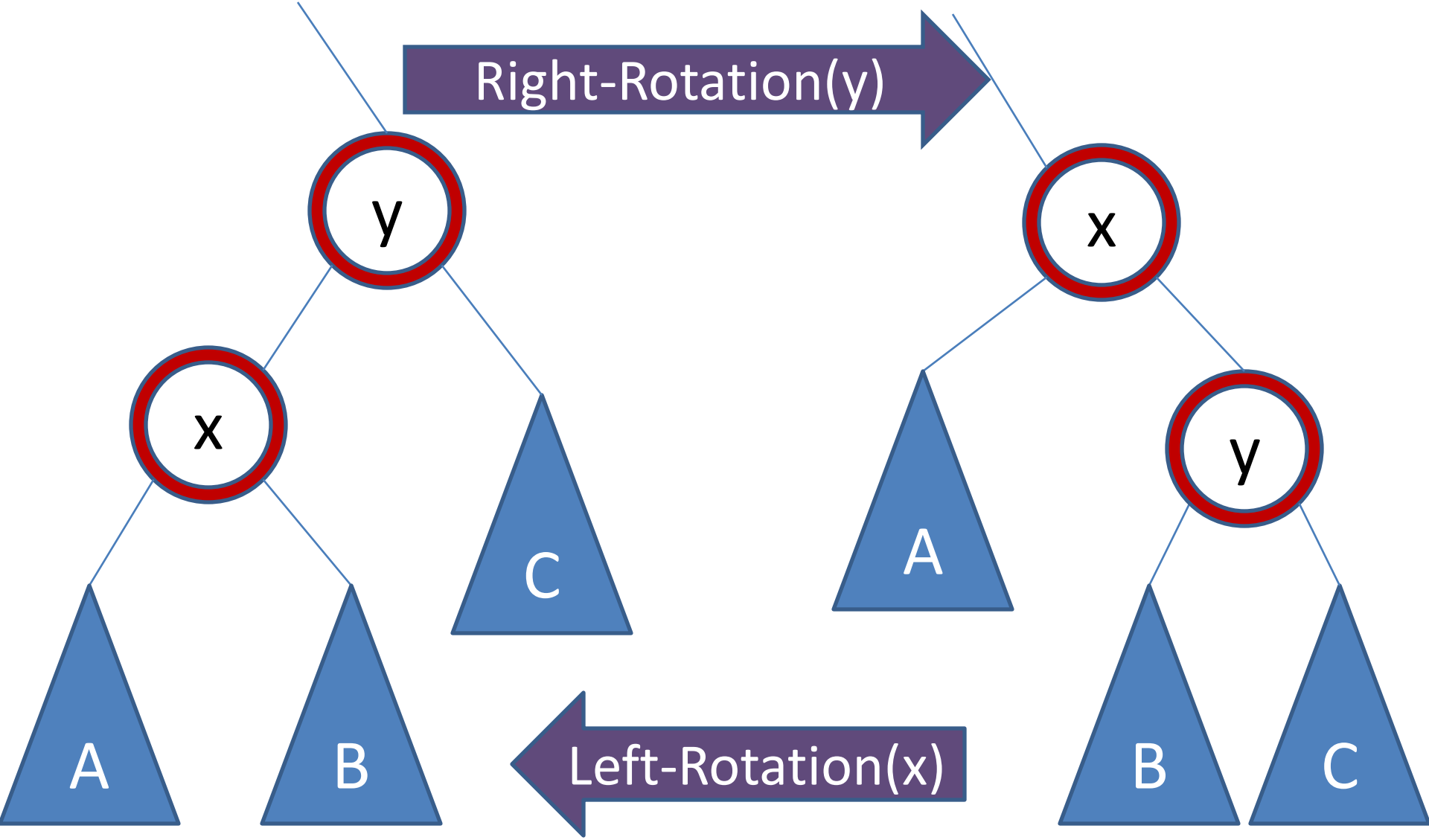
3110: Red-Black Trees

- Project?
- Homework – answer.
- Insertion
- An allude to Deletion

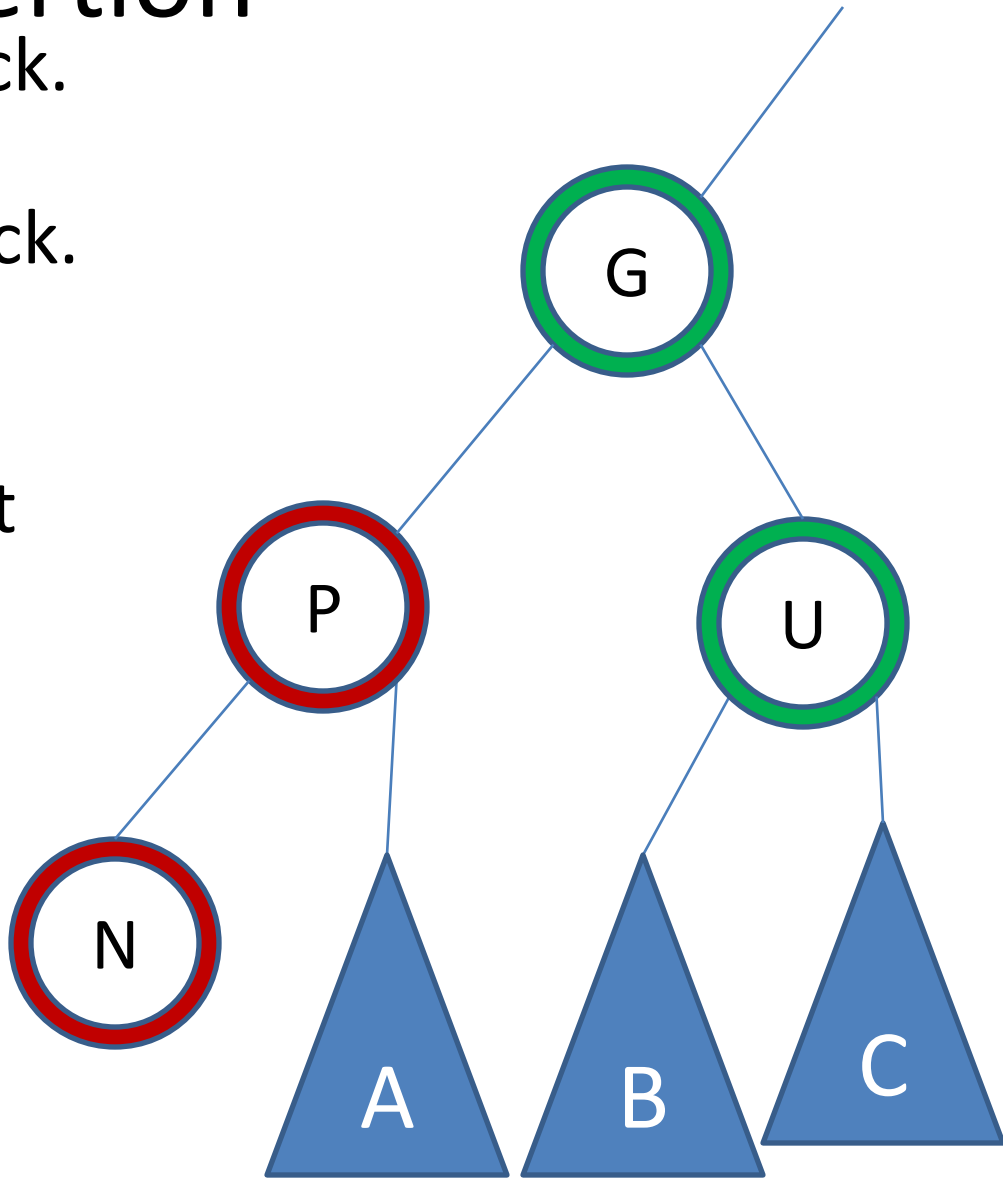
Rotations



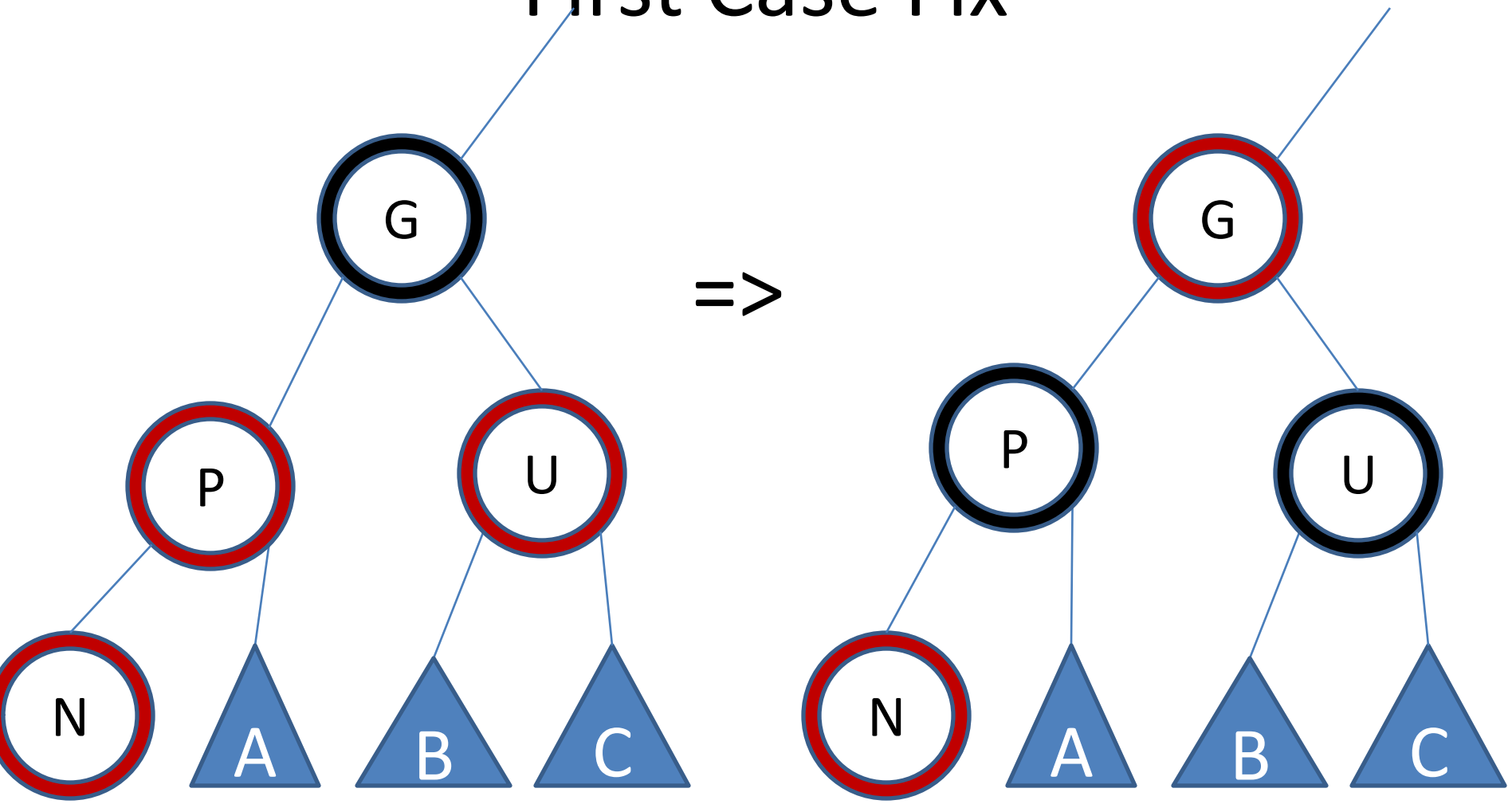
Insertion

- If inserting root, color black. Done.
- If parent of inserted is black. Done.

- Otherwise have to break it down into cases.
- Consider the structure of new, parent, grandparent, and uncle:

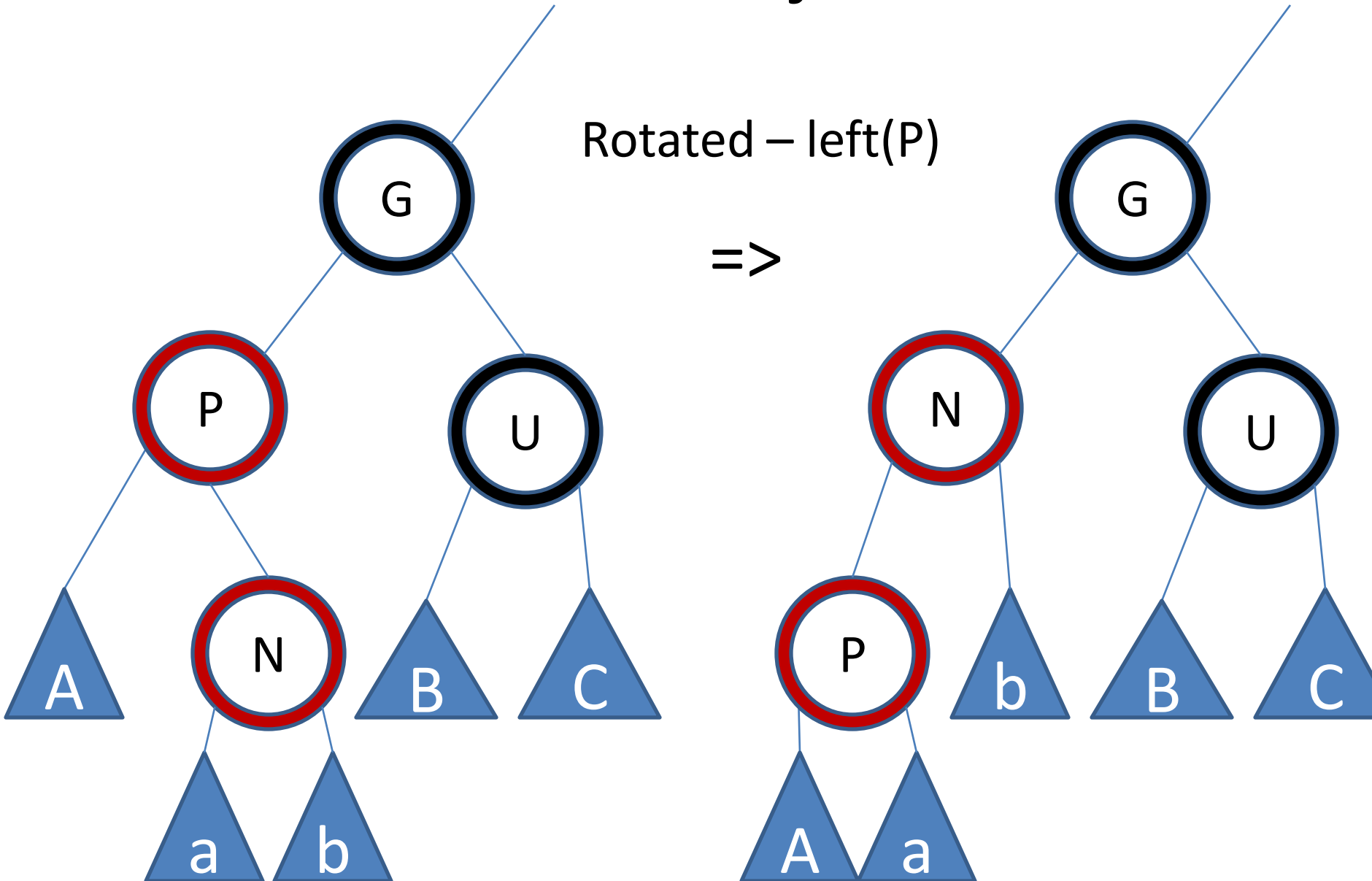


First Case Fix



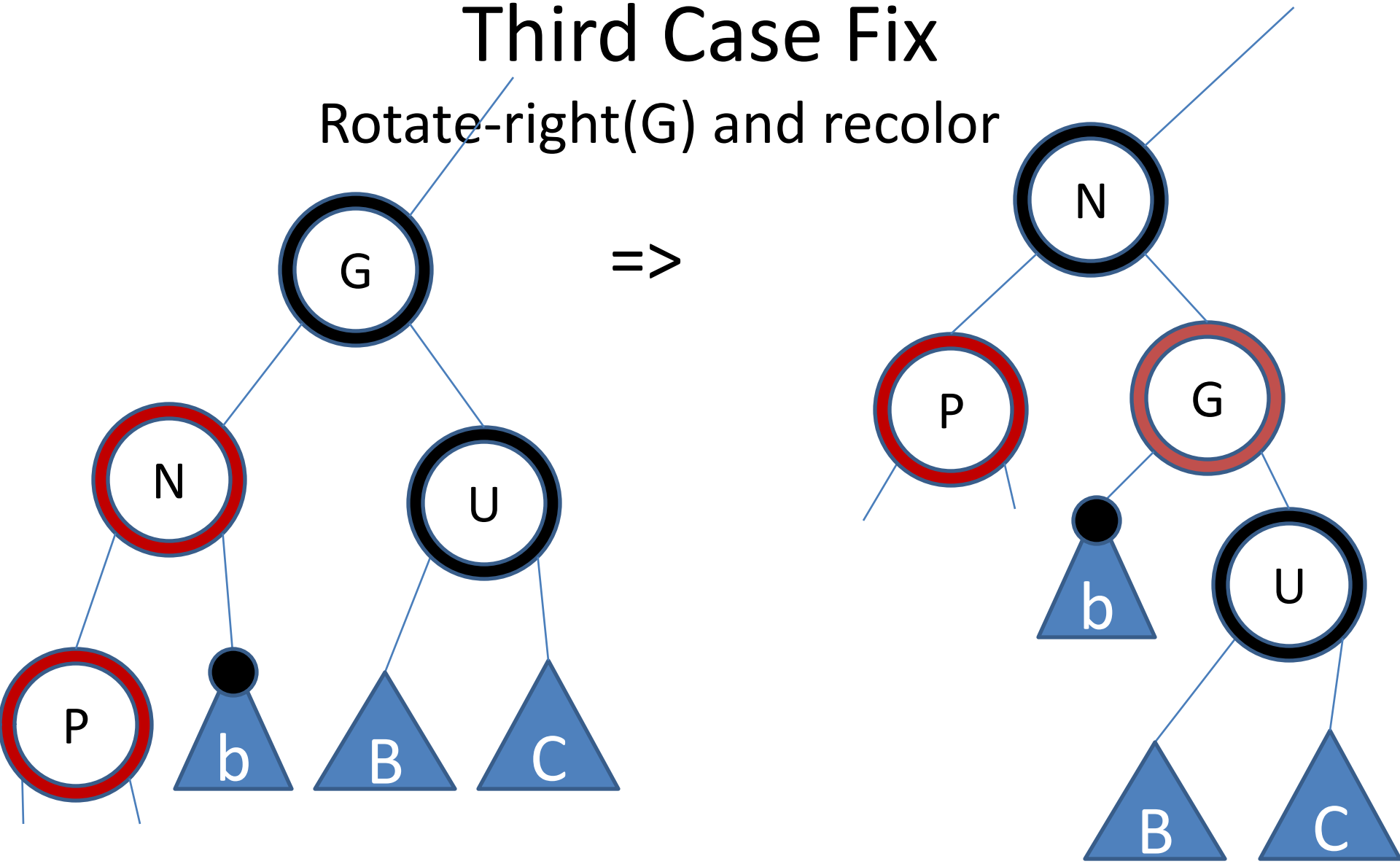
If parent of G is black, done. Otherwise consider G as the New node and recurse.

Second Case Adjustment



Third Case Fix

Rotate-right(G) and recolor



Case Progressions

First Case

Second Case

Third Case

=>

First Case

Second Case

Third Case

Done

Number of Operations

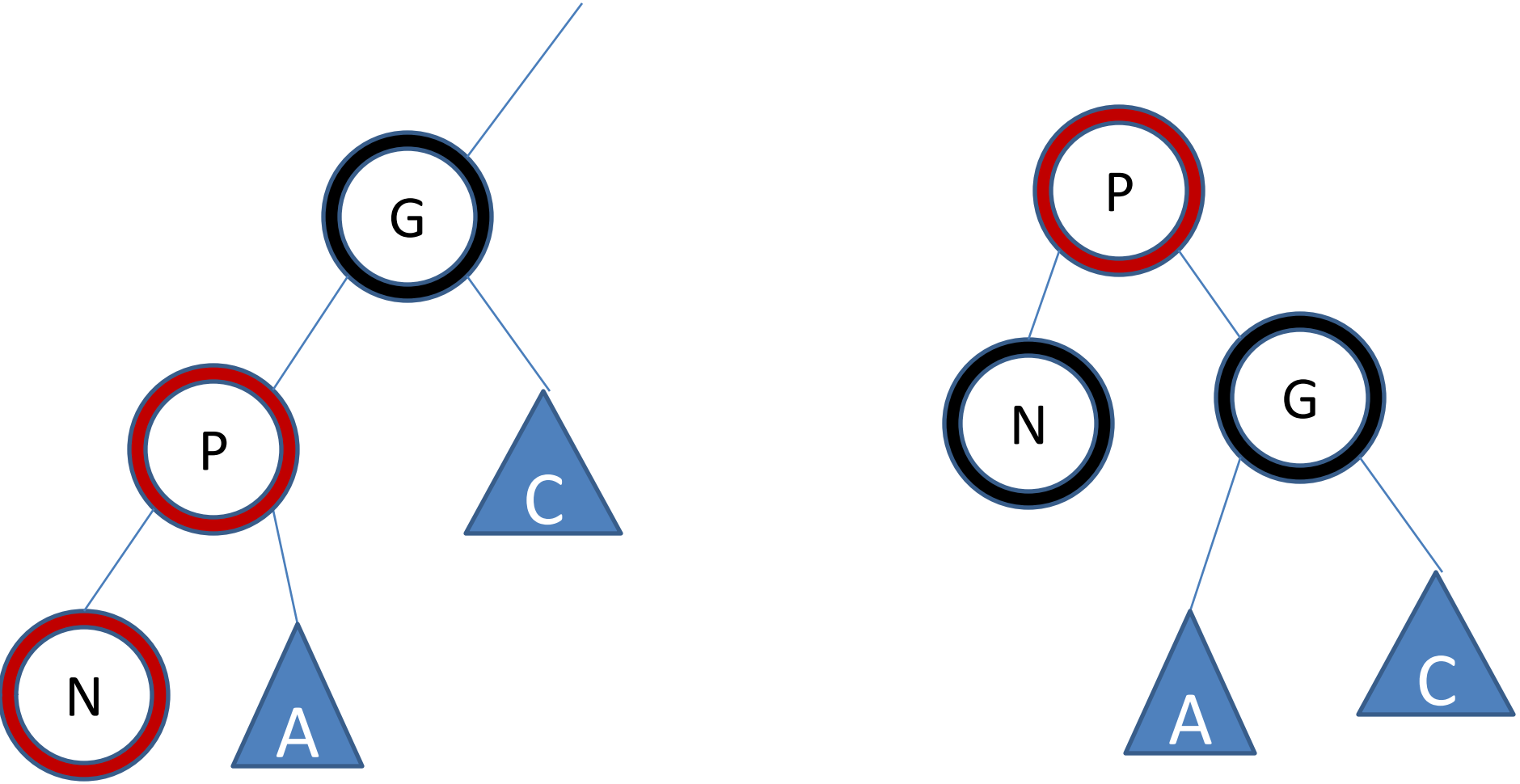
- First Case operations always move you up the tree and require a constant number of color flips, O(log(n))

- At most 2 rotations going through Second Adjustment and Third Case. Constant.

- O(log(n))

Alternative Method

- More rotations, but only 1 case, easier to code.
- http://portal.acm.org/citation.cfm?id=968583


G
P
C
N
A
P
N
G
A
C

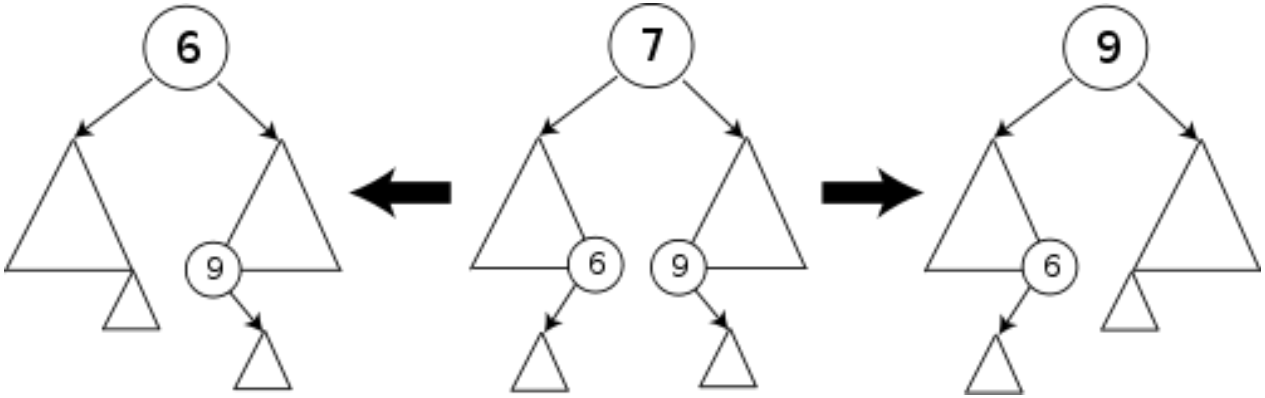
Youtube Example

- http://www.youtube.com/watch?v=vDHFF4wjWYU&feature=related

Deletion

First Step: find replacement value and copy. Use either max of left subtree or min of right subtree. Doing so maintains key order property.

Second Step: delete the node of the copied value. Note, this node will have at most 1 child and can moved up in its place.



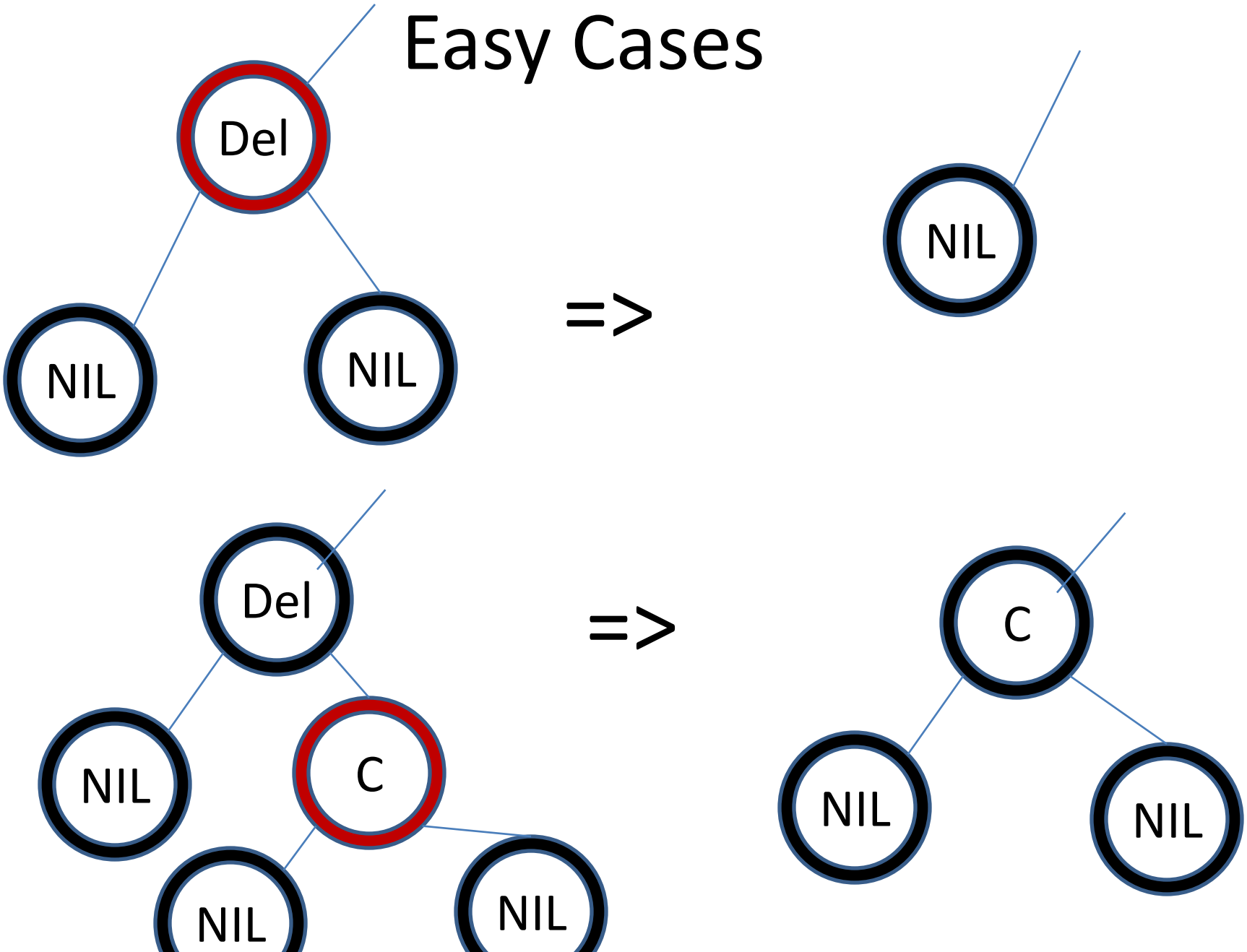
Correcting

- Note, the value copy does not destroy the properties of the red-black tree. Only the deletion of ‘foot’ node may.

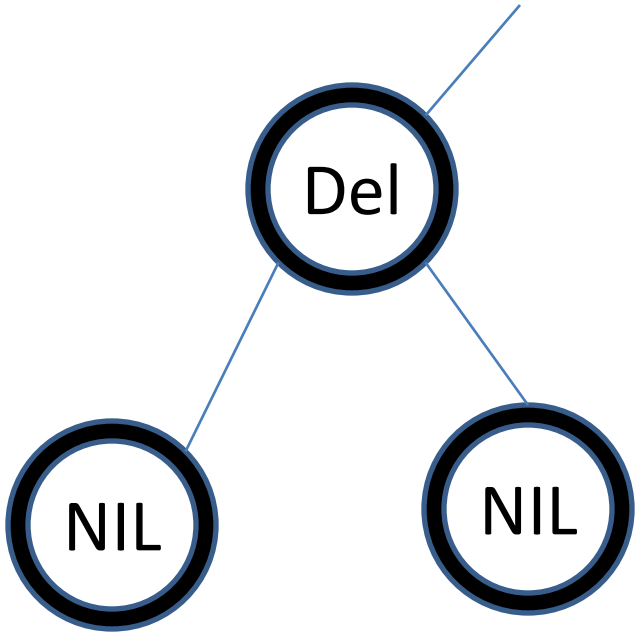
- If ‘foot’ node is red – no problem from deletion.

- Otherwise:

Easy Cases



Hard Case



Perform a series of rotations until we come across a red node and can convert it.

See CLRS or wikipedia for a list of all cases.

Schemes

- Insertion algorithm destroys the coloring condition and seeks to rotate preserving black-height, pushing the bad coloring up the tree until it can recolor to satisfy.

- Deletion algorithm destroys the black-height and seeks to rotate preserving the coloring condition, pushing the unbalanced black height up the tree until it can color a red to black to satisfy.